Maintaining Large-Scale Rechargeable Sensor Networks Perpetually via Multiple Mobile Charging Vehicles

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Wireless energy transfer technology based on magnetic resonant coupling has been emerging as a promising technology for wireless sensor networks (WSNs) by providing controllable yet perpetual energy to sensors. In this article, we study the deployment of the minimum number of mobile charging vehicles to charge sensors in a large-scale WSN so that none of the sensors will run out of energy, for which we first advocate a flexible on-demand charging paradigm that decouples sensor energy charging scheduling from the design of sensing data routing protocols. We then formulate a novel optimization problem of scheduling mobile charging vehicles to charge life-critical sensors in the network with an objective to minimize the number of mobile charging vehicles deployed, subject to the energy capacity constraint on each mobile charging vehicle. As the problem is NP-hard, we instead propose an approximation algorithm with a provable performance guarantee if the energy consumption of each sensor during each charging tour is negligible. Otherwise, we devise a heuristic algorithm by modifying the proposed approximation algorithm. We finally evaluate the performance of the proposed algorithms through experimental simulations. Experimental results demonstrate that the proposed algorithms are very promising, and the solutions obtained are fractional of the optimal ones. To the best of our knowledge, this is the first approximation algorithm with a nontrivial approximation ratio for a novel scheduling problem of multiple mobile charging vehicles for charging sensors.

CCS Concepts: • **Networks** → **Cyber-physical networks**; **Sensor networks;**

Additional Key Words and Phrases: Rechargeable wireless sensor networks, wireless energy transfer, critical sensor lifetime, energy recharging scheduling, mobile vehicle routing problem, approximation algorithms, combinatorial optimization problem, tree decomposition

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1. INTRODUCTION

Sensors in conventional wireless sensor networks (WSNs) are mainly powered by batteries. Due to the limited energy capacity imposed on the batteries, the operational time of WSNs usually is limited. To prolong the lifetime of sensor networks, extensive studies have been conducted in the past decade, which include batch deployments of sensors and harvesting energy from surrounding environments, among others [Akyildiz et al. 2002; Anastasi et al. 2009; Chang and Tassiulas 2004; Hou et al. 2008; Lin et al. 2005; Liang et al. 2013; Yick et al. 2008]. Despite these intensive efforts, the network lifetime remains the main obstacle in large-scale deployments of WSNs.

To prolong the operational lifetime of a WSN, an obvious solution is to replace the expired batteries with new ones [Tong et al. 2011; Xu and Liang 2011; Xu et al. 2012; Yuan et al. 2008]. However, for large-scale WSNs, it is not only painstaking but also costly to do so. Worst of all, it is almost impossible to adopt this approach for some applications, where WSNs are deployed for monitoring dangerous or seriously polluted regions. Alternatively, another solution is to compensate the expired sensors by dispatching a batch of new sensors to the region of dead sensors. However, this is achieved at the expense of long-term environmental contamination, as most batteries are made with poisonous chemical materials. Contrary to these solutions, a new technique for environmentally friendly WSN deployments has been explored in recent years. In other words, sensors are powered by renewable energy that is harvested from their surrounding environments, such as solar energy and wind energy [Jiang et al. 2005; Kansal et al. 2007; Liang et al. 2013; Ren et al. 2013, 2014; Shi and Hou 2008; Sudevalayam and Kulkarni 2011; Wang et al. 2008]. Although energy harvesting is an ideal solution, its success in WSNs remains limited in practice, as the time-varying nature of energy-harvesting sources poses a great challenge in sensor energy management. For example, in a solar energy harvesting system, statistics have shown that the differences of energy generating rates on shadowy, cloudy, and sunny days can be up to three orders of magnitude [Rahimi et al. 2003]. Furthermore, the size of an energy-harvesting device is also an important concern in its deployment, particularly when the size of a solar panel is of a much larger scale than the sensor node that it is attempting to power. Furthermore, the cost of energy-harvesting sensors will be significantly increased, especially for the deployment of a large-scale WSN.

Complementary to the energy-harvesting technique, the recent breakthrough of a wireless energy transfer technology has attracted a lot of attention that adds a new dimension to prolong the lifetime of sensor networks [Kurs et al. 2007, 2010; Xie et al. 2013a; Shu et al. 2014]. By exploiting a novel technique called *strongly coupled magnetic resonances*, Kurs et al. showed that the wireless energy transfer is not only efficient but also immune to its surrounding environment [Kurs et al. 2007]. Industry research further demonstrated that it is possible to transfer 60W of power over a distance of up to two to three feet with an energy transfer efficiency of 75% [Intel 2011], and several products based on the wireless energy transfer technology are now commercially available in markets, such as sensors [Powercast 2015], RFIDs [WISP 2015], cell phones [Powermat 2015], and automobiles [EV World 2015]. It is reported that the wireless energy transfer market is expected to grow from just $216 million in 2013 to $8.5 billion in 2018 [Yoo and Jeong 2012]. Armed with the advanced wireless energy transfer technology, mobile charging vehicles can be employed to charge sensors within their vicinities wirelessly [Shi et al. 2011]. The adoption of mobile charging vehicles for sensor charging can provide high and stable charging rates to sensors. Thus, less effort will be spent on the sensor energy management, and the manufacturing cost of sensors can be significantly reduced.

Most existing studies assumed that one mobile charging vehicle will have enough energy to charge all sensors in a WSN, and the proposed algorithms for vehicle charging scheduling thus are only applicable to small-scale WSNs [Shi et al. 2011; Xie et al. 2012, 2013a; Zhao et al. 2011; Xu et al. 2015a]. However, in a large-scale sensor network, the amount of energy carried by a single mobile charging vehicle may not be enough to charge all nearly expired sensors, as there are a large proportion of life-critical sensors to be charged to avoid their energy depletion. Thus, multiple mobile charging vehicles need to be employed, and new scheduling algorithms need to be devised.

In this article, we will study the use of multiple mobile charging vehicles to replenish energy to sensors for a large-scale WSN such that none of the sensors runs out of energy, and each sensor can be charged by a mobile charging vehicle within its vicinity through wireless energy transfer. We will adopt a flexible on-demand sensor charging paradigm that decouples sensor energy charging scheduling from the design of sensing data routing protocols and dispatch multiple mobile charging vehicles to charge life-critical sensors in an on-demand way. Specifically, in this article, we assume that each mobile charging vehicle can carry only a limited, rather than infinite, amount of energy. We will study a fundamental sensor charging problem in a large-scale WSN. In other words, given a set of life-critical sensors to be charged and the energy capacity constraint on each mobile charging vehicle, what is the minimum number of mobile charging vehicles needed to fully charge these sensors to save the operational cost of the WSN while ensuring that none of the sensors runs out of energy? To address this problem, not only should the number of charging vehicles be determined but also the charging tour of each mobile charging vehicle needs to be found so that all life-critical sensors can be charged prior to their energy expirations, where each vehicle consumes energy on charging sensors in its tour and its mechanical movement along the tour.

The main contributions of this work are as follows. We first consider the problem of sensor recharging by minimizing the number of mobile charging vehicles needed, subject to the energy capacity constraint on each of the mobile charging vehicles. We then devise an approximation algorithm with a provable performance guarantee if the energy consumption of each sensor during each charging tour can be ignored; otherwise, we propose a heuristic algorithm by modifying the proposed approximation algorithm. Finally, we conduct extensive simulation experiments to evaluate the performance of the proposed algorithms. Experimental results demonstrate that the proposed algorithms are promising, and the solutions delivered are fractional of the optimum. To the best of our knowledge, this is the first approximation algorithm with a constant approximation ratio for the multiple mobile charging vehicle scheduling problem to charge sensors wirelessly so that the number of mobile charging vehicles needed is minimized.

The remainder of the article is organized as follows. Section 2 reviews related studies. Section 3 introduces the system model, notions and notations, and problem definition. Section 4 details a 5-approximation algorithm for the optimal p closed tour problem (to be defined later), which serves as a subroutine of the proposed algorithms. Sections 5 and 6 propose an approximation algorithm and a heuristic algorithm, respectively. Section 7 evaluates the performance of the proposed algorithms, and Section 8 concludes the article.

2. RELATED WORK

With the advance in efficient wireless energy transfer technology, wireless energy replenishment to sensors has been studied for the lifetime prolongation of WSNs in literature [Li et al. 2011; Shi et al. 2011; Xie et al. 2012, 2013a; Zhao et al. 2011; Xu et al. 2015b]. However, applying this technology to sensor networks is still in its infancy stage. Several studies have been conducted in the past few years, and most of these studies considered sensor energy charging and dataflow routing jointly. For example, Shi et al. [2011] are the very first to conduct a theoretical study on the usage of wireless charging technique in WSNs by employing a mobile charging vehicle to periodically charge sensors such that the sensor network can operate perpetually. They formulated an optimization problem that maximizes the ratio of the vacation time of the vehicle over each charging cycle under the assumption that the data rates of all sensors are given in advance. They later extended their solution to a general case where either a mobile charging vehicle can charge multiple sensors at the same time if the sensors are within the vicinity of the vehicle [Xie et al. 2012] or the mobile vehicle charges sensors and collects the sensing data from being charged sensors simultaneously [Xie et al. 2013b]. Zhao et al. [2011] proposed a joint design of energy replenishment and sensing data collection by exploiting sink mobility. They designed an adaptive solution that jointly selects the sensors to be charged and finds an optimal data gathering scheme such that the network utility can be maximized and the perpetual operation of the sensor network can be maintained. Li et al. [2011] argued that existing charging schemes only passively replenish sensors that are deficient in energy supply and cannot fully leverage the strength of wireless energy transfer technology. They instead proposed a "charging-aware" routing protocol (J-RoC) by incorporating dynamic energy consumption rates of sensors into the design of data routing protocols. Although this schema can proactively guide the routing activities and charge energy where it is needed, it makes routing protocols design and management more complicated and may not be realistic in some application scenarios. For example, operational routing protocols in some sensor networks are required to be replaced (or updated) frequently due to the privacy and security of sensing data. There are also several recent studies on passive sensor energy replenishment. For example, Xu et al. [2014, 2015b] considered the problem of scheduling multiple mobile charging vehicles in a rechargeable sensor network for a given monitoring period of T, with an objective of maintaining the perpetual operations of sensors so that the sum of the traveling distances by all mobile charging vehicles in that period is minimized, for which they proposed an approximation algorithm with a guaranteed performance ratio. Xu et al. [2015a] also studied the problem of scheduling K mobile chargers to replenish a set of to-be-charged sensors such that the length of the longest charging tour among the K tours of the K chargers is minimized, for which they proposed constant approximation algorithms. Wang et al. [2013] devised a hybrid strategy for scheduling multiple charging vehicles to replenish sensor energy: active and passive energy replenishments. He et al. [2013] investigated a mobile charging problem using a nearest-job-next with preemption and provided analytical results on the number of charging requests served as well as the charging latency at each sensor, assuming that sensor charging request rates follow a Poisson distribution. However, their solution does not guarantee that all sensors will be charged prior to their energy expirations.

It must be mentioned that joint consideration of energy replenishment and dataflow routing in reality may have limited applications due to unrealistic assumptions, such as (1) the sensor energy consumption rate and/or sensing data rate do not change over time, (2) flow conservation at each sensor node is maintained, and (3) wireless communications between each pair of nodes are reliable. In reality, sensing data rates within a sensor network are usually closely related to the application scenario of the sensor network, whereas the flow conservation prevents any data aggregation at relay nodes. However, data aggregation is an important primitive operation in most sensor networks, as not only can it reduce network-wide data traffic, it also can save the energy consumption of sensors [Krishnamachari et al. 2002]. Furthermore, it is well known that wireless communication is notoriously unreliable [Zhao and Govindan 2003], and retransmissions at some nodes may lead to substantial energy consumptions of the nodes. In contrast, in this work, we advocate a new sensor charging paradigm—the *on-demand charging paradigm*. In other words, sensor energy replenishment should be decoupled from sensing data collection, particularly for real-time sensing monitoring and event surveillance applications. Since the frequencies between data collection and sensor energy charging are significantly different, the former is several orders of magnitude higher than the latter. The sensing data is required to be collected as soon as possible to better monitor the region of interest. Sensors will be charged according to their need, as energy consumption rates of different sensors may significantly vary. Thus, it is more economical to charge sensors that need to be charged. Furthermore, decoupling energy replenishment from sensing data routing protocol design can simplify the management of sensor networks. Routing protocol designers thus can concentrate only on routing protocol functionalities rather than taking sensor charging scheduling into account.

There are two closely related studies on minimizing the number of deployed vehicles [Nagarajan and Ravi 2011; Dai et al. 2014]. Specifically, Nagarajan and Ravi [2011] studied the *distance-constrained vehicle routing problem* (DVRP), in which given a set of nodes in a metric graph, a depot, and an integral distance bound D, the problem is to find the minimum number of tours rooted at the depot to cover all nodes such that the length of each tour is no more than D. For the DVRP problem, they presented a $(O(\log \frac{1}{\epsilon}), 1+\epsilon)$-bicriteria approximation algorithm for any constant ϵ with $0 < \epsilon < 1$—that is, the algorithm finds a set of tours in which the length of each tour is no more than $(1+\epsilon)D$, whereas the number of deployed vehicles is no more than $O(\log \frac{1}{\epsilon})$ times the minimum number of vehicles. On the other hand, Dai et al. [2014] investigated the problem of deploying the minimum number of charging vehicles to fully charge the sensors by making use of the approximation algorithm in Nagarajan and Ravi [2011], assuming that all sensors have identical energy consumption rates. However, there are two essential differences between these two mentioned studies and the work in this article. First, the cost of each found tour by the algorithms in Nagarajan and Ravi [2011] and Dai et al. [2014] may violate the travel distance constraint on the mobile vehicles. In contrast, in this work, the total energy consumption of each mobile charging vehicle per tour cannot exceed its energy capacity IE. Otherwise, the vehicle cannot return to the depot for recharging itself. In addition, a novel approximation algorithm for the minimum vehicle deployment problem is devised. Second, the study in Dai et al. [2014] assumed that there is not any energy consumption for any sensor during a mobile charging vehicle tour, and all sensors have identical energy consumption rates. Contrarily, our work does not require that all sensors have identical energy consumption rates, and the energy consumption rates of different sensors may be significantly different. Furthermore, the energy consumption or leaking of each sensor during its charging has also been considered. Therefore, the proposed algorithms in the two mentioned studies cannot be applicable to the problem in this article. New approximation and heuristic algorithms need to be devised, and new algorithm analysis techniques for analyzing the approximation ratio need to be developed as well.

3. PRELIMINARIES

In this section, we introduce the system model, sensor energy charging paradigm, and the problem definition.

3.1. System Model

We consider a large-scale WSN, $G_s = (V_s, E_s)$, deployed for environmental monitoring or event detection, where V_s is the set of sensors and a base station. There is an edge in E_s between any two sensors or a sensor and the base station if they are within the

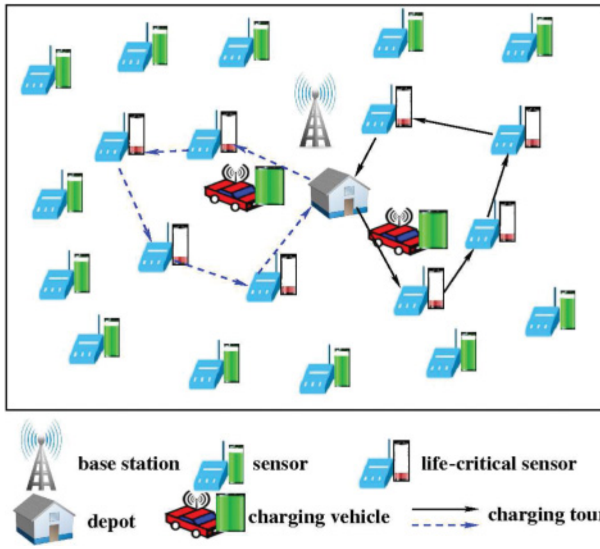


Fig. 1. A rechargeable WSN.

transmission range of each other. All sensing data will be relayed to the base station through multihop relays. Figure 1 illustrates such a network. Assume that there is sufficient energy supply to the base station. Each sensor $v_i \in V_s$ is powered by a rechargeable battery with energy capacity B_i. It consumes its energy when performing sensing, data processing, and data transmission and reception.

To maintain the long-term operation of a rechargeable sensor network, the sensors in the network will be charged at certain time points by mobile charging vehicles. We thus assume that there is a depot in the monitoring region, where there are a number of mobile vehicles available to meet sensor charging demands. Assume that each mobile charging vehicle has a full energy capacity IE and a charging rate μ for charging a sensor, and the vehicle travels at a constant speed s. We further assume that the mechanical movement of the vehicle is derived from its energy as well. Let η be the energy consumption rate of each vehicle on traveling per unit length. All mobile vehicles will start from the depot when performing their charging duties and return to the depot after finishing their charging tours. They will be recharged at the depot and wait for the next round of scheduling. Since the energy capacity of each mobile vehicle is limited, its total travel length and the number of to-be-charged sensors by the mobile vehicle must be constrained by its energy capacity IE.

The residual lifetime of each sensor $v_i \in V_s$ at time t is defined as

$$l_i(t) = \frac{RE_i(t)}{\rho_i(t)}, \tag{1}$$

where $RE_i(t)$ and $\rho_i(t)$ are the amount of residual energy and energy consumption rate of v_i at time t, respectively. The base station keeps a copy of the energy depletion rate $\rho_i(t)$ and the residual energy $RE_i(t)$ of each sensor $v_i \in V_s$.

We assume that each sensor is able to monitor its residual energy $RE_i(t)$ and estimate its energy consumption rate $\rho_i(t)$ in the near future through some prediction techniques, such as linear regressions. We further assume that the energy consumption rate of each sensor does not change within a charging round, or that such minor changes can be neglected as the duration of a charging round usually is short (e.g., a few hours).

But the energy consumption rate of each sensor is allowed to change at a different charging round. Thus, each sensor can estimate its residual lifetime $l_i(t)$ prior to the next charging round. Recall that for each sensor $v_i \in V$ there is a record of its energy consumption rate $\rho_i(t)$ at the base station, and this value is subject to be updated if the energy consumption profile of the sensor in the future will experience significant changes. To accurately measure the energy consumption rate of each sensor, each sensor adopts a lightweight prediction technique to estimate its energy consumption rate in the near future. For example, a sensor can make use of a linear regression $\hat{\rho}_i(t) = \omega\rho_i(t-1) + (1-\omega)\hat{\rho}_i(t-1)$, where $\hat{\rho}_i$ is the estimation, ρ_i is the actual value at that moment, and ω is a weight between 0 and 1 [Cox 1961]. Let $\theta > 0$ be a small given threshold. For each sensor $v_i \in V_s$, the updating of its energy consumption rate is as follows. If $|\hat{\rho}_i(t) - \hat{\rho}_i(t-1)| \leq \theta$, no updating report from sensor v_i will be forwarded to the base station; otherwise, the updated energy consumption rate and its residual energy of v_i will be sent to the base station through a charging request is issued by v_i. The base station then performs the update accordingly.

3.2. Sensor Energy Charging Paradigm

We notice that there is no need for every sensor to be charged at each round. In addition, sensor charging tours are not necessarily periodic; instead, sensors should be charged in an on-demand fashion. The rationale behind this is that in some applications, such as event detections, if there are no events happening in a monitoring area, sensors usually perform duty cycling to save energy, and thus they can run much longer than remaining in wake-up statuses. When an event does occur, the sensors within the event region will remain in wake-up statuses to capture the event and report their sensing results to the base station, whereas for the sensors not in the event region, they continue to maintain their wake-up and sleep duty cycling statuses, thus consuming much less energy. It can be seen from this case that not all sensors in the network need to be charged in each energy charging round; only the sensors in the regions where the event happened need to be charged.

Let l_{max} be the longest duration of a mobile vehicle tour for charging all sensors in the network. Consider that all sensors in the network will be charged by only one mobile charger. Then, l_{max} should be no more than the sum of the time spent on traveling and the time spent on charging sensors on its tour by a mobile charging vehicle. Thus, the value of l_{max} is upper bounded as follows:

$$l_{max} \leq \frac{L_{TSP}}{s} + \frac{\min\left\{IE, \sum_{v_i \in V_s} B_i\right\}}{\mu}, \tag{2}$$

where L_{TSP} is the length of a traveling salesman problem (TSP) tour including all sensors and the depot that can be approximately found by applying Christofides' algorithm [Christofides 1976], s is the travel speed of the charging vehicle, IE is the battery capacity of the vehicle, B_i is the battery capacity of sensor v_i, and μ is the charging rate for sensors. In other words, to ensure that none of the sensors fails due to its energy expiration, a sensor should be charged when its residual lifetime is no greater than l_{max}.

We define the *critical time point* of a sensor as the time point that the sensor can survive for the next l_{max} time units. We say that a sensor v_i at time t is in a *critical lifetime interval* if $l_{\max} \leq l_i(t) \leq \alpha \cdot l_{max}$ with a given constant $\alpha \geq 1$, where $l_i(t)$ has been defined in Equation (1). Following the definition of the critical lifetime interval, only the sensors within their critical lifetime intervals need to be charged to avoid running out of their energy completely. Without loss of generality, in the rest of this article, we assume that V is the set of sensors within their critical lifetime intervals—that is,

Table I. Notations and Their Descriptions

Notations	Descriptions
V_s	Set of sensors
B_i	Battery capacity of sensor v_i
$RE_i(t)$	Residual amount of energy of sensor v_i at time t
$\rho_i(t)$	Energy consumption rate of sensor v_i at time t
$l_i(t) = \frac{RE_i(t)}{\rho_i(t)}$	Residual lifetime of sensor v_i at time t
IE	Energy capacity of each mobile charging vehicle
μ	Charging rate
s	Travel speed of a charging vehicle
η	Energy consumption rate of each vehicle on traveling per unit length
l_{max}	Longest duration of a vehicle tour for charging all sensors
α	Parameter of critical lifetime interval
V	Set of to-be-charged sensors
r	Depot
$h(v)$	Amount of energy charged to sensor v
$w(u, v)$	Energy consumption of a vehicle traveling between sensors u and v

$V = \{v_i \mid v_i \in V_s, l_{max} \leq l_i(t) \leq \alpha \cdot l_{max}\}$, where $l_i(t)$ is the residual lifetime of sensor v_i at time t. Clearly, $V \subseteq V_s$.

We propose a flexible on-demand sensor energy charging paradigm as follows. Each sensor will send an energy-charging request to the base station for its energy replenishment when its the residual lifetime falls to the critical lifetime l_{max}. The energy-charging request contains the identity, the amount of residual energy, and the energy consumption rate of the sensor. Once the base station receives a set of such requests from the sensors, it then performs a scheduling to dispatch a number of mobile charging vehicles to charge the sensors in the set, where a sensor v_i at time t is in its critical lifetime interval if $l_{\max} \leq l_i(t) \leq \alpha \cdot l_{max}$. Hence, the result of each scheduling consists of the number of mobile charging vehicles needed, a closed tour for each of the mobile vehicle, and the charging duration at each to-be-charged sensor node along the tour. Finally, the mobile charging vehicles are dispatched from the depot to perform charging tasks. Table I lists the notations used in this article.

3.3. Problem Definition

Given a rechargeable sensor network $G_s = (V_s, E_s)$ consisting of sensors, one stationary base station, and a depot with multiple mobile vehicles, following the on-demand sensor energy charging paradigm, assume that at a specific time point the base station receives charging requests from the sensors within their critical lifetime intervals. The base station then starts a new round of scheduling by dispatching a certain number of mobile charging vehicles to charge these sensors so that none of sensors runs out of energy. Let V be the subset of sensors in G_s to be charged (within their critical lifetimes) in the next round ($V \subseteq V_s$) (see Figure 1). Assume that for each sensor $v_i \in V$, its energy consumption rate ρ_i does not change during each charging round (or such changes are marginal and can be ignored) and its residual energy RE_i is given (at the base station); the *minimum vehicle deployment problem* is to find a scheduling of mobile charging vehicles to fully charge the sensors in V by providing a closed tour for each vehicle such that the number of mobile vehicles deployed is minimized, subject to the energy capacity constraint IE on each mobile vehicle. For this defined problem, we further distinguish it into two different cases: one is that the energy consumption of each to-be-charged sensor during its charging round is not considered, and the other is that such energy consumption is taken into account. We will devise an approximation algorithm and a heuristic algorithm for them in Sections 5 and 6, respectively. The minimum vehicle deployment problem is NP-hard through a reduction from the well-known NP-hard TSP.

In the following, we define the p-optimal closed tour problem, which will serve as a subroutine of the proposed algorithms for the minimum vehicle deployment problem. Given a node and edge weighted complete metric graph $G = (V, E; h, w)$, a root node $r \in V$, and an integer $p \geq 1$, where $h : V \mapsto \mathbb{R}^{\geq 0}$ and $w : E \mapsto \mathbb{R}^{>0}$ (i.e., the node weight $h(v)$ of each sensor node $v \in V$ is the amount of energy to be charged to sensor v, and the edge weight $w(u, v)$ of each edge $(u, v) \in E$ represents the amount of energy consumed by a mobile vehicle traveling along the edge), the *p-optimal closed tour problem* in G is to find p node-disjoint closed tours covering all nodes in V, except the root r that appears in each of the tours, such that the maximum total cost among the p closed tours is minimized, where the *total cost* of a closed tour is the weighted sum of nodes and edges in it.

Notice that we assume the p found tours are node disjoint in the p-optimal closed tour problem, as there is no such need to include a node in multiple closed tours. In fact, the optimal cost of the problem under the node-disjoint requirement is identical to that of the problem without the node-disjoint requirement, which is shown as follows. On one hand, it can be seen that the optimal cost of the problem under the node-disjoint requirement is no less than the optimal cost of the problem without the node-disjoint requirement, as an optimal solution to the former is a feasible solution to the latter. On the other hand, given an optimal solution to the latter, we can find a feasible solution to the former by the removal of multiple appearances of a sensor from the charging tours in the optimal solution and performing short cutting to obtain shorter closed tours. The cost of the updated solution is no more than the cost of the optimal solution to the problem without the node-disjoint requirement. Therefore, the optimal costs of the problems with and without node-disjoint requirements are identical.

4. ALGORITHM FOR THE *P*-OPTIMAL CLOSED TOUR PROBLEM

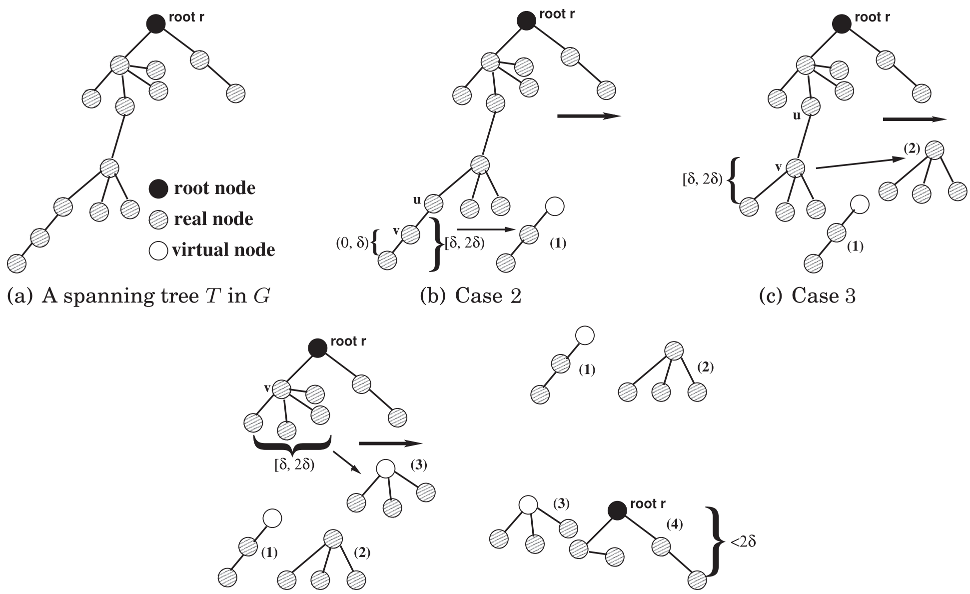
In this section, we devise a 5-approximation algorithm for the p-optimal closed tour problem in a node and edge weighted metric graph $G(V, E; h, w)$. This algorithm will be used as a subroutine for the minimum vehicle deployment problem in Section 5. As a special case of the p-optimal closed tour problem, when $p = 1$ is the well-known TSP problem that is NP-hard, the p-optimal closed tour problem is NP-hard as well. In the following, we start by introducing a popular technique to transform a tree into a closed tour in G. We then introduce a novel tree decomposition. We finally present an approximation algorithm for the problem based on the tree decomposition.

4.1. A Closed Tour Derived from a Tree

We first introduce the technique that transforms a tree in G to a closed tour by the following lemma.

LEMMA 4.1. *Given a node and edge weighted metric graph $G = (V, E; h, w)$ with sets V and E of nodes and edges, $h : V \mapsto \mathbb{R}^{\geq 0}$ and $w : E \mapsto \mathbb{R}^{>0}$, and the edge weight follows the triangle inequality, let $T = (V, E_T; h, w)$ be a spanning tree of G rooted at r. Let C be the traveling salesman tour of G derived from T through performing the pre-order traversal on T and pruning, then the total cost $WH(C)$ of C is no more than twice the total cost $WH(T)$ of T—that is, $WH(C) \leq 2WH(T) = 2(\sum_{v \in V} h(v) + \sum_{e \in E_T} w(e))$.*

PROOF. Let $H(X)$ be the weighted sum of nodes in X, and let $W(Y)$ be the weighted sum of edges in Y, as the weighted sum $W(C)$ of the edges in C is no more than $2\sum_{e \in E_T} w(e)$, and the weighted sum $H(C)$ of nodes in C is the same as the one in T.



(a) A spanning tree T in G (b) Case 2 (c) Case 3

(d) Case 4 (e) Obtained four subtrees

Fig. 2. An illustration of the tree decompositions.

Thus, the total cost of C is $WH(C) = W(C) + H(C) \le 2W(T) + H(T) \le 2(W(T) + H(T)) = 2WH(T)$. $\square$

4.2. Tree Decomposition

Given a metric graph $G = (V, E; h, w)$, let $T = (V, E_T; h, w)$ be a spanning tree in G rooted at node r, and let $\delta \ge \max_{v \in V}\{h(v), 2w(v, r)\}$ be a given value; then, both the node weight $h(v)$ of any node $v \in V$ and the edge weight $w(e)$ of any edge $e \in E_T$ in tree T are no more than δ—that is, $h(v) \le \delta$ and $w(e) \le \delta$. We decompose the tree into a set of subtrees such that the total cost of each subtree is no more than 2δ as follows.

Let (u, v) be a tree edge in T, where u is the parent of v and v is a child of u. Additionally, let T_v be a subtree of T rooted at node v. We perform a depth-first search on T starting from the tree root r until the total cost of the leftover tree rooted at r is less than 2δ—that is, $WH(T_r) < 2\delta$. Figure 2 demonstrates an example of the tree decomposition procedure. Assume that node v is the node that is currently visited; wedistinguish this into the following four cases:

—*Case 1*: If $WH(T_v) < \delta$ and $WH(T_v) + w(u, v) < \delta$, no action is needed and the tree decomposition procedure continues.

—*Case 2*: If $WH(T_v) < \delta$ and $WH(T_v)+w(u, v) \ge \delta$, then we must have $WH(T_v)+w(u, v) < 2\delta$, as the weight $w(u, v)$ of edge (u, v) is no more than δ. A new tree $T_v \cup \{(u', v)\}$ is created with a *virtual node* u' with $h(u') = 0$. Split the subtree $T_v \cup \{(u', v)\}$ from the original tree (see Figure 2(b)).

—*Case 3*: If $\delta \le WH(T_v) < 2\delta$, split the subtree T_v from the original tree and remove edge $(u, v) \in E_T$ from the original tree (see Figure 2(c)).

—*Case 4*: Let $v_1^c, v_2^c, \dots, v_k^c$ be the k children of v. Let l be the maximum children index so that $\delta \le \sum_{j=1}^{l}(WH(T_{v_j^c}) + w(v, v_j^c)) < 2\delta$ with $1 \le l \le k$; then, a new subtree $\cup_{j=1}^{l}(T_{v_j^c} \cup \{(v', v_j^c)\})$, rooted at the virtual node v', is created, which consists of these subtrees with $h(v') = 0$. Split off this subtree from the original tree (see Figure 2(d)).

As a result, a set of subtrees is obtained by the tree decomposition on T (see Figure 2(e)). The number of subtrees is bounded by the following lemma.

LEMMA 4.2. *Given a spanning tree $T = (V, E_T; h, w)$ of a graph $G = (V, E; h, w)$ with the total cost $WH(T)$ and a value $\delta \geq \max_{v\in V}\{2w(r, v), h(v)\}$, the tree T can be decomposed into p subtrees $T_1, T_2, \ldots, T_p$ with $WH(T_i) < 2\delta$ by the proposed tree decomposition procedure, $1 \leq i \leq p$. Then,*

$$p \leq \left\lfloor \frac{WH(T)}{\delta} \right\rfloor. \tag{3}$$

PROOF. Following the tree decomposition on T, subtrees with the total cost in $[\delta, 2\delta)$ are split away from T until the total cost of the leftover tree including root r is less than 2δ. Suppose that $T_1, T_2, \ldots, T_p$ are the split trees with $p \geq 2$. From the subtree construction, we know that $\delta \leq WH(T_i) < 2\delta$ for each i with $1 \leq i \leq p-1$. The only subtree with the total cost less than δ is T_p. Note that prior to splitting T_{p-1}, the total cost of the remaining tree is at least 2δ. Therefore, the average total cost of T_{p-1} and T_p is no less than δ. In other words, the average total cost of all T_i is at least δ. Thus, $p \cdot \delta \leq WH(T)$—that is, $p \leq \frac{WH(T)}{\delta}$. Since p is an integer, $p \leq \lfloor \frac{WH(T)}{\delta} \rfloor$. □

4.3. Algorithm for Finding *p*-Optimal Closed Tours

Given a metric graph $G = (V, E; h, w)$ with root r and a positive integer p, we now devise an approximation algorithm for the p-optimal closed tour problem in G as follows.

Let T be a minimum spanning tree (MST) of G rooted at r. The basic idea of the proposed algorithm is that we first perform a tree decomposition on T with $\delta = \max_{v\in V}\{WH(T)/p,\ 2w(v, r) + h(v)\}$ and later show that δ is a lower bound on the optimal cost of the p-optimal closed tour problem. As a result, p' subtrees are derived from such a decomposition, and p' closed tours are then derived from the p' subtrees. We finally show that $p' \leq p$ and the maximum total cost of any closed tour among the p' closed tours is no more than 5δ.

Specifically, T is decomposed into no more than p' edge-disjoint subtrees, except the root node r that appears in one of these subtrees. Let $T_1, T_2, \ldots, T_{p'}$ be the p' trees obtained by decomposing T. It can be observed that each T_i contains at least one real node and at most one virtual node, where a node v is a *real node* if $h(v) \neq 0$; otherwise, it is a virtual node. As a result, a forest $\mathcal{F}$ consisting of all trees is found through the tree decomposition, the number of trees in $\mathcal{F}$ is $p' \leq \lfloor WH(T)/\delta \rfloor$, and the total cost of each subtree is no more than 2δ by Lemma 4.2.

For each $T_i \in \mathcal{F}$, if it does not contain the root r, then a tree $T'_i = T_i \cup \{(v_i, r)\}$ rooted at r is obtained by including node r and a tree edge (v_i, r) into T_i, where node v_i is a node in T_i and $w(v_i, r) = \min_{v\in T_i}\{w(v, r)\}$. The total cost $WH(T'_i)$ of T'_i is

$$WH(T'_i) = WH(T_i) + w(v_i, r) \leq 2\delta + w(v_i, r) \leq 2.5\delta, \text{ as } w(v_i, r) \leq \delta/2.$$

Otherwise (T_i contains node r), $T'_i = T_i$ and $WH(T'_i) = WH(T_i) \leq 2\delta$. We thus obtain a forest $\mathcal{F}' = \{T'_1, T'_2, \ldots, T'_{p'}\}$. From the trees in $\mathcal{F}'$, p' edge-disjoint closed tours with each containing the root r can be derived. Let $\mathcal{C}' = \{C'_1, C'_2, \ldots, C'_{p'}\}$ be the set of p' closed tours obtained by transforming each tree in $\mathcal{F}'$ into a closed tour. For each C'_i, we have that $WH(C'_i) \leq 2 \cdot WH(T'_i) \leq 5\delta$ by Lemma 4.1. As there are some C'_is containing virtual nodes that are not part of a feasible solution to the problem, a feasible solution can be derived through a minor modification to the closed tours in $\mathcal{C}'$. In other words, for each C'_i, if it contains a virtual node (as each C'_i contains at most one virtual node), a closed tour C_i with a less total cost than that of C'_i is obtained by removing the virtual node and the two edges incident to the node from C'_i through short cutting, then $WH(C_i) \leq WH(C'_i)$, as the edge weight follows the triangle inequality. Otherwise, $C_i = C'_i$. Clearly, each of the p' closed tours $C_1, C_2, \ldots, C_{p'}$ roots at r. The detailed algorithm is described in `Algorithm` 1.

ALGORITHM 1: *Finding Closed Tours Rooted at r with Each Having the Bounded Total Cost*

Input: A metric graph $G = (V, E; h, w)$, a root $r \in V$, and a given value $\delta \geq \max_{v \in V}\{h(v), 2w(v, r)\}$.
Output: A set of node-disjoint closed tours covering all nodes in V with the shared root r so that the total cost of each tour is no more than 5δ.

1: Let T be an MST of G and $WH(T)$ be the total cost of T;
2: Let $\mathcal{F} = \{T_1, T_2, \ldots, T_{p'}\}$ be the forest obtained by performing the tree decomposition on T with the given value δ;
3: Let $\mathcal{F}' = \{T'_1, T'_2, \ldots, T'_{p'}\}$ be a forest, where $T'_i = T_i \cup \{(r, v_i)\}$ is derived by adding root r and an edge with the minimum edge weight between a node v_i in T_i and r if r is not in T_i; otherwise, $T'_i = T_i$, where $1 \leq i \leq p'$;
4: Let $\mathcal{C}' = \{C'_1, C'_2, \ldots, C'_{p'}\}$ be a set of p' closed tours, where C'_i is derived from T'_i;
5: Let $\mathcal{C} = \{C_1, C_2, \ldots, C_{p'}\}$ be a set of closed tours,where C_i is derived by removing the virtual node from $C'_i \in \mathcal{C}'$ if it does contain a virtual node. Otherwise, $C_i = C'_i$;
6: **return** $\mathcal{C}$.

4.4. Algorithm Analysis

We now show that `Algorithm` 1 delivers a 5-approximate solution by the following theorem.

THEOREM 4.3. *Given a metric graph $G = (V, E; h, w)$ and an integer $p \geq 1$, there is a 5-approximation algorithm for finding p-optimal closed tours. The time complexity of the proposed algorithm is $O(|V|^2)$.*

PROOF. In the following, we first show that `Algorithm` 1 delivers a feasible solution to the p-optimal closed tour problem. We then show that the total cost of each closed tour in the solution is no more than 5δ. Next, we show that δ $(= \max_{v \in V}\{WH(T)/p,\ 2w(v, r)+ h(v)\})$ is a lower bound on the optimal cost of the problem. Then, the total cost of each closed tour in the solution delivered by `Algorithm` 1 is no more than $5\delta \leq 5OPT$. We finally analyze the time complexity of `Algorithm` 1.

We first show that `Algorithm` 1 delivers a feasible solution to the p-optimal closed tour problem. Recall that T is an MST of G. Since $\delta = \max_{v \in V}\{WH(T)/p, 2w(v, r)+h(v)\}$, $\delta \geq \max_{v \in V}\{2w(v, r), h(v)\}$. A solution $\mathcal{C}$, which consists of p' closed tours rooted at r, can be obtained by applying `Algorithm` 1 on T, and

$$\begin{aligned} p' &\leq \lfloor WH(T)/\delta \rfloor \\ &\leq WH(T)/\delta \\ &= \frac{WH(T)}{\max_{v \in V}\{WH(T)/p, 2w(v, r) + h(v)\}} \\ &\leq \frac{WH(T)}{WH(T)/p} \\ &= p, \end{aligned} \tag{4}$$

by Lemma 4.2. Thus, $\mathcal{C}$ is a feasible solution.

We then show that the total cost of each closed tour in $\mathcal{C}$ is no more than 5δ. As each $C_i \in \mathcal{C}$ is derived from a $C'_i \in \mathcal{C}'$, we have $WH(C_i) \leq WH(C'_i) \leq 2WH(T'_i) \leq 2 \cdot 2.5\delta = 5\delta$ by Lemma 4.1.

Next, we prove that δ is a lower bound on the optimal cost of the problem. Given a node and edge weighted metric graph $G = (V, E; h, w)$ with root r, an integer $p \geq 1$, partition the nodes in V into p disjoint subsets $X_1, X_2, \ldots, X_p$, and let C_j be the closed tour containing all nodes in X_j and the root r. The optimal partitioning is a partitioning such that the maximum value $\max_{1\leq j\leq p}\{WH(C_j)\}$ is minimized. Let OPT be the total cost of the maximum closed tour in the optimal solution. We show that $\delta \leq OPT$ as follows.

Let $C_1^*, C_2^*, \ldots, C_p^*$ be the p closed tours in the optimal solution with the shared root r. Then, $WH(C_i^*) \leq OPT$. Let e_i be the maximum weighted edge in C_i^*. Then, a tree $T' = \cup_{i=1}^{p} C_i^* \setminus \cup_{i=1}^{p}\{e_i\}$ rooted at r can be obtained by removing e_i from each tour C_i^*. We then have

$$WH(T') = \sum_{i=1}^{p}(WH(C_i^*) - w(e_i)) \leq \sum_{i=1}^{p} WH(C_i^*) \leq p \cdot OPT. \tag{5}$$

It can be seen that T' is a spanning tree in G. Since T is an MST of G, $WH(T) \leq WH(T')$. We thus have

$$\frac{WH(T)}{p} \leq \frac{WH(T')}{p} \leq OPT. \tag{6}$$

On the other hand, each node $v \in V$ must be contained by one closed tour C_i^* in the optimal solution. Since tour C_i^* contains node v and the depot r, then the total cost of C_i^*, $WH(C_i^*)$, is at least $2w(v, r) + h(v)$, Thus,

$$2w(v, r) + h(v) \leq WH(C_i^*) \leq OPT, \quad \forall v \in V. \tag{7}$$

Combing inequalities (6) and (7), we have

$$\delta = \max_{v\in V}\left\{\frac{WH(T)}{p},\ 2w(v, r) + h(v)\right\} \leq OPT. \tag{8}$$

We finally analyze the time complexity of `Algorithm` 1. Following the algorithm, the MST T of G can be found in $O(|V|^2)$ time. The tree decomposition and the construction of $\mathcal{F}'$ can be done in $O(|V|)$ time. For each $C_i' \in \mathcal{C}'$, its corresponding C_i can be found in $O(|E_i'|)$ time, where E_i' is the set of edges in C_i'. We also know that $E_i' \cap E_j' = \emptyset$ if $i \neq j$. As C_i' is derived from tree $T_i' \in \mathcal{F}'$, $|E_i'| \leq 2|E(T_i')|$. Since $\sum_{i=1}^{p'} |E(T_i')| \leq |V| - 1$, $\sum_{i=1}^{p'} |E_i'| \leq \sum_{i=1}^{p'} 2|E(T_i')| \leq 2|V|$. Thus, `Algorithm` 1 takes $O(|V|^2)$ time. $\square$

5. APPROXIMATION ALGORITHM FOR THE MINIMUM VEHICLE DEPLOYMENT PROBLEM

In this section, we provide an approximation algorithm for the minimum vehicle deployment problem. As each mobile vehicle consumes energy on traveling and charging sensors per tour, the total amount of energy consumed by the mobile vehicle is bounded by its energy capacity IE.

5.1. Algorithm

The basic idea of the proposed approximation algorithm is to reduce the minimum vehicle deployment problem into a p-closed tour problem by bounding the total cost of each closed tour. A solution to the latter in turn returns a solution to the former as follows.

Recall that we assume the base station knows both the residual energy RE_i and the energy consumption rate ρ_i of each sensor $v_i \in V$, and μ is the wireless charging rate of a mobile vehicle. Assume that there are sufficient numbers of fully charged mobile vehicles available at the depot. Then, a mobile vehicle takes $\tau_i = \frac{B_i - RE_i}{\mu}$ time to charge sensor v_i to its full capacity B_i when it approaches the sensor. We thus construct a node and edge weighted metric graph $G = (V, E; h, w)$, where V is the set of sensors to be charged in this round. There is an edge in E between any two to-be-charged sensor nodes. For each edge $(u, v) \in E$, its weight is $w(u, v) = \eta \cdot d(u, v)$, which is the amount of energy consumed by a mobile vehicle traveling along the edge, where η is the energy consumption rate of a mobile vehicle for traveling per unit length and $d(u, v)$ is the Euclidean distance between sensor nodes u and v. For each node $v_i \in V$, its weight $h(v_i)$ $(= B_i - RE_i = \mu \cdot \tau_i)$ is the amount of energy needed to charge sensor v_i to reach its full capacity B_i. We assume that $IE \geq \max_{v \in V}\{2w(v, r) + h(v)\}$; otherwise, there are no feasible solutions to the problem, which will be shown by Lemma 5.1 later. The detailed algorithm is described in `Algorithm` 2. We refer to this algorithm as `NMV_without_Eloss`.

ALGORITHM 2: *Finding the Optimal Number of Mobile Vehicles and Their Closed Tours (NMV_Without_Eloss)*

Input: A metric graph $G = (V, E; h, w)$, a root r, and IE with $IE \geq \max_{v \in V}\{2w(r, v) + h(v)\}$.
Output: p-node-disjoint closed r-rooted tours $C_1, C_2, \ldots, C_p$ covering all nodes in V such that $WH(C_i) \leq IE$.

Let T be an MST of G. Denote by $W(T)$ and $H(T)$ the total costs of the edges and nodes in T, respectively;
if $IE \geq 2 \cdot W(T) + H(T)$ **then**
  One mobile vehicle suffices by Lemma 4.1; EXIT;
end
$A \leftarrow \max_{v \in V}\{2w(v, r)\}$;
if $IE/5 \geq A$ **then**
  $\delta \leftarrow IE/5$; /* δ is the average cost of each subtree after tree decomposition */
else
  $\delta \leftarrow \frac{IE-A}{4}$;
end
Perform the tree decomposition using δ. If there is a node v with $h(v) > \delta$, then the node itself forms a tree;
Let $\mathcal{C} = \{C_1, C_2, \ldots, C_p\}$ be the solution by applying `Algorithm` 1 for the tree decomposition on T with the given δ;
return $\mathcal{C}$ as a solution to the problem and $p = |\mathcal{C}|$.

5.2. Algorithm Analysis

In this section, we analyze the approximation ratio of `Algorithm` 2 and its time complexity. We start by Lemma 5.1, which says that there must be a feasible solution to the problem if and only if $IE \geq \max_{v \in V}\{2w(v, r) + h(v)\}$; otherwise, there are no solutions to the problem. Thus, in the rest of our discussions, we assume that $IE \geq \max_{v \in V}\{2w(v, r) + h(v)\}$.

LEMMA 5.1. *Given a metric graph $G = (V, E; h, w)$ and an energy capacity IE of each mobile charging vehicle, there is a feasible solution to the minimum vehicle deployment problem in G if and only if $IE \geq \max_{v \in V}\{2w(v, r) + h(v)\}$, where r is the depot of charging vehicles.*

PROOF. If $IE \geq \max_{v \in V}\{2w(v, r) + h(v)\}$, we can derive a feasible solution to the problem by dispatching one charging vehicle to charge only one of the $n = |V|$ sensors. Thus, n charging vehicles are deployed. On the other hand, assume that there is a feasible solution $\mathcal{C} = \{C_1, C_2, \ldots, C_p\}$ to the problem, where p charging vehicles are deployed to fully charge the n sensors and C_j is the charging tour of the j-th charging vehicle with $1 \leq j \leq p$. It is obvious that $WH(C_j) \leq IE$ for $1 \leq j \leq p$. Consider a sensor $v_i \in V$ such that $v_i = \arg\max_{v \in V}\{2w(v,r)+h(v)\}$. Let C_j be the charging tour containing sensor v_i in the solution. Since tour C_j must contain sensor v_i and depot r, the total cost of the tour, $WH(C_j)$, must be no less than $2w(v_i,r)+h(v_i)$—that is, $WH(C_j) \geq 2w(v_i,r)+h(v_i)$. Then, $IE \geq 2w(v_i,r)+h(v_i) = \max_{v \in V}\{2w(v,r)+h(v)\}$. □

THEOREM 5.2. *Given a metric graph $G = (V, E; h, w)$ and the energy capacity IE of each mobile charging vehicle with $IE \geq \max_{v\in V}\{2w(v,r)+h(v)\}$, there is an approximation algorithm,* `Algorithm` *2, with an approximation ratio of 8 for the minimum vehicle deployment problem in G if $IE \geq 2A$; otherwise, the approximation ratio of the algorithm is $4(1+\frac{A}{h_{min}})$. The algorithm takes $O(|V|^2)$ time, where r is the depot of charging vehicles, $A = \max_{v\in V}\{2w(r,v)\}$, and $h_{min} = \min_{v\in V}\{h(v)\}$.*

PROOF. We first show that `Algorithm` 2 can deliver a feasible solution $\mathcal{C} = \{C_1, C_2, \ldots, C_p\}$. Recall that $A = \max_{v\in V}\{2w(v,r)\}$, which is the maximum energy consumption of a charging vehicle on one round trip between a sensor v and the depot r in the sensor network. We distinguish it into three cases:

—*Case 1*: If $IE \geq 2 \cdot W(T) + H(T)$, then there is a closed tour C including all nodes in V derived from T, and the total cost of C, $WH(C)$ ($\leq 2\cdot W(T)+H(T) \leq IE$ by Lemma 4.1), is no more than the energy capacity of a mobile vehicle IE. Hence, one mobile charging vehicle suffices for charging all nodes in V.
—*Case 2*: If $IE/5 \geq A$, then $\delta = IE/5$, and the total cost of each closed tour in the solution is no more than $5\delta = IE$ by Theorem 4.3.
—*Case 3* ($IE/5 < A \leq IE$): Following `Algorithm` 2, we set $\delta = \frac{IE-A}{4}$. Clearly, $w(v,r) \leq A/2$ for any node $v \in V$ since $A = \max_{v\in V}\{2w(r,v)\}$. Then, the total cost of each closed tour C in the solution is analyzed as follows. First, C contains only one sensor node $v \in V$. The total cost of C is thus $WH(C) = 2w(r,v)+h(v) \leq IE$ by Lemma 5.1 and the input condition of the algorithm. Second, C consists of multiple sensor nodes and is derived from a tree T_i. Then, the total cost of tour C in the solution is $WH(C) \leq 2\cdot(2\delta + w(v_0,r)) \leq 4 \cdot \frac{IE-A}{4} + 2w(v_0,r) \leq IE - A + A = IE$, where $w(v_0,r) = \min_{v\in T_i}\{w(v,r)\}$ and T_i is the tree from which C is derived. Thus, the solution is a feasible solution to the problem.

We then analyze the approximation ratio of the proposed algorithm. Assume that the minimum vehicles needed is p_{min}. With a similar discussion in Theorem 4.3, a lower bound on the value of p_{min} is

$$p_{min} \geq \left\lceil \frac{WH(T)}{IE} \right\rceil. \tag{9}$$

Let p be the number of vehicles delivered by the proposed algorithm. We show the approximation ratio by the following four cases:

—*Case 1*: If $IE \geq 2\cdot W(T)+H(T)$, only one mobile vehicle suffices and this is an optimal solution.
—*Case 2*: If $IE/5 \geq A$, we have $\delta = IE/5$. Then, $\frac{p}{p_{min}} \leq \frac{\lfloor WH(T)/\delta \rfloor}{\lceil WH(T)/IE \rceil} \leq \frac{WH(T)/\delta}{WH(T)/IE} = IE/\delta = 5$ by Lemma 4.2.
—*Case 3* ($IE/5 < A \leq IE/2$): We have $\delta = \frac{IE-A}{4}$. Then, $\frac{p}{p_{min}} \leq \frac{\lfloor WH(T)/\delta \rfloor}{\lceil WH(T)/IE \rceil} = \frac{WH(T)/\delta}{WH(T)/IE} = \frac{IE}{\delta} = \frac{4\cdot IE}{IE-A} = \frac{4}{1-A/IE} \leq \frac{4}{1-A/2A} = 8$ by Lemma 4.2, Equation (9), and $IE \geq 2A$.
—*Case 4* ($IE/2 < A < IE$): We also have $\delta = \frac{IE-A}{4}$. Let $h_{min} = \min_{v\in V}\{h(v)\}$, which is the minimum amount of energy for fully charging an energy-critical sensor v in the sensor network. Then, $IE \geq \max_{v\in V}\{2w(r,v)+h(v)\} \geq 2w(r,v_i)+h(v_i) = A + h(v_i) \geq A + h_{min}$, where $v_i = \arg\max_{v\in V}\{2w(r,v)\}$. The approximation ratio for Case 4 then is

$\frac{p}{p_{min}} \le \frac{\lfloor WH(T)/\delta \rfloor}{\lceil WH(T)/IE \rceil} \le \frac{WH(T)/\delta}{WH(T)/IE} = \frac{IE}{\delta} = \frac{4 \cdot IE}{IE-A} = 4(1+\frac{A}{IE-A}) \le 4(1+\frac{A}{A+h_{min}-A}) = 4(1+\frac{A}{h_{min}}) = O(1)$, as each of the to-be-charged sensors has consumed a large portion of its energy already and h_{min} is thus proportional to the battery capacity of each sensor. The ratio $\frac{A}{h_{min}}$ is usually a constant, where A is the maximum energy consumption of a charging vehicle on one round trip between a sensor and the depot r, and h_{min} is the minimum amount of energy for fully charging an energy-critical sensor. Therefore, the approximation ratio for Case 4 is a constant. Notice that in practice, Case 4 rarely happens, as the energy capacity of a charging vehicle cannot be used just for its travel without charging sensors, or its energy is only enough to charge one or two sensors per tour.

In summary, the approximation ratio of Algorithm 2 is no more than 8 when $IE \ge 2A$; otherwise $(\max_{v \in V}\{2w(w,r)+h(v)\} \le IE < 2A)$, its approximation ratio is $4(1+\frac{A}{h_{min}})$. The dominant time of `Algorithm` 2 is the invoking of `Algorithm` 1, which takes $O(|V|^2)$ time by Theorem 4.3. □

6. HEURISTIC ALGORITHM WITH SENSOR ENERGY CONSUMPTION PER CHARGING TOUR

Thus far, we have provided an approximate solution to the minimum vehicle deployment problem by assuming that the energy consumption of each sensor during a charging tour is several orders of magnitude less than its full capacity, and therefore such a small amount of energy consumption can be neglected. However, if this energy consumption has to be considered, the problem becomes much more complicated. For example, if a critical lifetime sensor has not been charged in the current tour, the sensor may run out of energy prior to the next round of charging. For this extreme case, the mobile charging vehicle must charge the sensor no later than its critical time point; otherwise, the sensor is dead. Thus, the charging tour of each mobile charging vehicle may not be the shortest one, as it must charge sensors in the order of their survival time. To minimize the energy consumption of each mobile charging vehicle on traveling, in this section we assume that all to-be-charged sensors have sufficient residual energy for any charging tour. Since the residual lifetime $l_i(t)$ of each to-be-charged sensor v_i at time point t is no less than l_{max} and l_{max} is the longest duration of a mobile vehicle tour for charging all sensors in the network, the charging order of the sensors in a charging tour does not matter. Under this assumption, we propose a novel heuristic algorithm for the problem by incorporating sensor energy consumption during each charging tour as follows.

The basic idea is that we first apply the approximation algorithm with the mobile vehicle energy capacity IE to deliver a solution. This solution may or may not be feasible. If it is feasible, done. Otherwise, we then use a portion of the energy capacity of a mobile vehicle for sensor charging and the rest of the energy capacity to compensate for the total energy consumption of sensors during its charging tour. This procedure continues until a feasible solution is found. Specifically, we apply the approximation algorithm with vehicle energy capacity IE to deliver a solution. Let $\mathcal{C} = \{C_1, C_2, \ldots, C_p\}$ be the solution. For each closed tour $C \in \mathcal{C}$, we start from the root node r by indexing its nodes clockwise. Let $v_1, v_2, \ldots, v_{|C|}$ be the node sequence in C with $r = v_1$, and let $e_1, e_2, \ldots, e_{|C|}$ be the edge sequence of C, where $e_i = (v_i, v_{(i+1) \bmod |C|})$. Let t_i be the charging time spent on node v_i that consists of two parts: the time τ_i on charging the sensor just prior to the current tour and the time Δt_i for compensating the energy consumption of the sensors due to the travel delay and the delay on charging other nodes by the mobile charging vehicle in the tour so far. Notice that we ignore the energy consumption of a sensor in the period during which it is being charged. Thus, we have

$t_1 = 0 + \Delta t_1 = 0$ as $\Delta t_1 = 0$,
$t_2 = \tau_2 + \Delta t_2$ as $\Delta t_2 = \frac{\rho_2(t_1 + d(v_1, v_2)/s)}{\mu}$,
$t_3 = \tau_3 + \Delta t_3$ as $\Delta t_3 = \frac{\rho_3(t_1 + t_2 + (d(v_1, v_2) + d(v_2, v_3))/s)}{\mu}$,
$\vdots$
$t_i = \tau_i + \Delta t_i$ as $\Delta t_i = \frac{\rho_i \sum_{j=1}^{i-1}(t_j + d(v_j, v_{j+1})/s)}{\mu}$ for each i with $1 \le i \le |C|$, where s is the traveling speed of each vehicle.

The total amount of time spent on tour C by a mobile vehicle is $\sum_{j=1}^{|C|} t_j + \sum_{j=1}^{|C|} \frac{d(v_j, v_{j+1})}{s}$. And the total amount of energy needed for tour C is

$$E(C) = \sum_{j=1}^{|C|}(\mu t_j + \eta d(v_j, v_{j+1})) = \sum_{j=1}^{|C|}(\mu(\tau_j + \Delta t_j) + \eta d(v_j, v_{j+1}))$$
$$= \sum_{j=1}^{|C|}(\mu \tau_j + \eta d(v_j, v_{j+1})) + \Delta E(C), \quad (10)$$

where $\Delta E(C) = \mu \sum_{j=1}^{|C|} \Delta t_j = \sum_{j=1}^{|C|} \rho_j \sum_{k=1}^{j-1}(t_k + d(v_k, v_{k+1})/s)$. Notice that $\Delta E(C)$ is the extra amount of energy needed compared to the one in the previous section without taking the sensor energy consumption during each charging tour into account. It can be seen that the value of $\Delta E(C)$ is determined by the mobile vehicle traveling distance, the energy consumption rate ρ_j, and the charging duration τ_j of each node v_j in C.

We now estimate an upper bound of $\Delta E(C)$ in tour C as follows. We know that $\Delta E(C) = \sum_{j=1}^{|C|} \rho_j \sum_{k=1}^{j-1}(t_k + d(v_k, v_{k+1})/s)$, where $\sum_{k=1}^{j-1}(t_k + d(v_k, v_{k+1})/s)$ is the time duration from the time a vehicle departs from the depot to the time the vehicle begins charging sensor v_j. Recall that $l_{max} = \frac{L_{TSP}}{s} + \frac{\min\{IE,\ \sum_{v_i \in V_s} B_i\}}{\mu}$ is the longest duration of a mobile vehicle tour for charging all sensors in the network. Thus, $\sum_{k=1}^{j-1}(t_k + d(v_k, v_{k+1})/s) \le l_{max}$. Let $\rho_{max} = \max_{v_i \in V}\{\rho_i\}$ be the maximum energy consumption rate among the sensors in V. Then, $\Delta E(C) \le \sum_{j=1}^{|C|}(\rho_{max} \cdot l_{max})$. Since the number of sensors in any tour C is no more than $|V|$, $\Delta E(C) \le |V| \cdot \rho_{max} \cdot l_{max}$.

To find a feasible solution to the problem, it must ensure that the total amount of energy consumed per tour in the solution is no more than the vehicle energy capacity IE. To this end, let $E_{max} = \max_{1 \le i \le p}\{E(C_i)\}$ be the maximum amount of energy needed among the p closed tours delivered by the approximation algorithm. If the solution is feasible ($E_{max} \le IE$), done. Otherwise, a new energy capacity $IE - E_{extra}$ for each mobile vehicle is assigned, and apply the approximation algorithm again, where $E_{extra} = \rho_{max} \cdot l_{max}$. If the new solution is still infeasible, then increase E_{extra} by $\rho_{max} \cdot l_{max}$. The increase on E_{extra} will reduce the vehicle energy capacity to $IE - E_{extra}$ in later invoking the approximation algorithm; this implies that the number of mobile vehicles used for the current round charging will be increased. This procedure continues until a feasible solution is found. It can be seen that a feasible solution will be found within $|V|$ times of increasing E_{extra} since the extra amount of energy needed in each tour $\Delta E(C)$ is no more than $|V| \cdot \rho_{max} \cdot l_{max}$. The detailed algorithm for the minimum vehicle deployment problem under this general setting is described in `Algorithm` 3. We refer to this algorithm as `NMV_with_Eloss`.

THEOREM 6.1. *Given a metric graph $G = (V, E; h, w)$, there is a heuristic algorithm,* `Algorithm` *3, for the minimum vehicle deployment problem in G if the energy*

ALGORITHM 3: *Finding the Optimal Number of Mobile Vehicles and Their Closed Tours Under the Sensor Energy Consumption Assumption (NMV_With_Eloss)*

Input: A metric graph $G = (V, E; h, w)$, a root $r \in V$, the energy capacity IE, the charging rate μ of each mobile vehicle, and the energy depletion rate ρ_i of each node $v_i \in V$.
Output: p-node-disjoint closed tours with a shared node r covering all nodes in V such that the total cost of each tour is no more than IE.

infeasible $\leftarrow$ '*false*'; /* determine the solution is a feasible solution to the problem */
Find a solution $\mathcal{C} = \{C_1, C_2, \ldots, C_p\}$ by applying `Algorithm` 2 with vehicle energy capacity IE;
for *each closed tour* $C_i \in \mathcal{C}$ **do**
  Compute $E(C_i)$ by Equation (10);
  if $E(C_i) > IE$ **then**
    infeasible $\leftarrow$ '*true*';
  end
end
$E_{extra} \leftarrow \rho_{max} \cdot l_{max}$, where $\rho_{max} = \max_{v_i \in V}\{\rho_i\}$;
while *infeasible* **do**
  A new solution $\mathcal{C}'$ is obtained by invoking `Algorithm` 2 with vehicle energy capacity $IE' = IE - E_{extra}$;
  $\mathcal{C} \leftarrow \mathcal{C}'$;
  if *the new solution is infeasible* **then**
    $E_{extra} \leftarrow E_{extra} + \rho_{max} \cdot l_{max}$;
  else
    infeasible $\leftarrow$ '*false*';
  end
end
return $\mathcal{C}$ and the number of mobile vehicles $p = |\mathcal{C}|$.

consumption of each sensor during a charging tour is also taken into account. The time complexity of the proposed algorithm is $O(|V|^3)$.

PROOF. Following `Algorithm` 3, it can be seen that a feasible solution can be found by invoking `Algorithm` 2 with $O(|V|)$ times, and each invoking takes $O(|V|^2)$. The proposed heuristic algorithm thus takes $O(|V|^3)$ time. □

7. PERFORMANCE EVALUATION

In this section, we evaluate the performance of the proposed algorithms through experimental simulations. We also investigate the impact of several important parameters on algorithm performance, including the network size n, the variance of energy consumption rates, the energy capacity IE of mobile charging vehicles, and the critical lifetime interval parameter α.

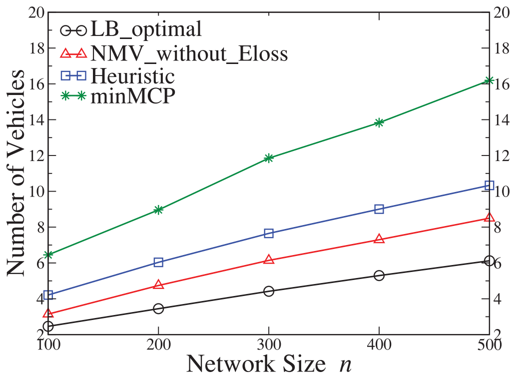
7.1. Simulation Environment

We consider a wireless rechargeable sensor network consisting of from 100 to 500 sensors that are randomly deployed in a 500×500 m square. The battery capacity B_i of each sensor $v_i \in V_s$ is set to be 10.8 kiloJoules (kJ) by referring to a regular *NiMH* battery [Shi et al. 2011]. A base station is located at the center of the square, and a depot of mobile vehicles is co-located with the base station. The energy capacity of each mobile charging vehicle IE ranges from 1,000 to 5,000 kJ. We assume that each of them travels at a constant speed of $s = 5$m/s with an energy consumption rate of $\eta = 0.6$kJ/m [Xie et al. 2013b]. The energy charging rate of each charging vehicle is $\mu = 5$W [Kurs et al. 2007]. The default value of α is 5.

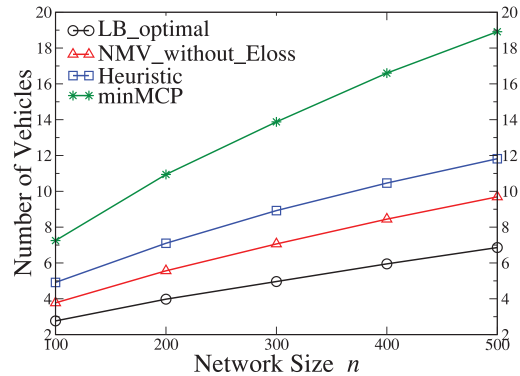
We consider two different distributions of energy consumption rates of sensors: the linear distribution and the random distribution. In the *linear distribution*, the energy consumption rate ρ_i of sensor v_i is proportional to its distance to the base station. The sensors nearest to and farthest from the base station have the maximum energy consumption rates ρ_{max} and the minimum energy consumption rates ρ_{min}, respectively, where $\rho_{min} = 1$mJ/s and $\rho_{max} = 10$mJ/s. The linear distribution models the energy consumptions of sensors in WSNs where the main energy consumption of sensors is on the data transmission and relays. Sensors close to the base station must relay the sensing data for other remote sensors, thus consuming much more energy than the others. Furthermore, by adjusting the energy consumption ratio of each sensor from ρ_{max} to ρ_{min}, this model can be used to model data aggregations at relay sensor nodes (i.e., a smaller ratio $\frac{\rho_{max}}{\rho_{min}}$ implies a higher data aggregation). On the other hand, in the *random distribution*, the energy consumption rate ρ_i of each sensor $v_i \in V_s$ is randomly chosen from a value interval $[\rho_{min}, \rho_{max}]$. The random distribution captures the energy consumption of heterogeneous sensors. For example, video camera sensors in multimedia sensor networks typically consume plenty of energy on image processing [Akyildiz et al. 2007]. Thus, the energy consumption rates of sensors in such sensor networks do not closely correlate to the distances between the sensors and the base station. We further assume that the energy charging rate μ of each mobile vehicle is several orders of magnitude of the energy depletion rate of sensors (i.e., $\mu >> \max_{v_i \in V}\{\rho_i\}$). A fully charged sensor can survive from 10 days up to 4 months. We put 1 year as our monitoring period of the sensor network. Each value in the figures is the mean of the results by applying each mentioned algorithm to 50 different network topologies with the same network size.

To evaluate the performance of the proposed algorithms, we have also implemented three benchmarks: `LB_optimal`, algorithm `Heuristic`, and algorithm `minMCP` [Dai et al. 2014; Nagarajan and Ravi 2011], in which *LB_optimal* is a lower bound on the minimum number of mobile vehicles, which is an approximate estimation of the optimal solution—that is, `LB_optimal` $= \lceil WH(T)/IE \rceil$ by Equation (9), where $WH(T)$ is the total cost of the MST T of the metric graph G induced by the to-be-charged sensors and IE is the energy capacity of each mobile charging vehicle. Algorithm `Heuristic` is described as follows. Given n to-be-charged sensors $v_1, v_2, \ldots, v_n$ indexed by their appearance in the area, we assume that the depot is the origin and index the sensors in counterclockwise order. Algorithm `Heuristic` assigns the vehicles to the sensors one by one until all sensors are charged. Specifically, assume that the first $K - 1$ mobile vehicles have been assigned to sensors $v_1, v_2, \ldots, v_{i-1}$ already. We now assign the K-th mobile vehicle to charge the sensors in the sequence $v_i, v_{i+1}, \ldots, v_n$. Initially, $K = 1$ and $i = 1$. The set of sensors charged by vehicle K will be $v_i, v_{i+1}, \ldots, v_j$ if the total cost of a shortest closed tour C_K including depot r and sensors $v_i, v_{i+1}, \ldots, v_j$ is no more than the energy capacity IE, whereas the total cost of a shortest closed tour C'_K including depot r and sensors $v_i, v_{i+1}, \ldots, v_j, v_{j+1}$ is larger than IE—that is, $WH(C_K) \le IE$ and $WH(C'_K) > IE$, where $i \le j \le n$. This procedure continues until all n sensors are charged.

To compare our work to two closely related works, we adopt a variant of algorithm `minMCP` in Nagarajan and Ravi [2011] and Dai et al. [2014], as the total energy consumption of some of the closed tours delivered by their algorithms may violate the energy capacity constraint IE, and the amount of energy consumed on each such a tour can be up to $IE(1 + \epsilon)$ with $\epsilon > 0$ being a constant. To ensure that the energy consumption of any charging tour is no greater than the energy capacity IE of each mobile vehicle when applying algorithm `minMCP`, we set the energy capacity of mobile vehicles as $\frac{IE}{1+\epsilon}$ when invoking the algorithm. Thus, the total energy consumption of a



(a) Linear distribution



(b) Random distribution

Fig. 3. Performance of algorithms `NMV_without_Eloss`, `Heuristic`, and `minMCP` by varying network size under two different distributions of energy consumption rates when $IE = 1{,}000$kJ, $\rho_{min} = 1$mJ/s, and $\rho_{max} = 10$mJ/s.

charging vehicle per tour will be no more than $\frac{IE}{1+\epsilon} \cdot (1+\epsilon) = IE$, and we set $\epsilon = 0.1$ in all of our experiments to the default setting.

7.2. Performance Evaluation of Algorithms

In this section, we evaluate the performance of algorithms `NMV_without_Eloss`, `NMV_with_Eloss`, `Heuristic`, and `minMCP`, where algorithm `NMV_without_Eloss` does not take into account the sensor energy consumption during each charging tour, whereas algorithm `NMV_with_Eloss` does take such sensor energy consumption into consideration.

We first evaluate the performance of algorithms `NMV_without_Eloss`, `Heuristic`, and `minMCP` under the assumption that sensor energy consumption rates follow linear and random distributions by varying the network size from 100 to 500 sensors. Figure 3(a) plots their performance curves, from which it can be seen that the solution delivered by algorithm `NMV_without_Eloss` is fractional of the optimal one. Specifically, the number of mobile vehicles delivered by it is around 35% more than the lower bound `LP_optimal`, whereas the number of mobile vehicles by it is about 20% and 45% less than that by algorithms `Heuristic` and `minMCP`, respectively. The rationale behind is as follows. Given a set of to-be-charged sensors, algorithm `Heuristic` first sorts the sensors in counterclockwise order, where the depot is the origin. The algorithm then assigns the mobile vehicles to sensors one by one until all sensors are charged. There may be some cases in which some sensors charged by a mobile charging vehicle are far away from each other. Then, the charging vehicle consumes more energy on traveling rather than on charging the sensors. As a result, more charging vehicles are needed. In contrast, the proposed algorithm `NMV_without_Eloss` will schedule a mobile charging vehicle to replenish a set of sensors whose locations are close to each other. Therefore, fewer charging vehicles are required. Figure 3(b) indicates that the four algorithms have similar behaviors under both linear and random distributions of energy consumption rates.

We then investigate the average total travel distance of dispatched charging vehicles by algorithms `TSP`, `NMV_without_Eloss`, `Heuristic`, and `minMCP`, where algorithm `TSP` finds a closed tour visiting all to-be-charged sensors and the depot by applying Christofides' algorithm [Christofides 1976]. We can see that the length of the tour found by algorithm `TSP` is a lower bound on the minimum total travel distance of employed vehicles for the minimum vehicle deployment problem, as there is no energy capacity constraint on the vehicle in the algorithm. Figure 4(a) shows that the average

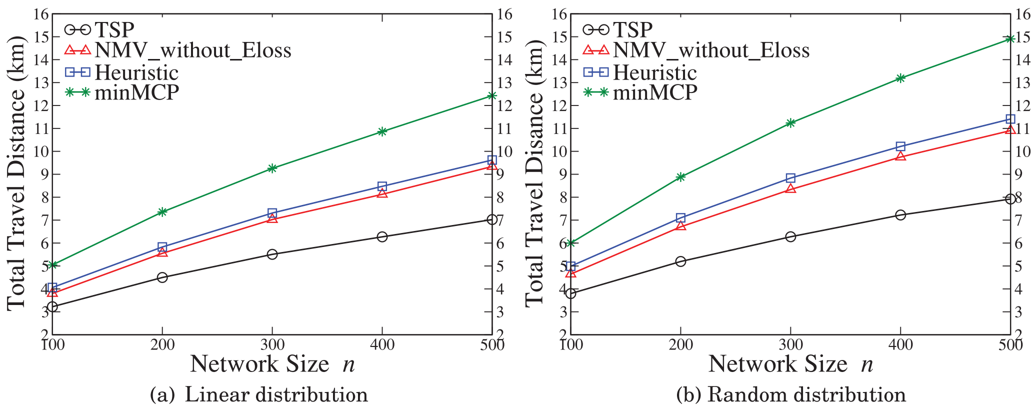


Fig. 4. The average total travel distance of dispatched charging vehicles by algorithms `TSP`, `NMV_without_Eloss`, `Heuristic`, and `minMCP` by varying network size under two different distributions of energy consumption rates when $IE = 1{,}000$kJ, $\rho_{min} = 1$mJ/s, and $\rho_{max} = 10$mJ/s.

total travel distance by the proposed algorithm `NMV_without_Eloss` is only from 18% to 32% longer than that by algorithm `TSP`, whereas it is about 5% and 25% shorter than that by algorithms `Heuristic` and `minMCP`, respectively. Again, Figure 4(b) implies that the four algorithms have the similar behaviors under both linear and random distributions. Thus, in the rest, we only investigate the impact of several parameters on the performance of these algorithms under the linear distribution of energy consumption rates.

Next, we study the impact of the energy capacity of mobile charging vehicle IE on the performance of algorithms `NMV_without_Eloss`, `Heuristic`, and `minMCP` by varying IE from $1{,}000$kJ to $5{,}000$kJ. Figure 5 shows that with the growth of the energy capacity IE, the number of mobile charging vehicles delivered by algorithm `NMV_without_Eloss` decreases, and the gap between the solution and the lower bound of the optimal solution becomes smaller and smaller, which implies that the performance of algorithm `NMV_without_Eloss` is near optimal. On the other hand, the number of vehicles delivered by algorithm `NMV_without_Eloss` is up to 50% less than that by algorithm `Heuristic`.

We further investigate the impact of the variance among energy consumption rates of sensors on the performance of algorithms `NMV_without_Eloss`, `Heuristic`, and `minMCP` by varying ρ_{max} from 1mJ/s to 10mJ/s while fixing ρ_{min} at 1mJ/s. Figure 6 indicates that the number of mobile vehicles needed by each of the three algorithms, `NMV_without_Eloss`, `Heuristic`, and `minMCP`, decreases, followed by them slowly growing. The rationale behind this is that when the variance is quite small (i.e., the gap between ρ_{max} and ρ_{min} is small), the solution delivered by algorithm `NMV_without_Eloss` will include almost all sensors in each charging round, and thus a large number of mobile vehicles are required. With the increase on the variance, the number of to-be-charged sensors in each charging round significantly decreases. On the other hand, when the maximum energy consumption rate ρ_{max} becomes large, the average energy depletion rate of the sensors will be faster, and the solution by algorithm `NMV_without_Eloss` will include more sensors to be charged per charging round, as more sensors are within their critical lifetimes. In the following, we do not compare the performance of algorithm `minMCP`, as its performance is the worst one among the four algorithms, `LB_optimal`, `NMV_without_Eloss`, `Heuristic`, and `minMCP`, which has been shown in Figures 3 through 6.

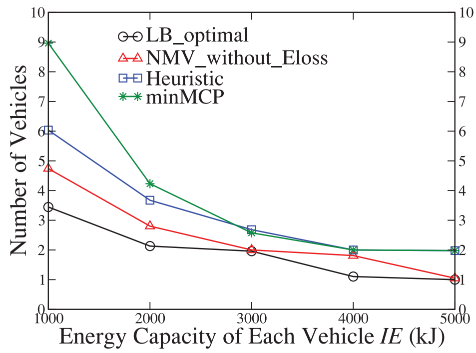


Fig. 5. Performance of algorithms `NMV_without_Eloss`, `Heuristic`, and `minMCP` by varying the energy capacity of each mobile vehicle *IE* when $n = 200$, $\rho_{min} = 1$mJ/s, and $\rho_{max} = 10$mJ/s.

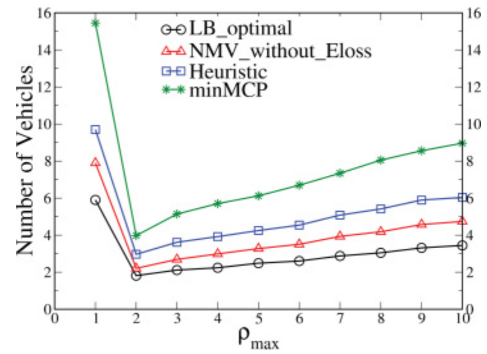
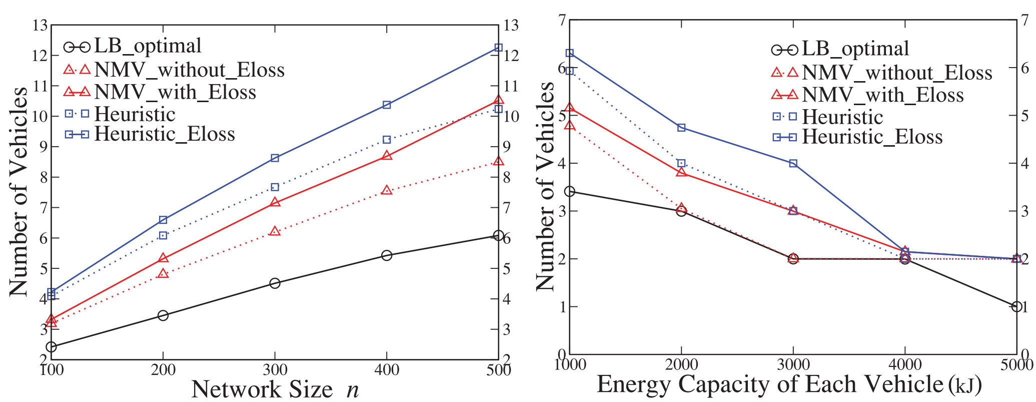


Fig. 6. Performance of algorithms `NMV_without_Eloss`, `Heuristic`, and `minMCP` by varying the maximum energy consumption rate ρ_{max} from 1mJ/s to 10mJ/s when $n = 200$, $IE = 1{,}000$kJ, and $\rho_{min} = 1$mJ/s.



(a) Network size n ($IE = 1{,}000$kJ) (b) Energy capacity of each vehicle IE ($n = 200$)

Fig. 7. Performance of algorithms `NMV_with_Eloss` and `Heuristic_Eloss` by varying network size n and energy capacity *IE* when $\rho_{min} = 50$mJ/s, $\rho_{max} = 100$mJ/s, and $\alpha = 3$.

We finally evaluate the performance of algorithms `NMV_with_Eloss` and `Heuristic_Eloss` under the assumption that sensor energy consumptions during each charging tour are taken into account against the ones of algorithms `NMV_without_Eloss` and `Heuristic` by varying network size n and energy capacity *IE* while keeping the high sensor energy consumption rates ($\rho_{min} = 50$mJ/s and $\rho_{max} = 100$mJ/s). Note that we omit the experimental results under the low sensor energy consumption rates (e.g., $\rho_{min} = 1$mJ/s and $\rho_{max} = 10$mJ/s), as the number of charging vehicles deployed by each of the four mentioned algorithms is almost identical in these two cases (i.e., with and without taking energy consumption during charging tours into consideration). Figure 7 illustrates the performance curves of different algorithms, from which it can be seen that the number of mobile vehicles delivered by algorithm `NMV_with_Eloss` is around 7% more than that by algorithm `NMV_without_Eloss`.

7.3. The Impact of α on Algorithmic Performance

We now evaluate the impact of critical lifetime interval parameter α on the performance of the proposed algorithms by varying the value of α from 1 to 7. A smaller α implies

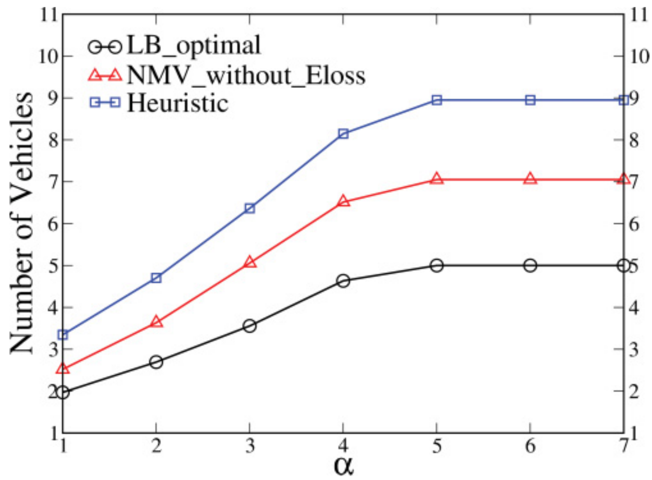


Fig. 8. Performance of algorithms `NMV_without_Eloss` and `Heuristic` by varying α when $n = 200$, $IE = 1{,}000$kJ, $\rho_{min} = 50$mJ/s, and $\rho_{max} = 100$mJ/s.

that more frequent schedulings are needed and fewer numbers of mobile vehicles are employed per charging round. With the growth of α, more and more sensors will be included in V, and more sensors will be charged by mobile charging vehicles per charging round. Figure 8 implies that with the growth of α, more charging vehicles are needed by algorithms `NMV_without_Eloss` and `Heuristic` in each charging round, as more sensor nodes fall in the defined critical lifetime interval. However, it is interesting to see that no more mobile vehicles are required when the value of α is greater than 6, as all sensors will be charged in each charging round.

8. CONCLUSION

In this article, we studied the use of the minimum number of mobile charging vehicles to charge sensors in a large-scale WSN so that none of the sensors will run out of energy, subject to the energy capacity constraint imposed on each mobile charging vehicle. We first proposed an on-demand energy charging paradigm for sensors. We then formulated the minimum vehicle deployment problem. Since the problem is NP-hard, we instead devised an approximation algorithm with a provable performance guarantee, assuming that the energy consumption of each sensor during each charging tour is neglected; otherwise, we proposed a novel heuristic by invoking the approximation algorithm iteratively. We finally conducted extensive experiments by simulations to evaluate the performance of the proposed algorithms. Experimental results demonstrate that the proposed algorithms are promising, and the solutions obtained by them are fractional of the optimal ones. In our future work, we will study the minimum vehicle deployment problem when the residual lifetime of each to-be-charged sensor is very short (i.e., less than l_{max}), for which we will devise new algorithms, and we believe that the charging order of sensors in each charging tour will be the key in the design of such algorithms.

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