Using Modified Alpha-cut Based Fuzzy Interpolation in Petrophysical Properties Prediction

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ABSTRACT: *In petrophysical properties prediction, fuzzy rules are normally set up using some fuzzy rule extraction techniques. After the fuzzy rule base has been set up, it is then used to perform prediction at uncored depths or other wells around the region. However, when generating fuzzy rules from the available core data, it may result in a sparse fuzzy rule base. This is the problem of missing rules when used to make prediction, which is caused by a fuzzy rule set that does not cover the whole universe of discourse leaving gaps in-between. Fuzzy rule interpolation techniques have been well established to solve the problems created by the sparse rule base. This paper presents the use of the modified α -cut based fuzzy interpolation (MACI) technique to interpolate the membership function. A case study and results will also be presented in this paper. This work will extend the applicability of fuzzy systems in well log analysis.*

Introduction

The cost of developing a petroleum reservoir now requires exploration to be a very carefully managed and controlled process. The initial phase normally involves a series of wells being drilled at different locations around the region believed to hold the reservoir. Then, well logging instruments are lowered into each well to collect data, typically every 150mm or so of depth. These data are known in the industry as well log data. After this follows a very intense processing of this data in order to commence an evaluation of the reservoir's potential. Well logging instruments used for this data acquisition broadly fall into three categories: electrical, nuclear and acoustic [1]. Gamma Ray (GR), Resistivity (RT), Spontaneous Potential (SP), Neutron Density (NPHI) and Sonic interval transit time (DT) are examples of the measurements obtained.

Physical rock samples from various depths are obtained by using a coring barrel to recover intact cylindrical samples of reservoir rock. These samples are then sent to a laboratory and examined using various physical and chemical processes. Data obtained from this phase are known as core data in the log analysis process. Although core data is the most accurate way of assessing the hydrocarbon of a well, they are very difficult and expensive to obtain. Means of providing good prediction of the petrophysical properties is necessary to avoid spending excessive amounts of money on coring. Therefore it is important to establish an accurate well log data analysis procedure to provide reliable information for the log analyst. Two key issues in reservoir evaluation using well log data are the characterisation of formation and the prediction of petrophysical properties. Examples of petrophysical properties are porosity, permeability and volume of clay. While a core data set gives an accurate picture of the petrophysical properties at specific depths, it takes a lengthy process and incurs great expense to obtain such data. Hence, only limited core data are available at selected wells and depths. The objective of well log data analysis is therefore to establish an accurate interpretation model which can be used to predict the petrophysical properties for uncored depths and wells around that region [2,3].

Recently, a method that can express the underlying characteristics of a system in human understandable rules (known as Fuzzy Logic (FL)) is used to establish an interpretation model [4,5]. A fuzzy set allows for the degree of membership of an item in a set to be any real number between 0 and 1 [6]. This allows human observations, expressions and expertise to be modelled more closely. Once the fuzzy sets have been defined, it is possible to use them in constructing rules for fuzzy expert systems and in performing fuzzy inference. This approach seems to be suitable to well log analysis as it allows the incorporation of intelligent and human knowledge to deal with each individual case. However, the extraction of fuzzy rules from the data can be difficult for analysts with little experience. This could be a major drawback for use in petrophysical properties prediction. However, if a fuzzy rule extraction technique can be used, then fuzzy systems can still be used for permeability determination [4,5].

In most fuzzy extraction techniques, the fuzzy rule base is set up using any available core data. Quite often, the core data provided are not enough to construct a complete and comprehensive fuzzy rule base. Depending on the nature of the wells, in some cases a fuzzy rule base contains gaps, which is a sparse rule base and classical fuzzy reasoning methods can no longer be used. This is due to the lack of an inference mechanism in the case when observations find no fuzzy rule to fire in uncored depths or wells around the region [7]. This is undesirable when using a fuzzy interpretation model. If more than half the input instances in the prediction well cannot find any rule to fire, this interpretation model is considered useless. This paper examined the practical application of the modified α-cut based fuzzy interpolation (MACI) technique to interpolate the membership function [8,9]. A case study and results have shown that with the use of MACI, sparse fuzzy rule bases generated from the core data can still be used for petrophysical properties prediction.

Modified Alpha-Cut Based Fuzzy Rule Interpolation

Fuzzy rule interpolation techniques provide a tool for specifying an output fuzzy set whenever at least one of the input spaces is sparse. Kóczy and Hirota [10] introduced the first interpolation approach known as (linear) KH interpolation. This is based on the Fundamental Equation of Rule Interpolation (refer to equation (1)). This method determines the conclusion by its α-cuts in such a way that the ratio of distances between the conclusion and the consequents should be identical with that among observation and the antecedents for all important α-cuts (breakpoint levels). This is shown in the equation as follow (refer to Figure 1 for notations):

$$d(A^*, A_1) : d(A^*, A_2) = d(B^*, B_1) : d(B^*, B_2) \qquad (1)$$

The KH interpolation possesses several advantageous properties. Firstly, it behaves approximately linearly between the breakpoint levels. Secondly, its computational complexity is low, as it is sufficient to calculate the conclusion for the breakpoint level set. Moreover, its extension is found to be a universal approximator [11]. However, for some input situation it fails to results in a directly interpretable fuzzy set, because the slopes of the conclusion can collapse as shown in Figure 1**.** Several approaches were proposed in the last decade to alleviate this inconvenience [12, 13, 14, 15]. These approaches either determine conditions with respect to the input sets [12, 13] or implement conceptually different methods to avoid abnormal conclusions [14, 15]. The new concepts, however, do not preserve the low computational complexity of the original KH method. Recently, a modification of the original method has been proposed which solves the problem of abnormal conclusions while maintaining its advantageous properties [8, 9]. This is known as the modified α-cut based fuzzy interpolation (MACI).

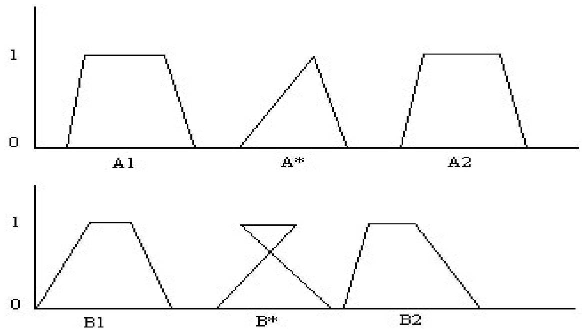

Figure 1: Problem of linear KH fuzzy interpolation.

MACI works with the vector description of fuzzy sets. The fuzzy set A is represented by a vector $a = [a_{-m}, ..., a_0, ..., a_n]$ where $a_k (k \in [-m, n])$ are the characteristic points of A and a_0 is the reference point of A with membership degree one. This means that $a_L = [a_{-m}, ..., a_0]$, and $a_R = [a_0, ..., a_n]$ are the left flank and right flank of A, respectively. Coordinate transformation is used to avoid the abnormality. The basic idea of the method is that it transforms the space of the consequent sets to another space, where any abnormality can be excluded. The calculation of the conclusion is performed in the transformed space, and finally, the resulting set is transformed back to the original space.

The left and right flanks of the conclusion are calculated separately, but their calculations are similar. E.g., for the right flank, the coordinates of the conclusion can be obtained as:

$$b_k^* = {}^{KH}b_k^* + \sum_{i=0}^{k-1} (\lambda_i - \lambda_{i+1})(b_{2i} - b_{1i}) \qquad (2)$$

$$k \in [0, n],$$

and as for the left flank as:

$$b_k^* = {}^{KH}b_k^* + \sum_{i=0}^{k-1} (\lambda_{-i} - \lambda_{-(i+1)})(b_{2i} - b_{1i}) \qquad (3)$$

$$k \in [-m, 0],$$

where b_{1i}, and b_{2i} are the ith coordinate of the consequent B_1, and B_2, respectively; furthermore

$$\lambda_i = \frac{a_i^* - a_{1i}}{a_{2i} - a_{1i}}$$

are the ith ratio factor derived from the appropriate coordinates of the A_1, A_2, and A_*.

$${}^{KH}b_k^* = (1 - \lambda_k) b_{1k} + \lambda_k b_{2k}$$

is the value of the kth coordinate calculated by the α-cut based original KH approach. In this case, since only triangular membership functions are concerned. Due to the formula (2) and (3), the left and right flanks of the conclusion are connected at the reference point, b_0^*. Under this situation, the MACI will only a yield singleton conclusion if and only if the consequents are singleton.

With the above interpolation characteristics of the modified α-cut based fuzzy interpolation technique, any input instances which fall into the gaps in between the rule antecedents could provide some form of interpolated results. This will not only ensure that all input variables can generate reasonable output predictions, it could also expand the usage of the fuzzy rules inference system to be used in the field of well log analysis.

Self-generating Fuzzy Rules Inference System

The self-generating fuzzy rules inference system [4] has shown successful results in establishing the petrophysical properties prediction model, and is used to extract fuzzy rules in the case study presented in this paper. This fuzzy extraction technique basically is to aid the user in setting up a fuzzy rules interpretation model by mapping the available core data to their corresponding memberships. After this has been done, the log analyst can examine the interpretation model from the fuzzy rules. The log analyst can then modify and add-on to the rule base easily. The steps involved in the self-generating fuzzy rules inference system are summarised as follows:

(1) Normalise the data between 0 and 1 by using linear or logarithmic transformations depending on the nature of the well log data.
(2) Define the number of fuzzy regions and fuzzy terms for all data. For ease of extraction, only triangular types of membership functions are used.
(3) The space associated with each fuzzy term over the universe of discourse for each variable is then calculated and divides them evenly.
(4) For each available core data, a fuzzy rule is established by directly mapping the physical value of the variable to the corresponding fuzzy membership function.
(5) Go through Step (4) with all the available core data and generate one rule for each input-output core data pair.
(6) Eliminate repeated fuzzy rules.
(7) The set of remaining fuzzy rules together with the centroid defuzzification algorithm now forms the fuzzy interpretation model.

Case Study and Results

In this case study, data from two wells in the same region are used. The input well logs used in this case study are gamma ray (GR), deep induction resistivity (ILD) and sonic travel time (DT). They are used to predict the petrophysical property, porosity (PHI). Core data from one well are used to establish a prediction model based on the self-generating fuzzy rules inference system. The model is then used to predict the porosity in the second well. All the variables are normalised between the values of 0 and 1. The first well has a total of 71 core data and is used to establish the fuzzy rules. The second well has 51 core data and is used as the testing well to test the prediction accuracy. A few membership functions (3,5,7,9) have been tested, and 9 membership functions appear to give the best prediction results. The total number of rules extracted from the training well is 63. After all the fuzzy rules have been set up, the input instances from the second well are used to infer the predicted PHI. Figure 2 shows the output plot of the predicted testing output (solid line on the plot) as compared to the core data (dots on the plot) in the second well. Figure 3 shows the warning message when no rule is fired for the two input instances around point 31 in Figure 2.

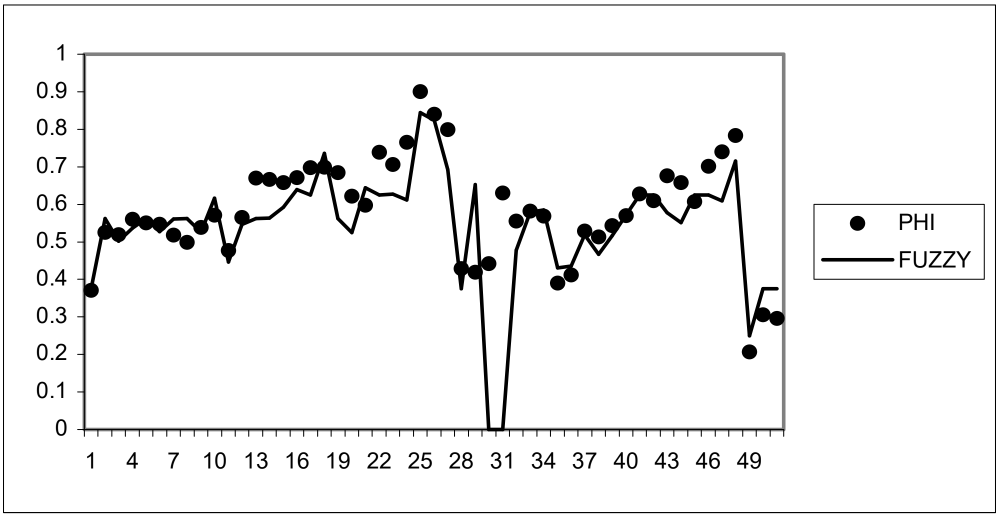

Figure 2: Output plot of testing well (Well 2) without MACI.

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Warning: no rule is fired for input [0.271000 0.367000 0.506000 ]!
0 is used as default output!

Warning: no rule is fired for input [0.360000 0.322000 0.599000 ]!
0 is used as default output!
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Figure 3: Warning message for input instances without rule to fire.

After the two input instances have been picked up which do not have any fuzzy rule to fire, the nearest fuzzy rules in the established fuzzy rule base need to be selected. From the observation and Euclidean distance measured on each input variable, the nearest fuzzy rules of the two input instances are determined for use by MACI. For ease of manipulation, all the values have been normalised between 0 and 100 when performing fuzzy rule interpolation.

The parameters used to interpolate the first input instance are as follow (refer to Figure 4):

A_{11}: 25, 38, 38, 50	A_{12}: 0, 13, 13, 25	A_1*: 27, 27, 27, 27
A_{21}: 50, 63, 63, 75	A_{22}: 13, 25, 25, 38	A_2*: 37, 37, 37, 37
A_{31}: 63, 75, 75, 88	A_{32}: 25, 38, 38, 50	A_3*: 51, 51, 51, 51
B_1: 38, 50, 50, 63	B_2: 25, 38, 38, 50	

After the MACI processing, the interpolated conclusion, B*: 30, 42, 42, 54

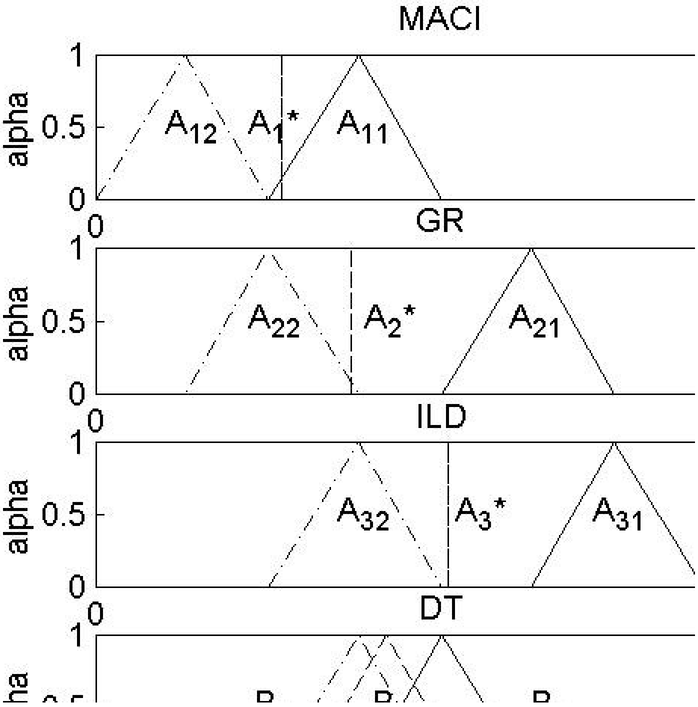

Figure 4: Fuzzy Interpolation for the first input instance.

The parameters used to interpolate the second input instance are as follow (refer to Figure 5):

A_{11}: 25, 38, 38, 50 A_{12}: 25, 38, 38, 50 A_1*: 36, 36, 36, 36
A_{21}: 38, 50, 50, 63 A_{22}: 13, 25, 25, 38 A_2*: 32, 32, 32, 32
A_{31}: 38, 50, 50, 63 A_{32}: 63, 75, 75, 88 A_3*: 60, 60, 60, 60
B_1: 25, 38, 38, 50 B_2: 38, 50, 50, 63

After the MACI processing, the interpolated conclusion, B*: 33, 45, 45, 58

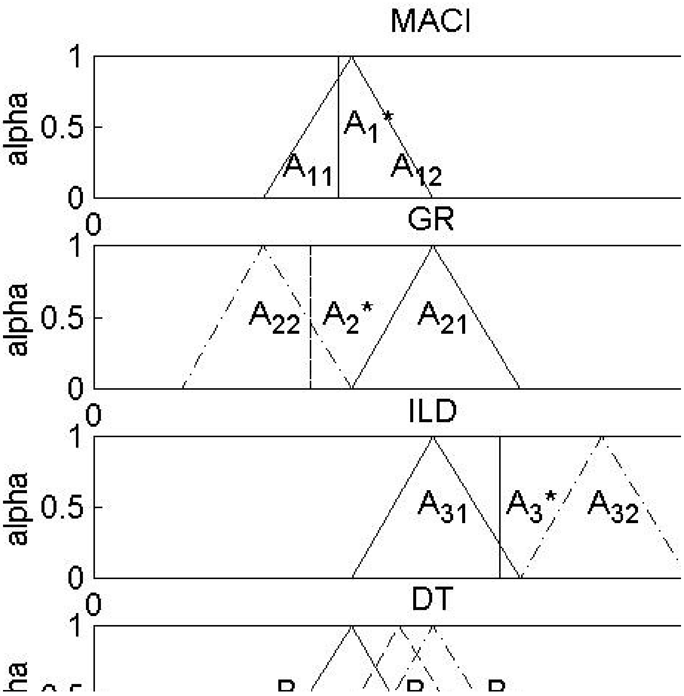

Figure 5: Fuzzy Interpolation for the second input instance.

After the fuzzy rules have been interpolated, the two input instances leads to two interpolated fuzzy rules added into the original fuzzy rule base. This time, all the input instances can find rules to fire, as all the available fuzzy rules together with the two interpolated fuzzy rules can cover all the input instances. The graphical plot of the predicted PHI as compared to the core PHI are shown in Figure 6.

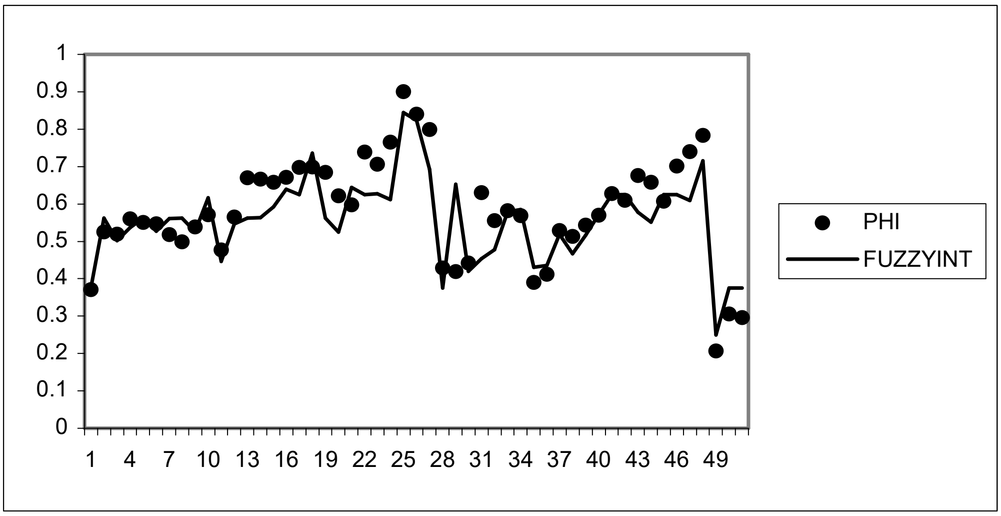

Figure 6: Output plot of testing well (Well 2) with MACI.

The prediction accuracy for this case study is calculated using the correlation factor as follow, the results are shown in Table 1.

$$\ell_{x,y} = \frac{\text{cov}(X,Y)}{\sigma_x . \sigma_y} \qquad (4)$$

where $-1 \leq \ell_{x.y} \leq 1$

and $$\text{cov}(X,Y) = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu_x)(y_i - \mu_y)$$

Table 1: Comparison of prediction accuracy

Prediction Measure	**Predicted PHI and core PHI for Well 2 (without MACI)**	**Predicted PHI and core PHI for Well 2 (with MACI)**
Correlation factor	0.6545	0.8603

From Table 1, we can see that with the assistance of MACI, the correlation factor of the predicted PHI as compared to the core PHI has increased quite significantly. This is also partly due to the default prediction output for the two input instances are set to zero. In this case study the number of input instances that cannot find any fuzzy rule to fire is considered minimal. In cases where more than half input instances in the prediction well cannot find any rule to fire, the fuzzy interpretation model could not be used for petrophysical properties prediction at all. With the fuzzy rule interpolation technique, MACI, the number of fuzzy rules is considered the same, as the extra two fuzzy rules added into the system later are interpolated. However, the prediction ability has improved. This is a desirable characteristic for fuzzy petrophysical properties prediction, as an increase in the number of fuzzy rules would result in an increase in complexity which would make the examination of the fuzzy rule base more difficult.

Conclusion

In this paper, the practical applicability of the fuzzy rule inference system with a rule extraction technique used in petrophysical properties prediction has been examined. The problem of a sparse rule base and insufficient core data may cause undesirable prediction outcomes. This is mainly due to input instances that could not find any rule in the fuzzy rule base. To provide a solution to this problem, the modified α-cut based fuzzy interpolation (MACI) technique has been applied. This method can be used to interpolate the gaps between the rules. This ensures that the set of sparse fuzzy rules generated by the fuzzy rule extraction technique will be useable in a practical system. This is significant as this will allow the use of a fuzzy system as an alternative for petrophysical properties prediction, at the same time without increasing the number of fuzzy rules that allows more human control.

Acknowledgment

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References

[1] M., Rider (1996) *The Geological Interpretation of Well Logs*, Second Edition, Whittles Publishing.
[2] E.R., Crain (1986) *The Log Analysis Handbook Volume 1: Quantitative Log Analysis Methods*, Penn Well Publishing Company.
[3] G.B., Asquith, and C.R., Gibson (1982) *Basic Well Log Analysis for Geologists*, The American Association of Petroleum Geologists.
[4] C.C., Fung, K.W., Wong, and P.M., Wong (1997) “A Self-generating Fuzzy Rules Inference Systems for Petrophysical Properties Prediction,” *Proceedings of IEEE International Conference on Intelligent Processing Systems*, Beijing, pp. 205 – 208.
[5] H., Kuo, T.D., Gedeon, and P.M., Wong (1999) "A Clustering Assisted Method for Fuzzy Rule Extraction and Pattern Classification," *Proceedings of 6th International Conference on Neural Information Processing*, vol. 2, Perth, pp. 679-684.
[6] L.A., Zadeh (1965) "Fuzzy Sets," *Information and Control*, vol. 8, pp. 338-353.
[7] K.W., Wong, and T.D., Gedeon (2000) "Petrophysical Properties Prediction Using Self-generating Fuzzy Rules Inference Systems with Modified Alpha-cut Based Fuzzy Interpolation," *The Seventh International Conference of Neural Information Processing ICONIP 2000*, Korea, in press.
[8] Y., Yam, P., Baranyi, D., Tikk, and L.T., Kóczy (1999) "Eliminating the Abnormality Problem of α-cut Based Interpolation," *Proceedings of the 8th IFSA World Congress*, vol. 2, pp. 762-766.
[9] D., Tikk, and P., Baranyi (2000) "Comprehensive Analysis of a New Fuzzy Rule Interpolation Method," *IEEE Transactions on Fuzzy Systems*, vol. 8, no. 3, pp. 281-296.
[10] L.T., Kóczy, and K., Hirota (1993) "Approximate Reasoning by Linear Rule Interpolation and General Approximation," *Int. J. Approx. Reason*, vol. 9, pp.197-225.
[11] D., Tikk, I., Joó, L.T., Kóczy, P., Várlaki, B., Moser, and T.D., Gedeon, "Stability of Interpolative Fuzzy KH-Controllers," *Fuzzy Sets and Systems*, in press.
[12] L.T., Kóczy, and Sz, Kovács (1994) "The Convexity and Piecewise Linearity of the Fuzzy Conclusion Generated by Linear Fuzzy Rule Interpolation," *BUSEFAL*, Vol 62, pp23-29.
[13] Y., Shi, and M., Mizumoto, (1995) "Some Considerations on Kóczy's Interpolative Reasoning Method," *Proceedings of the FUZZ-IEEE/IFES '95*, Yokohama, pp. 2117-2122.
[14] P., Baranyi, T.D., Gedeon, and L.T., Kóczy (1996) "A General Interpolation Technique in Fuzzy Rule Bases With Arbitrary Membership Functions," *Proceedings of the IEEE Int. Conf. of SMC*, Beijing, pp. 510-515.
[15] T.D., Gedeon, and L.T., Kóczy, (1996) "Conservation of Fuzziness in Rule Interpolation," *Intelligent Technologies, International Symposium on New Trends in Control of Large Scale Systems*, vol. 1, Herlany, pp. 13-19.