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Online multicasting in WDM networks with shared light splitter bank

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Abstract We study online multicasting in WDM networks with shared light splitter bank. Our objective is either to maximize the network throughput or to minimize the blocking probability. Due to the nature of dynamic requesting for network resources by online multicast requests, the network usually is unable to allocate the resources needed for each request in advance. Instead, it either accepts the request by building an economic multicast tree for the request, in terms of the utilization of the network resources if it has sufficient resources available, or rejects the request, otherwise. It is desirable that the cost of realizing each multicast request be minimized, and the network throughput will be maximized ultimately through the cost saving on each individual request. Since optical light splitting and wavelength conversion switching in optical networks is cost expensive and its fabrication is difficult, it is assumed that only a limited number of light splitters and wavelength converters are installed at a node, which will be shared by all the incoming signals at the node. In addition, it is further assumed that only a fraction of nodes in the network are installed with such optical switches. In this article we first propose a cost model for realizing an online multicast request under such network environments with limited light splitters and wavelength converters, which models the cost of utilization of network resources, particularly in modeling the light splitting and wavelength conversion ability at nodes. We then show that finding a cost-optimal multicast tree for a multicast request under the proposed cost model is NP-complete, and instead devise approximation and heuristic algorithms for it. We finally conduct experiments to evaluate the performance of the proposed algorithms. The results show that the proposed algorithms are efficient and effective in terms of network throughput.

Keywords Multicast · Multicast routing and wavelength assignment · Shared light splitter bank · Optimal multicast trees · WDM networks · Approximation algorithms · Heuristic algorithms

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1 Introduction

Optical networks with Wavelength-Division Multiplexing (WDM) are now widely regarded as the most promising candidates for next-generation Internet, due to their ability to meet the ever-increasing huge bandwidth demands. A WDM network consists of nodes and fiber links, in which nodes are connected by optical fiber links. On each fiber link there are multiple distinct wavelengths carrying different data. Nodes are equipped with optical switches. An optical switch at a node is usually responsible for receiving optical signals from the incoming links and forwarding them to the outgoing links of the node. If optical signals from two incoming links of a node are forwarded to one of its outgoing links using the same wavelength, it will cause a wavelength collision, which can be resolved either by dropping one of the signals or by converting one of them to a different wavelength using a wavelength converter. It is obvious that the benefit of using wavelength conversion is that the blocking probability can be reduced by eliminating or reducing the effects of the so-called *wavelength continuity constraint*. In order to accommodate the unicast function in optical layer, some nodes in the network are equipped with optical crossconnect (OXC) devices, which can switch a optical signal from any input link to any output link, and make it possible to establish a *lightpath* between any pair of nodes. In order to accommodate the multicast function in optical layer, the *light-tree* concept was proposed [1], which requires that an incoming optical signal at an internal node in the tree can be split into multiple outgoing optical signals along the tree links. Thus, to support multicasting in an optical network, a fraction of the nodes in the network need to be equipped with Multicast-Capable OXC (MC-OXC) devices that can split an incoming optical signal into multiple identical outgoing optical signals. However, light splitters are the fundamental optical devices contributing to power loss [2]. Even in the ideal case the power of each output of a splitter is only $1/n$ of that of the input signal, where n is the fanout of the splitter. Some devices such as erbium-doped fiber amplifiers can be used to keep the power level of an optical signal above some threshold so that the signal is able to be detected. The use of amplifiers would increase the cost of WDM networks. It is predicted that the cost associated with OXC and MC-OXC devices will still be expensive in the near future [3]. Another factor increasing the cost of WDM networks is the use of wavelength converters that allow the wavelength of outgoing signals different from that of their incoming counterpart. Therefore, the number of light splitters and the number of wavelength converters in a network should be taken into account [2,4], and the light splitters and wavelength converters installed at a node should be shared by all its incoming signals in a power-efficient and cost-effective WDM network.

Light splitters are the key components to implement multicast. A *multicast request* typically involves the transport of information between a single sender (source) and multiple receivers (terminals). A special case of a multicast request is *broadcast*, where the set of receivers consists of all the nodes in the network except the source. Multicast applications includes video conferencing, entertainment distribution, remote educations, and distributed data processing, etc. [5,6]. Multicast routing and wavelength assignment (MC-RWA) is a fundamental problem in WDM networks, which aims at finding a set of links and wavelengths on these links to establish the connection from the source to the terminals. MC-RWA includes the building of a routing tree (light-tree) and the assignment of wavelengths to the links in the tree. Since the combined multicast routing and wavelength assignment is a hard problem, the most adopted strategy is to decouple the problem into two separate subproblems: the light-tree routing problem and the wavelength assignment problem [2,3,7]. The former aims to build a routing tree for a multicast request, while the latter aims to assign the available wavelengths to the links in the tree.

There are several studies on different MC-OXC switching architectures for multicasting in WDM networks with various objectives like minimizing the cost of establishing a network or maximizing the network throughout [2,3,8]. The

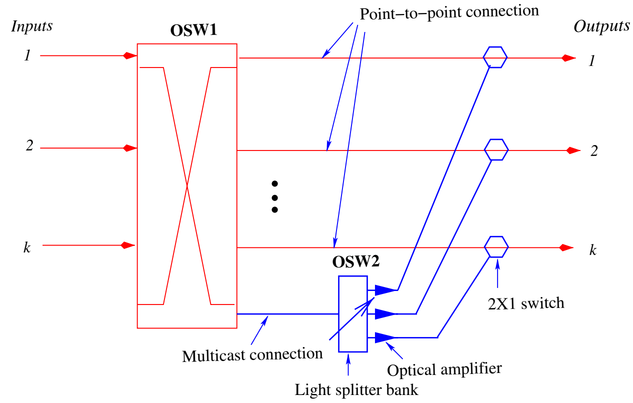

Fig. 1 Light splitter-sharing switch [2]

previous research focusing on the dedicated splitter switching (MC-OXC) requires a large number of light splitters and optical amplifiers, resulting in a network that is cost expensive and fabrication complex [3]. Instead, a new switching architecture called *light splitter-sharing bank* is proposed [2,3], as shown in Fig. 1, which is designed for low cost and power loss. The light splitters at a node are shared by all its incoming signals. The information on the incoming link is first demultiplexed into separate wavelength signals, which then are switched to outgoing links. Signals that do not need multicast are sent directly to the corresponding outgoing links by an optical subswitch OSW1, while those signals that need multicast are sent to another optical subswitch OSW2—the light splitter bank. The signals sent to the light splitter bank may be enhanced by signal amplification. The splitters then route different copies of an incoming signal to their outgoing links respectively. Due to splitter sharing, this architecture significantly reduces the cost of routing multicast requests and simplifies the fabrication complexity of splitting switches. In this article we will adopt this splitter sharing switching architecture and further assume that the MC-OXC and OXC nodes have also wavelength conversion ability.

1.1 Related work

Consider a multicast request with terminal set D in a WDM network. The objective is to find a cost-optimal multicast tree under different cost models to realize the request. Much effort on this problem has been taken in the past decade. For example, several studies have been carried out under the cost model in which the cost of a multicast tree is defined as the cost sum of wavelength conversion at nodes and wavelengths used at links, where different conversion costs are applied to different pairs of wavelengths at nodes, and different costs are charged by using different wavelengths to reflect the bandwidth consumption as well as the communication delay on links [9,10]. Sometimes, the routing congestion factor on links is also incorporated into the cost. Liang and Shen [9] proposed an approximation algorithm for the problem. Sahasrabuddhe and Mukherjee [1] approached the problem by formulating it into a mixed-integer linear programming. Chen and Wang [10] provided an exact solution to the problem in a very special network—the tree network, using dynamic programming. Znati et al. [11] dealt with the problem by decoupling the delay cost from the other cost of network resources, and presented several heuristic algorithms for finding a multicast tree meeting both delay and cost optimization objectives. Jia et al. [12] considered the routing congestion issue in a single hop (all-optical) network by proposing two heuristic algorithms for a multicast problem that aims to minimize the total cost of a multicast tree under the end-to-end delay constraint. Libeskind-Hadas and Melhem [13] investigated multicast communication in circuit-switched multihop networks by showing that it is polynomially solvable when the optimization objective is the wavelength assignment only, despite the fact that the general multicast problem is NP-hard. In addition, there have been several other studies for constructing constrained multicast trees in WDM networks. For example, Bermond et al. [14] investigated routing and wavelength assignment in WDM networks with only unicast-capable switches. Libeskind-Hadas [15] extended the unicast communication (point-to-point communication model) by proposing a multipath routing model, in which the multicast problem is to find a set of paths from the source to the destination nodes such that each path contains a subset of destination nodes, the nodes in the set of destination nodes are included by these paths, and the cost sum of these paths is minimized.

There are also several studies focusing on the physical constraints on optical switches like light splitting ability. Sahin and Azizolgu [16] considered the multicast problem under various fanout polices and Malli et al. [17] dealt with the problem under a sparse splitting model. Zhang et al. [8] considered it by focusing on the limited splitting power of optical switches, and provided several heuristic solutions. Xin and Rouskas [18] studied the splitting power loss in the signal propagation path by introducing the *split ratio of a node* concept, which represents the residual power of a light signal received at a node after the splits along the path, and a Balance-Light-Tree (BLT) algorithm for finding a multicast tree that meets the minimum power threshold is proposed. Zhang and Yang [7] considered the problem in a tree network with an objective of minimizing the number of wavelength conversions by providing an approximation algorithm for it. In addition, Rouskas [3] and Zhou and Poo [4] provided excellent surveys on the optical multicast problem under various cost models.

1.2 Motivations

Motivated by recent works on unicasting and multicasting in WDM networks with shared light splitter bank by Ali and Deogun [2], Zhang et al. [8], Rouskas [3], and Zhang and Yang [7], we here consider the online multicast routing and wavelength assignment problem in the networks in which light splitters and wavelength converters are installed only on a fraction of the nodes and shared by incoming signals. Since there are efficient algorithms for wavelength assignment in tree structures available [7,10], we focus on the routing problem under this shared light splitter bank architecture. Specifically, we consider the following online multicasting problem.

Assume that there is a sequence of multicast requests that is unknown in advance and the requests arrive one by one. Once a multicast request arrives, the response by the system to the request is to either realize the request by building a multicast tree for it or reject the request due to lack of network resources. The objective is to maximize the network throughput or minimize the blocking probability. Due to the unknown pattern of future requests, we focus on realizing each individual multicast request by building an economic multicast tree for the request.

1.3 Contributions

In this article, we approach the online multicasting in WDM networks with shared light splitter bank by building a series of optimal multicast trees for a sequence of multicast requests. We first propose a node cost model for the networks of concern to model the cost of light splitting/wavelength conversion resources and show that the optimal multicast tree problem is NP-complete. We then present approximation and heuristic algorithms for the problem. We finally conduct experiments by simulations to evaluate the performance of the proposed algorithms against that of two existing algorithms in terms of network throughput. The experimental results demonstrate that the proposed algorithms outperform the existing algorithms significantly.

2 Preliminaries

In this section, we first introduce a model of WDM networks with shared light splitter bank. We then propose a node cost model that characterizes the utilization of these network resources. We finally define the optimal multicast tree problem and the online multicast request maximization problem.

2.1 Shared light splitter bank model

The WDM network with shared light splitter bank is modeled by an *undirected graph* $G = (V, E, \Lambda, w)$, where V is a set of nodes (vertices), E is a set of bidirectional optical fiber links (edges), Λ is a set of wavelengths in G, and w is a function from V to $\mathbb{R}^+$, $n = |V|$, $m = |E|$, and $|\Lambda| = K$. Let $\Lambda = \{\lambda_1, \lambda_2, \ldots, \lambda_K\}$. Some of the nodes in the network are installed with MC-OXC devices. Associated with each MC-OXC node, a light splitter bank is shared by all its incoming links. The light splitting/wavelength conversion ability of a node is determined by whether any MC-OXC switches are installed and how many MC-OXC switches are installed at the node. In the dynamic routing (online setting), the light splitting/wavelength conversion ability of a node is also determined by the current traffic load at the node. The weight $w(v)$ at node v represents the light splitting/wavelength conversion ability of node $v \in V$. Associated with each link $e \in E$, there is a set $\Lambda(e)$ $(\subseteq \Lambda)$ of wavelengths available on e initially.

2.2 Node cost model

We propose a node cost model, which models the light splitting/wavelength conversion ability of each node. The weight $w(v)$ is inversely proportional to the light splitting/wavelength conversion ability at node v, that is, a larger value $w(v)$ of v means that the light splitting/wavelength conversion ability of v is weak, otherwise, a smaller value $w(v)$ of v indicates that the light splitting/wavelength conversion ability of v is quite high. For example, w can be such a function defined as follows: $w(v) = 1 - f(v)$ if $f(v) \neq 0$; otherwise $w(v) = \infty$, where $f(v)$ is the ratio of the residual light splitting/wavelength conversion ability to the initial light splitting/wavelength conversion capacity at v with $0 \leq f(v) \leq 1$. When $f(v) = 0$, all messages entering v will be trapped at v and there is no outgoing flow from v. Thus, we may set $w(v)$ to be a sufficiently large positive value, and v is unlikely to be included as an internal node into multicast trees. When $f(v) = 1$, which means that there is no traffic load at v at all or the light splitting/wavelength conversion ability of v is full, we may simply set $w(v) = 0$. For a node v with $0 < f(v) < 1$, its light splitting/wavelength conversion ability is limited. It can be seen that node v has some light splitting/wavelength conversion ability if $0 \leq w(v) < 1$. In order to ensure a multicast tree does not contain any unnecessary node v with $w(v) = 0$ as its internal node, each node v with $w(v) = 0$ can be assigned a small value, e.g., $w(v) = \epsilon = 1/(n+1)$, where n is the number of nodes in the network. Since each leaf node in the multicast tree only receives messages from its parent, no light splitting/wavelength conversion is needed at the node. For a given WDM network G and a multicast request, a multicast tree rooted at the source and spanning all the terminals is built if there are sufficient network resources to realize the request. The cost $C(T)$ of a multicast tree T in G is defined as the weighted sum of all the internal nodes in T. We refer to this model as *the node cost model*, which aims to be used in minimizing the utilization of the light splitting/wavelength conversion resources in the multicast tree per request.

2.3 Problem definition

The multicast tree for a given multicast request $(s; D)$ in G is such a tree rooted at s and spanning all the nodes in D that all its leaf nodes are terminals, where the source $s \in V - D$ and the terminal set $D \subset V$.

The optimal multicast tree for a given multicast request $(s; D)$ is such a multicast tree that the weighted sum of the internal nodes in the tree is minimized.

The optimal multicast tree problem is to find an optimal multicast tree for a given multicast request $(s; D)$ in G. The optimal multicast tree problem is referred to as *the optimal broadcast tree problem* when $D = V - \{s\}$.

The online multicast request maximization problem for a sequence of multicast requests is to maximize the number of the realized requests in the sequence until the system is unable to accommodate any further requests.

Due to the nature of unforeseen future requests, it is very difficult to provide an exact solution to the online multicast request maximization problem, instead, in this article, we focus on finding a cost-optimal multicast tree for each request under the node cost model. We must mention that we here deal with the WDM networks with shared light splitter bank, the availability of light splitters/wavelength converters at a node is the major concern and the link traffic load will not be taken into account in the node cost modeling.

3 Algorithms based on the node cost model

In this section, we first show that the optimal multicast tree problem under the proposed node cost model is NP-complete. We then provide approximation and heuristic algorithms for the problem of concern.

3.1 NP-hardness of the optimal multicast tree problem

In the following, we show that the optimal broadcast tree problem is NP-complete by a reduction from the maximum leaf spanning tree problem (MLST for short) in G, which is to find a spanning tree in G such that the number of leaves in the tree is maximized. MLST has been shown to be NP-complete [19]. In fact, in terms of computational hardness, the optimal broadcast tree problem and MLST are equivalent within polynomial time. In addition, the optimal broadcast tree problem is a special case of the optimal multicast tree problem, thus, the optimal multicast tree problem is also NP-complete.

Theorem 1 *The optimal broadcast tree problem in a WDM network $G(V, E, w)$ with shared splitter bank is not only NP-complete, but also complete for MAX-SNP.*

Proof Given an instance $G(V, E)$ of MLST and an integer k, the decision version of MLST is to determine whether there is a spanning tree in G such that the number of leaf nodes in the tree is no less than k.

We now construct an instance—a WDM network $G(V, E, w)$ of the optimal broadcast tree problem, where each node v in V has identical light splitting/wavelength conversion ability $w(v) = r > 0$. Let T_{opt} be the optimal broadcast tree in G and n_1 the number of leaf nodes in T_{opt}. Then, the weighted sum of the internal nodes in the tree is $r^*(n - n_1)$, which is the minimum when n_1 is maximized.

Given the instance $G(V, E)$ of MLST, we can see that there is a corresponding instance $G(V, E, w)$ of the optimal broadcast tree problem with an integer $r^*(n - k)$, and thus there is a broadcast tree in $G(V, E, w)$ such that the weighted sum of its internal nodes is no more than $r^*(n - k)$.

Clearly, to verify whether a given tree is a solution to the optimal broadcast tree problem can be done within polynomial time. Thus, the optimal broadcast tree problem is NP-complete. It is easy to show that the optimal broadcast tree problem and the MLST problem are equivalent in terms of computational complexity under polynomial time reduction. It has been shown that the MLST problem is not only NP-complete [19], but also complete for MAX-SNP [20], which means that it does not permit a fully polynomial-time approximation schema unless $P = \text{NP}$ [21]. Thus, it is unlikely to have a fully approximation schema to the optimal broadcast tree problem unless $P = \text{NP}$. □

3.2 A simple approximation algorithm

Due to the NP-hardness of the optimal multicast tree problem, we provide a simple approximation algorithm for it, which is referred to as *algorithm* `SA`.

The edge-weighted directed Steiner tree problem for a source s and a terminal set D is to find a tree in G rooted at s and spanning all the nodes in D such that the weighted sum of the edges in the tree is minimized.

Now, we approach the optimal multicast tree problem by reducing it to the edge-weighted directed Steiner tree problem for the source s' and the terminal set D' in an auxiliary directed graph $G'(V', E', w')$, which is defined as follows:

$$V' = \{v_1, v_2 \mid v \in V\},$$
$$E' = \{\langle v_1, v_2\rangle \mid v \in V\} \cup \{\langle v_2, u_1\rangle, \langle u_2, v_1\rangle \mid (u, v) \in E\},$$
$$w'(\langle v_1, v_2\rangle) = w(v) \text{ and } w'(\langle v_2, u_1\rangle) = w'(\langle u_2, v_1\rangle) = 0,$$
$$s' = s_1,$$
$$D' = \{v_1 \mid v \in D\}.$$

Theorem 2 *Given a WDM network $G(V, E, w)$, a source s and a terminal set D, $s \in V - D$, $D \subset V$, assume that $G'(V', E', w')$ is the corresponding auxiliary graph of G. Let T' be a solution to the edge-weighted directed Steiner tree problem for s' and D' in G'. Then, T is a solution to the optimal multicast tree problem for $(s; D)$, where $V(T) = \{v|v_1 \in V(T')\}$ and $E(T) = \{\langle v, u\rangle | v_1$ is an internal node in T' and $\langle v_2, u_1\rangle \in E(T')\}$.*

Proof If v_1 is an internal node in T', then $v_2 \in V(T')$ because $\langle v_1, v_2\rangle$ is the only edge starting from v_1 in G'. Since $v_2 \notin D'$, v_2 is not a leaf node in T', then there exists a node u_1 in T' such that $\langle v_2, u_1\rangle \in E(T')$. Thus, T is a tree.

If $\langle v_1, v_2\rangle$ is an edge in T', v is an internal node in T. Then, we have $C(T) = W(T')$, where $C(T)$ is the weighted sum of the internal nodes in T, and $W(T')$ is the weighted sum of the edges in T'. Now, we prove that $C(T)$ is minimized. If there is another tree T_1 rooted at s and spanning all the nodes in D and $C(T_1) < C(T)$. We define T_1' as follows.

$$V(T_1') = \{v_1|v \in V(T_1)\} \cup \{v_2 \mid v \text{ is an internal node in } T_1\},$$
$$E(T_1') = \{\langle v_1, v_2\rangle, \langle v_2, u_1\rangle \mid \langle v, u\rangle \in E(T_1)\}.$$

Then, T_1' is a tree in G' rooted at s' and spanning all the nodes in D', and $W(T_1') = C(T_1)$. Thus, we have $W(T_1') = C(T_1) < C(T) = W(T')$, which contradicts to the assumption that T' is a solution to the edge-weighted directed Steiner tree problem for s' and D'. □

Following Theorem 2, an approximation solution to the edge-weighted directed Steiner tree problem in G' can be transformed into an approximation solution to the optimal multicast tree problem in G. It is known that the best possible approximation solution for the directed Steiner tree problem so far is $O(|D'|^\delta)$ times of the optimum [22], where δ is a constant with $0 < \delta \leq 1$, $|D'| = |D|$. We thus have the following theorem.

Theorem 3 *Given a WDM network $G(V, E, w)$ with shared light splitter bank and a multicast request $(s; D)$, there is an approximation solution to the optimal multicast tree problem, which is $O(|D|^\delta)$ times of the optimum, where δ is a constant with $0 < \delta \leq 1$.*

3.3 A heuristic algorithm

In the following sections, we propose a heuristic algorithm for the optimal multicast tree problem. The proposed heuristic is similar to the approximation algorithm for the node-weighted Steiner tree problem, referred to as *algorithm* `KR`, by Klein and Ravi [23] but with some important modifications. *The node-weighted Steiner tree problem* is to find a tree in G spanning all the nodes in terminal set D such that the weighted sum of the nodes in the tree is minimized.

The algorithm KR maintains a forest F that consists of a node-disjoint set $\{T_1, T_2, \ldots, T_k\}$ of trees and contains all the terminals, $1 \le k \le |D|$. Initially, each terminal by itself is a tree. The algorithm uses a greedy strategy to iteratively merge several current trees into a larger tree until there is only one tree left in the forest. In each iteration, the algorithm selects a node and a subset of the current trees of size at least two so as to minimize the ratio

$$\frac{w(v) + \sum_{T_j \in S} d(v, T_j)}{|S|}, \quad (1)$$

where $S \subseteq F$, $|S| \ge 2$, $d(v, T_j)$ is the distance from v to T_j. The distance along a path in algorithm KR does not include the weights of the two endpoints of the path. Thus, the choice minimizes the average node-to-tree distance. The algorithm uses the shortest paths between the node and the selected trees to merge the trees into a larger one.

In order to implement each iteration, for each node v, the *quotient cost* of v is defined to be the minimum value of (1), taken over all subsets of the current trees of size at least two. In order to find the quotient cost of v, the algorithm computes the distance d_j from v to each T_j, assuming without loss of the generality that the trees are numbered so that $d_1 \le d_2 \le \ldots \le d_k$. In computing the quotient cost of v, it is sufficient to consider subsets of the form $\{T_1, T_2, \ldots, T_i\}$, $2 \le i \le k$. The quotient cost for a given node can be calculated in polynomial time, and the minimum quotient cost can then be determined. Thus, each iteration can be carried out within polynomial time. The solution delivered by the algorithm for node-weighted Steiner tree problem is $2 \ln |D|$ times of the optimum. Note that the approximation of the solution is within a constant factor of the best possible approximation achievable in polynomial time unless $\tilde{P} \supseteq NP$ [24]. Guha and Khuller [25] later provided an improved algorithm for the problem with a better approximation ratio at the expense of a longer running time. Their improved algorithm delivers a solution within $1.35 \ln |D|$ times of the optimum.

It should be emphasized that the problem we deal with is different from the one discussed by Klein and Ravi [23], despite there being some similarities between them. Their approximation analysis is based on an assumption that the weight of each terminal is zero, since all the terminals will be included into the Steiner tree. However, for the optimal multicast tree problem, we treat each terminal differently, depending on whether or not it is an internal node in a multicast tree. If it is, its node weight should be taken into account; otherwise its node weight can be ignored because a leaf node is only a receiver of messages and no light splitting/wavelength conversion is needed at it. Thus, the solution delivered by algorithm KR is not an approximation solution for the optimal multicast tree problem.

Now, we propose a heuristic for the optimal multicast tree problem based on some modifications to algorithm KR. The differences between our heuristic and algorithm KR are at the following crucial steps in defining the length of a path between two nodes and calculating the quotient cost of a node.

Assume that there are k trees $T_1, T_2, \ldots, T_k$ currently, $k \le |D|$. In order to compute the quotient cost of a given node v, we need to compute the distance from v to T_j, which is in turn reduced to computing the *length* of the shortest paths between v and every node u in T_j. In our algorithm, the length of a path between v and a tree node u is the weighted sum of the nodes in the path except u and v if u is not a leaf node in T_j or $T_j = \{u\}$; otherwise, the length of the path is the weighted sum of all the nodes in the path except v. While computing the quotient cost of a node v in (1) in our algorithm, $w(v)$ is not taken into account if v is an internal node in one of the current trees; otherwise $w(v)$ is included in the calculation of the quotient cost of v. We refer to this heuristic as *algorithm* MKR.

4 Performance study

In this section, we evaluate the performance of the proposed approximation algorithm SA and heuristic algorithm MKR against that of two existing algorithms KR and SPT by conducting experimental simulations, where SPT is the edge-weighted shortest path tree algorithm in which each edge has identical weight. We use network throughput as the main metric in our simulations, where network throughput is the number of the realized multicast requests in a given sequence. We found that the performance of the proposed algorithms MKR and SA is much better than that of algorithms KR and SPT.

4.1 Simulation environment

We assume that 100 nodes are deployed randomly in a region of $10 \times 10\,\mathrm{m}^2$ using the NS-2 simulator. For each pair of nodes u and v, a random number $r_{u,v}$ is generated, $0 \le r_{u,v} < 1$. Whether or not u and v are connected is determined by $r_{u,v}$ and the edge probability [26,27]

$$P(u, v) = \beta e^{\frac{-d(u,v)}{L\alpha}},$$

where $d(u, v)$ is the Euclidean distance between u and v, L is the maximum distance between nodes in the region, and α and β are the parameters governing the edge density in the network, $0 < \alpha, \beta \le 1$. There is an edge between u and v if and only if $r_{u,v} < P(u, v)$. Different values of α and

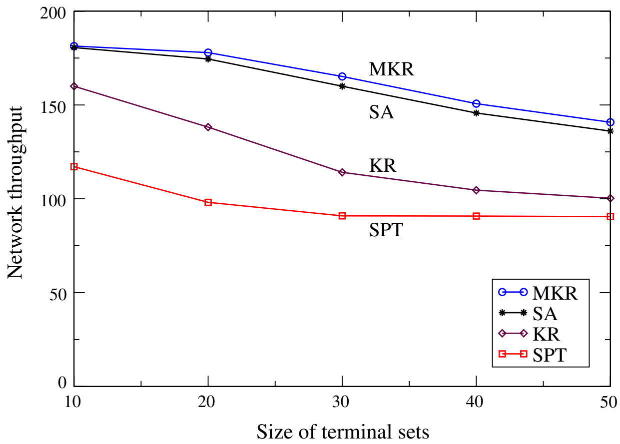

Fig. 2 Comparison of the network throughputs with $\alpha = 0.3$ and $\beta = 0.3$

β result in different network topologies even with the same node distribution.

We assign a weight to each node in the network to model its light splitting/wavelength conversion ability. Initially, the weight $w(v)$ is a random number between zero and one. $w(v)$ increases by c if v is an internal node of the multicast tree built for a multicast request that consumes light splitting/wavelength conversion resources of amount c. Node v has no light splitting/wavelength conversion ability when its current weight is greater than or equal to one.

We assume that there is a sequence of randomly generated multicast requests that is unknown in advance. The multicast requests in the sequence arrive one by one and are processed successively. Once a multicast request arrives, it must be realized by building a multicast tree for it if there are sufficient resources available or rejected otherwise. The sequence of multicast requests consists of 200 requests. Each request consists of the source, the terminal set, the arrival time, the duration and the cost consumption. We randomly select the source and the terminal set for a multicast request, and the size of the terminal set ranges from 10 to 50 with increments of 10. We also assume that each multicast request lasts for a period of time and consumes a certain amount of light splitting/wavelength conversion resources. For simplicity, we further assume that the consumption of these resources is identical for all the internal nodes in a multicast tree.

We simulated various algorithms on 10 different randomly generated network topologies for different problem size. For each size of the network instance, the value shown in the graphs is the mean of 10 individual values obtained by running each algorithm on these 10 randomly generated network topologies.

4.2 Performance evaluation of various algorithms

We first evaluate the network throughputs delivered by different algorithms with various sizes of the terminal sets when

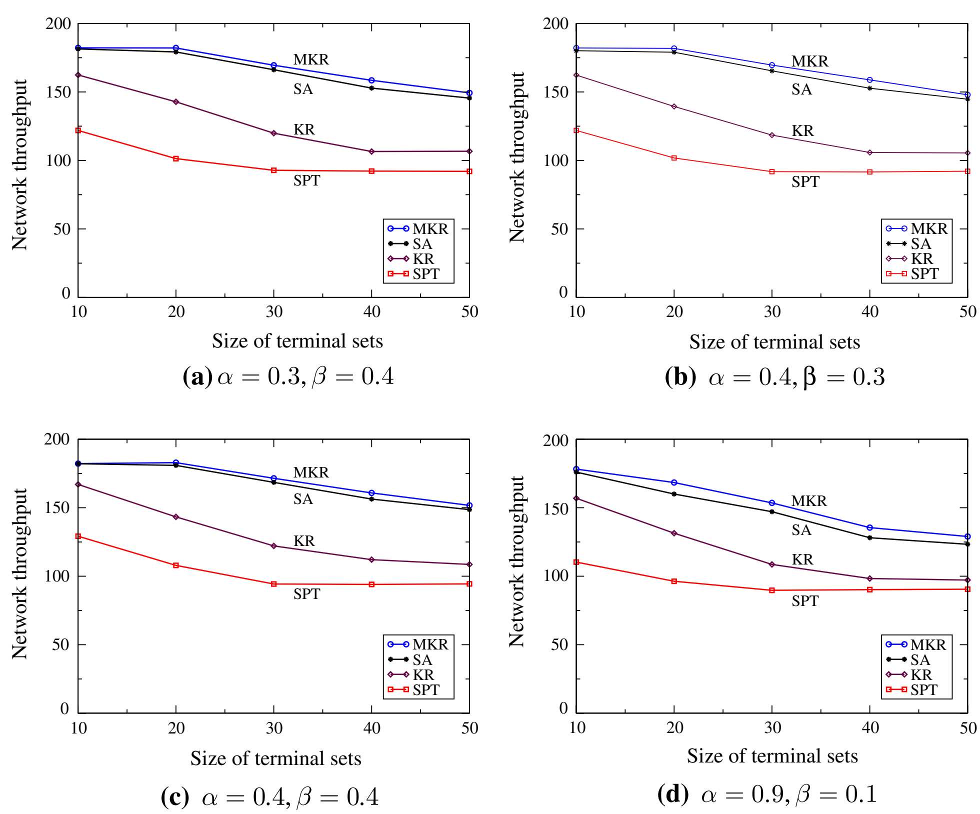

Fig. 3 Comparison of the network throughputs with various values of α and β

$\alpha = 0.3$ and $\beta = 0.3$. It can be seen from Fig. 2 that algorithms MKR and SA outperform algorithms KR and SPT significantly with the sizes of the terminal sets varying from 10 to 50 with increments of 10. When the terminal set consists of 10 nodes, more than 90% of the requests can be realized by algorithms MKR and SA, whereas the realization ratios of algorithms KR and SPT are only 80% and 60%, respectively. In addition, the network throughput delivered by algorithm KR drops faster than those delivered by the other algorithms with growth of the size of the terminal sets. When the size of the terminal sets reaches 50, the realization ratio of algorithm KR is only 50%, whereas algorithms MKR and SA can still realize around 70% of the requests. It also can be observed from Fig. 2 that the network throughput delivered by algorithm MKR is always greater than that delivered by algorithm SA for various sizes of terminal sets. We then change the edge density in various network topologies by varying the values of α and β. It is indicated in Fig. 3 that there is no significant difference among the algorithms in terms of the performance, compared with the case where $\alpha = 0.3$ and $\beta = 0.3$, i.e., the performance of algorithms MKR and SA is constantly better than that of algorithms KR and SPT.

5 Conclusions

In this article we have studied online multicasting in WDM networks with shared light splitter bank aiming at maximizing the network throughput. We first proposed a node cost model for multicast trees that models the cost of utilizing the network resources such as light splitters/wavelength converters at nodes. We then showed that finding a cost-optimal multicast tree under the proposed cost model is NP-complete, and instead devised approximation and heuristic algorithms for finding such cost-optimal multicast trees. We finally conducted experiments to evaluate the performance of the proposed algorithms. The experimental results show that the proposed algorithms are efficient and effective in terms of network throughput.

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