Polymorphic Fuzzy Signatures

B.S.U. Mendis and T.D. Gedeon

***Abstract*— The fuzzy signature [1], [2] approach is aimed at finding a hierarchically decomposed solutions by adding new elements to Zadeh's approach [3]. It tackles the problem by splitting the problem into hierarchically organized local sub-models and by applying more complex and heterogenous descriptors, more fit for the identification of extremely complex models. However, the computational time complexity still affects the fuzzy signatures as we were attempt to create an atomic fuzzy signature for each data point we get. Importantly, the atomic fuzzy signatures we store has properties we can make use of to make search over this structure computationally efficient. In this paper we introduce a new approach that uses the meta-data about a set of fuzzy signatures to extract a Polymorphic Fuzzy Signature. Productively, a polymorphic fuzzy signature represents its base set of fuzzy signatures in a higher meta level which also allows search/inference, and so can reduce the computational time complexity of the inference process.**

I. Introduction

Fuzzy systems are a general form of expert control using fuzzy sets representing vague / linguistic predicates, modelling a system by If ... then rules. In the classical approaches of Zadeh [3] and Mamdani [4], the essential idea is that an observation will match partially with one or several rules in the model. The conclusion is calculated by evaluation of the degree of these matches and by the use of the matched rules. The rules represent relations in the multidimensional input-output state space. Mamdani's technique is based on orthogonal projections, while Zadeh's method relates to arbitrary and fully known relations. A serious problem is caused by the high computational time and space complexity of rule bases describing systems with multiple inputs with proper accuracy. The latter is much more computationally expensive. The complexity allows little general systems application (or real time application) of classical fuzzy algorithms, where the inputs exceed 6 or so. Traditional fuzzy systems deal with simple data: the number of inputs is well defined, and values for inputs occur for most or all data items. This further reduces their general applicability.

The most important aspect of the construction of sub-symbolic models of very complex systems are the trade-offs between accuracy and computability, just like biological and human solutions to formally intractable problems. Fuzzy Signature [1], [2] approach is aimed at finding a solutions by adding new elements to Zadeh's approach, by splitting the problem into local sub-models and by applying more complex and heterogenous descriptors, more fit for the identification of extremely complex models.

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However, the computational time complexity still affects the fuzzy signatures [5] as we create an atomic fuzzy signature for each data point we get. Importantly, the atomic fuzzy signatures we store has properties we can make use of to make search over this structure computationally efficient. This is intuitively obvious if this is a set of data which a human being would understand. In this paper we discuss our proposed approach that uses the meta-data about a set of fuzzy signatures to extract a Polymorphic Fuzzy Signature. This polymorphic fuzzy signature represents its base set of fuzzy signatures in a higher level of search/inference to reduce the computational time complexity of the inference process. This can be seen to be analogous to information retrieval indexes, or video compression codecs which keep key frames and changes to the key frame. The differences between polymorphic fuzzy signature instances which are judged to be the same or similar enough are significant properties of many of the techniques below. Search on the extracted meta-data will be quick, and we can then drill down to the actual data.

II. Fuzzy Signatures

In this section we describe the initial concept of the Fuzzy signatures [1], [5]. Fuzzy signatures can be considered to be fuzzy n-dimensional objects, which can be compared with each other.

A. Vector Valued Fuzzy Sets

In 1965 L.A. Zadeh published the idea of fuzzy set theory [6] for better modeling of uncertainty. Fuzzy set theory defines the membership function of a fuzzy set A as,

$$q_{\underline{A}} : X \rightarrow [0, 1] \, . \tag{1}$$

Goguen [7] considered that optimality, relative to the situation, of an object in the real world can not be always organized into a total ordered set based on possible criteria. As an example: selection of an optimal shopping list is based on all possible shopping lists as well as several conflicting criteria of optimality, such as cost, nutritional value, quality, and so on. Thus we can assign only a partial ordering among them, and need to select the best shopping list according to possible partial ordering criteria. Therefore, Goguen [7] generalized the concept of fuzzy sets to *L*-fuzzy sets to allow the partial ordering of sets in fuzzy systems. The membership function of a *L*-fuzzy set is defined as,

$$q_{\underline{A}} : X \rightarrow L. \tag{2}$$

where L is a lattice [8], [9].

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The early work of Kóczy introduced the Vector Valued Fuzzy Sets (VVFS) concept [10] is a special form of an L-fuzzy set, and can be denoted in the following form:

Definition 1: A Vector Valued Fuzzy Set, $\underline{A}$, on $X = X_1 \times \ldots \times X_n$ can be written as:

$$\underline{A} = \left(X, q_{\underline{A}}\right) \quad (3)$$

The membership function $q_{\underline{A}}$ can be defined as:

$$q_{\underline{A}} : X \rightarrow [0,1]^n \quad (4)$$

B. Fuzzy Signatures

Intelligent decision making systems need to describe, compare and classify objects with complex structure and interdependent features. The structural complexity expresses the correlation of different dimensions, and can be organized into different branches and levels in a hierarchical structure. The global preference among the set of input dimensions is a relation, which can be approximated using an aggregation function. Hierarchical Fuzzy Signatures are fuzzy multidimensional descriptors of real world objects that inherit multi-aggregateness from VVFSs. Now, we recall the definition of fuzzy signature concept [1], [11]:

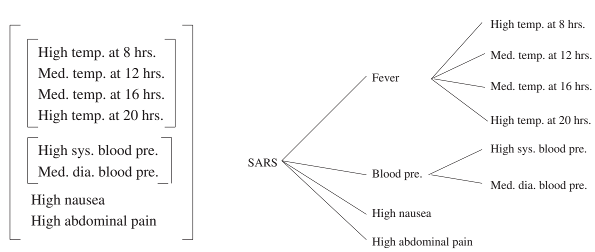
Definition 2: A Fuzzy Signature is a VVFS, where each vector component is another VVFS (branch) or an atomic value (leaf), and denoted by:

$$q_{\underline{A}} : X \rightarrow [a_i]_{i=1}^{k} \left(\equiv \prod_{i=1}^{k} a_i \right). \quad (5)$$

$$where\ a_i = \begin{cases} [a_{ij}]_{j=1}^{k_i} & ;if\ branch\ (k_i > 1) \\ [0,1] & ;if\ leaf \end{cases}$$

and Π describes the Cartesian product.

The Fig. 1(a) shows an example fuzzy signature [12]. This fuzzy signature describes an individual SARS patient, which is a data point in the SARS data collected in 2003 [13], [14], [15]. Fig. 1(b) shows the hierarchical view of the fuzzy signature shown in Fig. 1(a).



(a) As a Vector (b) As a Tree

Fig. 1: Example Fuzzy Signature

Fig. 2 shows an example of a fuzzy signature structure with two arbitrary levels g and $(g+1)$. Now, we use the following notation for the description of all concepts described in later sections.

The aggregation of a_0 in level 0 of the fuzzy signature structure can be written as $a_0 = @_0\{a_i\}$, where $@_0$ is an

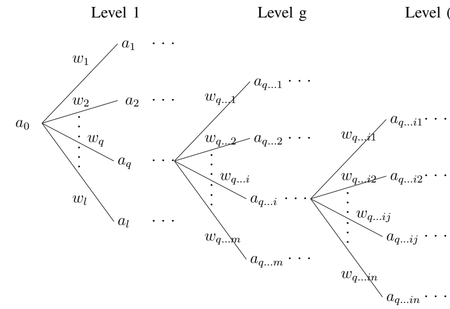


Fig. 2: An Arbitrary Fuzzy Signature

arbitrary aggregation function, $i = 1, \ldots, l$, and $a_0, a_i \in [0,1]$. Also, aggregation of an arbitrary branch $a_{q\ldots i}$ in level g (Fig. 2) can be written as, $a_{q\ldots i} = @_{q\ldots i}\{a_{q\ldots ij}\}$, where $@_{q\ldots i}$ is an arbitrary aggregation function, $j = 1, \ldots, n$ and $a_{q\ldots i}, a_{q\ldots ij} \in [0,1]$.

Fig. 3 shows an example aggregation of a fuzzy signature using *max* and *min* as the aggregation functions.

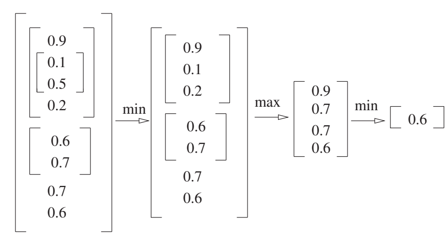


Fig. 3: Aggregation of fuzzy signatures

III. Polymorphic Fuzzy Signature (PFS)

In this section we introduce the concept of Polymorphic Fuzzy Signatures (PFS). Further we proposed the use of generalized Weighted Relevance Aggregation Operator (WRAO) in PFSs. Polymorphic fuzzy signatures have the ability to reduce the computational time for searching the large number of individual fuzzy signature structures required for a complex decision making model by introducing meta information.

A. Polymorphic Fuzzy Signatures

We observe that in some situations we may be able to find a single meta fuzzy signature for a set of individual data points, by reducing the number of fuzzy signatures required to search in a decision making model. We call such a fuzzy signature a polymorphic fuzzy signature for the base set of data points it represents.

Lemma 1: A polymorphic fuzzy signature is a fuzzy signature according to definition 2. However leaf nodes are fuzzy sets.

As the definition 1 describe, unlike in fuzzy signatures in polymorphic fuzzy signature's leaf nodes are fuzzy sets.

Fig. 4 shows an example of a polymorphic fuzzy signature structure with two arbitrary levels g and $(g+1)$.

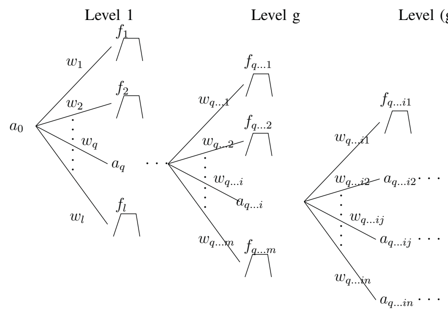


Fig. 4: An Arbitrary Polymorphic Fuzzy Signature (PFS)

Below, we formulate an objective function to optimize a polymorphic fuzzy signature for automation learning from data.

Definition 3: Let $A = \{S_1, S_2, \ldots, S_n\}$ be a collection of fuzzy signatures for a certain problem and let $\{d_1, d_2, \ldots, d_n\}$ be the collection of data points[1] they represent respectively. Now, let S_i be the corresponding fuzzy signature of the data point d_i. Further, let $S_i(d_i)$ be the degree[2] of match of the data point d_i with fuzzy signature S_i. Then S is said to be a polymorphic fuzzy signature of the set A if

$$\sum_{i=1}^{n} |S(d_i) - S_i(d_i)| \leq \epsilon \qquad (6)$$

Where ϵ is a small number close to zero.

IV. Generalized Weighted Relevance Aggregation Operator (WRAO)

An interesting research direction in the fuzzy signature context is what kinds of aggregations are applicable for combining data with partly different substructures. This sub-section we propose the use of generalized Weighted Relevance Aggregation Operator (WRAO) [16] to aggregate polymorphic fuzzy signatures. WRAO enhances the adaptability of the fuzzy signature model to different applications and simplifies the learning of fuzzy signature models from data by combining both weight and aggregation functions into one operator.

Now, we recall the definition of WRAO in [16].

Definition 4: The generalized Weighted Relevance Aggregation Operator (WRAO) of an arbitrary branch $a_{q...i}$ with n sub branches $s_{q...i1}, s_{q...i2}, \ldots, s_{q...in} \in [0,1]$, and weighted relevancies $w_{q...i1}, w_{q...i2}, \ldots, w_{q...in} \in [0,1]$, for a fuzzy signature, is a function $g : [0,1]^{2n} \rightarrow [0,1]$ such that

$$a_{q...i} = \left[\frac{1}{n} \sum_{j=1}^{n} (s_{q...ij} \bullet w_{q...ij})^{p_{q...i}} \right]^{\frac{1}{p_{q...i}}} \qquad (7)$$

Following 3 properties must be satisfied:

(i) $w_{q...ij} \in [0,1]$
(ii) $\bigvee_{j=1}^{n} w_{q...ij} \leq 1$
(iii) $p_{q...i} \neq 0$

In [16], we prove the following two theorems for WRAO.

Theorem 1: Let $a_{q...i}$ be an arbitrary branch with n sub-branches $a_{q...i1}, \ldots, a_{q...in}$, and weighted relevancies $w_{q...i1}, \ldots, w_{q...in}$, for an arbitrary fuzzy signature (see figure 2). Then WRAO in definition (4) holds the following properties:

(i) Idempotency
(ii) Commutativity
(iii) Monotonicity

In the literature [17], [18] monotonicity has been considered in this way. It is adequate to satisfy the requirement to be a meaningful aggregation function [19] as weights, $w_{q...i1}, w_{q...i2}, \ldots, w_{q...in}$, in WRAO are fixed for any instance of a fuzzy signature in the decision making phase, and both weights and aggregation operators vary simultaneously only in the learning phase.

Theorem 2: The WRAO in equation (7) holds the following characteristics:

(a) $p_{q...i} \rightarrow 0$ then WRAO $\rightarrow$ geometric mean
(b) $\lim_{p_{q...i} \rightarrow +\infty} g(s_{q...i1}, \ldots s_{q...in}; w_{q...i1}, \ldots, w_{q...in})$
$$= max(s_{q...i1} w_{q...i1}, \ldots s_{q...in} w_{q...in})$$
(c) $\lim_{p_{q...i} \rightarrow -\infty} g(s_{q...i1}, \ldots s_{q...in}; w_{q...i1}, \ldots, w_{q...in})$
$$= min(s_{q...i1} w_{q...i1}, \ldots s_{q...in} w_{q...in})$$
(d) $p = 1$ then WRAO$\rightarrow$ arithmetic mean
(e) $p = -1$ then WRAO$\rightarrow$ harmonic mean

The WRAO has been derived similarly to the form of the generalized weighted means function discussed in [17] in order to satisfy the Weighted Relevance Aggregation concept in definition 4. The WRAO is a more generalized version of weighted mean as it has weaker constraint on weights $\left(\bigvee_{j=1}^{n} w_{q...ij} \leq 1 \right)$ compared to that of weighted mean. Thus, WRAO is different from the generalized weighted means function [17]. Further, WRAO aggregates the weighted input whereas generalized weighted means aggregates the non weighted input and uses the weights as parameters to achieve the desired aggregation function.

A. Method of Extracting WRAO from Data

In this section we explain the method of learning WRAO from real world data briefly with more detailed explanations to be found in [16]. First, to avoid the first 2 constraints on the weighted relevance factor $w_{q...ij}$ in definition 4, we replaced it by the following sigmoid function,

[1]*In the fuzzy signature concept, a data point means a collection of data which represents an event, e.g. in medical applications, a patient's data record of a whole day can be considered as a data point [12].*

[2]*The degree of match is the final result of aggregating the input data point d_i using the fuzzy signature S_i*

$$w_{q...ij} = \frac{1}{1+e^{-\lambda_{q...ij}}} \quad (8)$$

where $\lambda_{q...ij} \in \Re$. Now, the equation (7) can be modified as follows,

$$a_{q...i} = \left[\frac{1}{n} \sum_{j=1}^{n} \left(s_{q...ij} \left[\frac{1}{1+e^{-\lambda_{q...ij}}} \right] \right)^{p_{q...i}} \right]^{\frac{1}{p_{q...i}}} \quad (9)$$

The $p_{q...i}$ and $\lambda_{q...ij}$ are called the aggregation factors of branch $q \ldots i$ and weighted relevance factor of sub branch $q \ldots ij$ of the fuzzy signature in figure 4, respectively. This form of WRAO (9) can easily be used for gradient based learning.

The parameters we need to learn are the aggregation factor $p_{q...i}$ and weighted relevance factors $\lambda_{q...ij}$ for each WRAO at each node of the fuzzy signature structure shown in figure 4. First we can obtain the partial derivatives of the equation (9) w.r.t. $p_{q...i}$:

$$\frac{\partial a_{q...i}}{\partial p_{q...i}} = \left[\frac{a_{q...i}^{1-p_{q...i}}}{n p_{q...i}^2} \right] \left\{ \sum_{j=1}^{n} t \ln(t) - nt' \ln\left(t'\right) \right\} \quad (10)$$

where $t = (a_{q...ij} w_{q...ij})^{p_{q...i}}$ and $t' = a_{q...i}^{p_{q...i}}$. Similarly we obtain the partial derivatives of the equation (9) w.r.t. $\lambda_{q...ik}$:

$$\frac{\partial a_{q...i}}{\partial \lambda_{q...ik}} = \left[\frac{1}{n} \sum_{j=1}^{n} (s_{q...ij} w_{q...ij})^{p_{q...i}} \right]^{\frac{1}{p_{q...i}}-1} T \quad (11)$$

where $w_{q...ij} = \frac{1}{1+e^{-\lambda_{q...ij}}}$ and $T = \left\{ \frac{d([s_{q...ik} w_{q...ik}]^{p_{q...i}})}{d\lambda_{q...ik}} \right\}$.

The Levenberg-Marquardt (LM) method [20], [21] has been used for the learning of WRAO. The LM algorithm is a widely used advanced optimization algorithm that outperforms simple gradient descent and other gradient methods when applied in a wide variety of problems. The LM algorithm is a pseudo-second order, Sum of Square Errors (SSE) based optimization method, in which the Hessian matrix is estimated using the gradients [21], [20]. The two equations (10) and (11) above, together with the chain rule for derivation have been used to calculate the Jacobian, which is used to approximate the Hessian matrix of the LM learning. A detailed discussion of the method of using LM for learning WRAO can be found in [22].

B. *Experiment 3: LM Learning of WRAO form Data*

Now, we explain the results of 2 experiments to extract the WRAO from data. As the first experiment we take the High Salary Selection problem of employees [23] and the SARS patient classification problem as the second experiment .

Next, before go to the experiments, we briefly introduce the concept of Fuzzy Classification Error (FYCLE) that can be used to better illustrate and compare the results of the experiments.

1) Fuzzy Classification Error: We formulate the Fuzzy Classification Error (FYCLE) in the following way. First, we specify that both *desired* output and *predicted* output of an experiment are in the range $[0, 1]$. Next we define a set of rules for the classification, and these rules are visualized in the following Fig. 5.

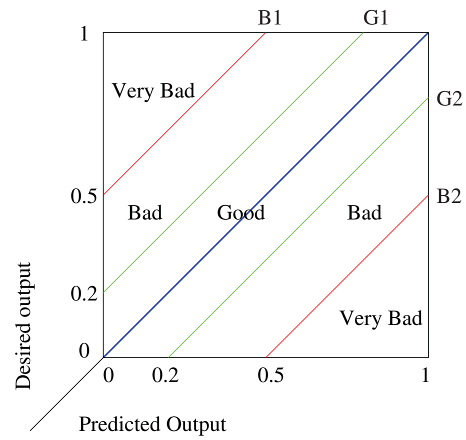


Fig. 5: Fuzzy Classification Error Rules

According to figure 5, there are 3 categories of classifications that can occur, being *Good*, *Bad* and *Very Bad (VB)*. Now we assume the pair of *predicted* and *desired* values of the ith input, respectively, are taken as *x* and *y* coordinates of the point P_i on the 2 dimensional classification error space in figure 5. The Fuzzy Classification Error of an arbitrary point P_i can be written as,

$$FYCLE(P_i) = \begin{cases} 0 \;\; if \;\; P_i \in Good \\ 0.5 \;\; if \;\; P_i \in Bad \\ 1 \;\; if \;\; P_i \in Very\ Bad \end{cases}$$

Let us consider the 4 straight lines, $B1$, $B2$, $G1$, and $G2$, in figure 5. In this experiment, they are equivalent to,

$B1 \equiv y - x - 0.5 = 0$
$G1 \equiv y - x - 0.2 = 0$
$G2 \equiv y - x + 0.2 = 0$
$B2 \equiv y - x + 0.5 = 0$

Now, The Fuzzy Classification Error of an arbitrary point P_i can be calculated as,

$$FYCLE(P_i) = \begin{cases} 0 \; if \; G1(P_i) \le 0 \;\&\; G2(P_i) \ge 0 \\ 0.5 \; if \; (B1(P_i) \le 0 \;\&\; G1(P_i) > 0) \\ \qquad OR \\ \qquad (G2(P_i) < 0 \;\&\; B2(P_i) \ge 0) \\ 1 \; if \; B1(P_i) > 0 \; OR \; B2(P_i) < 0 \end{cases}$$

Next, the Sum of Fuzzy Classification Error (SYCLE) for a set of data with m records can be calculated as,

$$SYCLE(P) = \sum_{i=1}^{m} FYCLE(P_i) \;\; where \; m \in \mathbb{N} \quad (12)$$

Further, we can define the Average Fuzzy Classification Error (AVGFYCLE) as follows:

$$AVGFYCLE(P) = \frac{1}{m} \sum_{i=1}^{m} FYCLE(P_i) \;\; where\, m \in \mathbb{N} \quad (13)$$

The AVGFYCLE, is more useful when we need to compare the results of the same experiment carried out with two or more different data sets and the number of data points in these data sets are different. In this paper, we always reprise the same data sets for all experiments. Therefore, the Sum of Fuzzy Classification Error (SYCLE) is adequate to visualize and classify the results of the experiments in this paper.

2) High Salary Selection PFS: The High Salary Selection problem has been discussed in [23]. This problem is to find the degree of likelihood of having a high salary based on the *contacts*, *age* and *work experience* of an employee. Figure 6 shows the polymorphic fuzzy signature which is obtained using domain experts knowledge for the high salary selection problem.

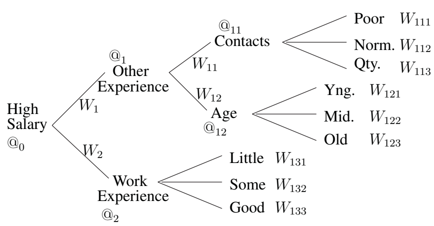


Fig. 6: High Salary Selection PFS

Note that in figure 6 $@_i$ and w_i represent the aggregation function and weighted relevance of node i, respectively.

We learnt the WRAOs for each node in the High Salary Selection fuzzy signature structure in figure 6 automatically [22]. Figures 7 and 8 show training and test results of the experiment. The *Desired* values plot shows the desired degree of relevance of each employee's salary to the high salary category. The *"Cls. Corct"* results plot shows the correctly predicted results of the High Salary Selection fuzzy signature, w.r.t. the Fuzzy Classification Error in section IV-B.1. The *"Cls. Error"* results plot shows the incorrectly predicted results of the High Salary Selection fuzzy signature. Table I shows the training and test results of this experiment numerically.

TABLE I: Test Results of High Salary Selection PFS

	MSE	SYCLE
Train	0.0130	8.5
Test	0.0130	6

According to the results illustrated in figures 7 and 8 and table I, we can conclude that the Polymorphic fuzzy signature for High Salary Selection together with learnt WRAOs can very well predict the desired results. Also that the LM method can learn both aggregations and weights for the High Salary Selection PFS for good results.

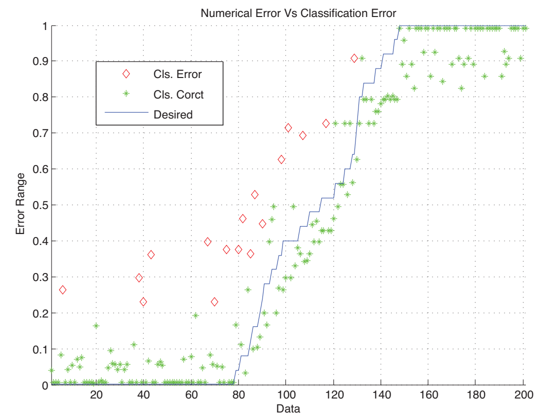


Fig. 7: Training Results of Salary Signature

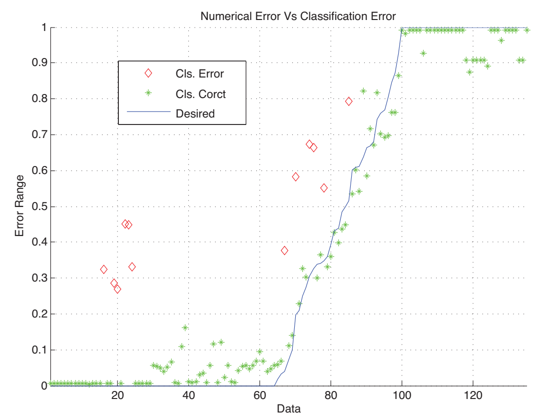


Fig. 8: Testing Results of Salary Signature

3) SARS Patient Classification PFS: Next, as the second experiment again the SARS Patients classification problem has been used. The first part of the second experiment is to learn to classify the SARS patient data into 2 classifications, that is other patients and SARS patients. Figure 9 shows the test result of the experiment. The degree of abnormal condition of all SARS patients data (range 3,000 to 4,000) remains above 0.5 and all non-SARS patients data (range 1,000 to 3,000) remain close to 0. Table II compares the results of SARS patients classification PFS with non-weighted, WRA, and WRAO methods respectively.

During the second part of this experiment we found that the LM based learning method can also find aggregation functions and weighted relevancies to classify data into 3 categories, namely SARS, other and normal patients. Extended experiments show that the learning method can learn WRAOs which can separate SARS data into 4 output target categories,

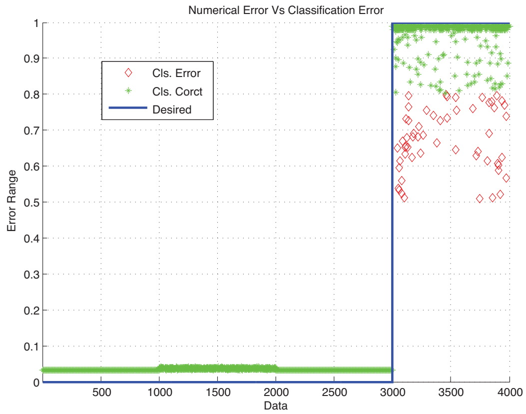


Fig. 9: Test Results of SARS Signature

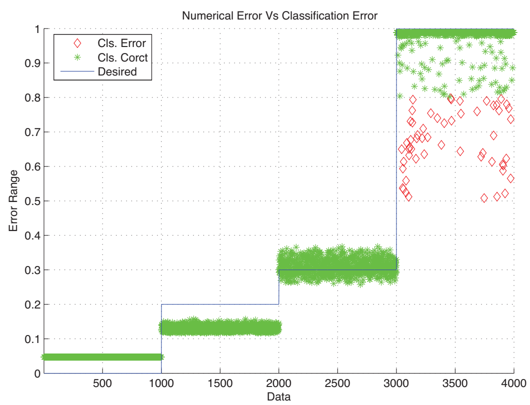


Fig. 10: Test Results of SARS Signature for 4 classifications

namely SARS, pneumonia, hypertension and normal patients. Figure 10 illustrates the results of the extended experiment to learn 4 output target categories. It can be seen that for the normal patient data in range 0 - 1000 the degree of abnormality of their condition keeps close to 0, for the pneumonia patient data in range 1000-2000 it keeps to 0.13, for the hypertension patients in range 2000-3000 it keeps to 0.3 and for almost every SARS patient data range in 3000 - 4000 it keeps above 0.5. Also, the *"Cls. Corct."* and *"Cls. Error"* plots indicate the individual patient predictions that are correct or wrong classifications respectively.

V. Conclusion

Polymorphic Fuzzy Signatures (PFS) was introduced. Next we proposed Weighted Relevance Aggregation Operator (WRAO) for aggregation of Polymorphic Fuzzy Signatures. We used an example to show that the WRAO enhances the results as well as the optimality of Polymorphic Fuzzy Signatures. Experiments with two real world problems, namely High Salary and SARS patients classifications, show

TABLE II: Test Results of SARS PFS

	Train		Test	
	MSE	SYCLE	MSE	SYCLE
non-weighted PFS	-	-	0.2332	1000
PFS with WRA	-	-	0.0026	45
PFS with WRAO	0.0020	33.5	0.0018	25.5

that the Levenberg-Marquardt method based algorithm can successfully learn WRAO for both problems. Further, for the SARS patients classification problem WRAO not only classifies input data into SARS and non-SARS patients, but it also can classify data into 4 classifications, namely SARS, pneumonia, hypertension and normal patients.

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