Maintenance of local fuzziness in rule interpolation

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Abstract — Approximate reasoning using fuzzy rule based systems have wide application in for example industrial control, property prediction, and in pattern recognition areas.

In this paper we introduce our method which is conservative with respect to the degree of local fuzziness in the rule base, and demostrate its utility on a petroleum engineering problem.

I. INTRODUCTION

Sparse rule bases which do not contain redundant information can provide computational advantages to comprehensive rule bases. Also, sometimes there are natural gaps in the knowledge base.

Fuzzy rule interpolation can be used to provide conclusions for observations for which there may be no overlap with even the supports of existing rules in the rule base. All of the methods developed are descendants of the Kóczy and Hirota [1, 2] method of linear interpolation for sparse rule bases, and have various advantages and disadvantages.

The criteria for evaluating interpolation methods must include elements such as the ability to form conclusions where it is appropriate, the formation of intuitively acceptable conclusions, and a computational complexity which would allow it to be useful in reducing the size of fuzzy rule bases.

In this paper we introduce our method which is conservative [3] with respect to the degree of local fuzziness in the rule base. The notion of being intuitively acceptable is important, and can be said to be already inherent in the field of fuzzy logic. That is, the use of linguistic variables which are used as fuzzy rules is in some sense intuitive and qualitative.

II. PHILOSOPHICAL BACKGROUND

This section introduces the philosophical basis of our approach, based on our previous work [3]. Fig. 1 illustrates a pair of rules:

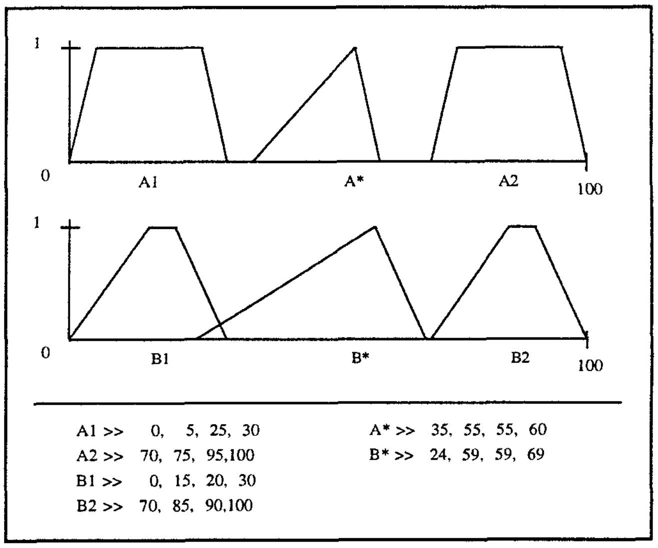

Fig. 1. Rule set p2.

The two rules are *if x is A1 then y is B1*, and *if x is A2 then y is B2*. The observation A* does not overlap with the antecendents of either of the flanking two rules. An interpolated conclusion B* is shown, between the flanking consequents of the two rules. For simplicity, both the antecedent and consequent universes of discourse have been scaled to the 0 to 100 range above, and rules are represented by 2 support and 2 core points (to allow trapezoidal forms).

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The initial starting point is that we do assume as little homogeneity in the rule base as possible. Thus, we treat the nearest core points of rules as all that is visible. Thus, A* is in a valley between A1 and A2, and whether A1 or A2 are actually plateaux is not visible, and is not used. The core points of B* are derived by simple linear interpolation between the nearest core points of A1, A2 and B1, B2.

Once the core points of B* are determined, we reduce further the assumption of homogeneity in the rule base. That is, the interpolation of the right of B* is only between the the rightmost core point of B* and the leftmost core point of B2. This is consistent with the premise that A* need not be symmetrical, and with the notion that determining the right side of B* should be based on 'nearby' information such as the right side of A* and the left sides of A2 and B2.

This is in contrast to the initial method which derived the right side of B* from the right sides of A1, B1, A2, B2 and A*. All of these except the latter are further away from the right side of A* or B* than the 'nearby' information we propose to use.

The use of such 'nearby' information has real world plausibility, while there are few domains where it could be readily proven that there is justification for the assumption of handedness of the the rule shapes. In view of the claims to intuitive acceptability we make, we will first describe and derive our formulation geometrically before expressing the method in terms of the more usual equations using fuzzy distances.

A Geometric Description of Method

Given the subsection of the previous figure, below:

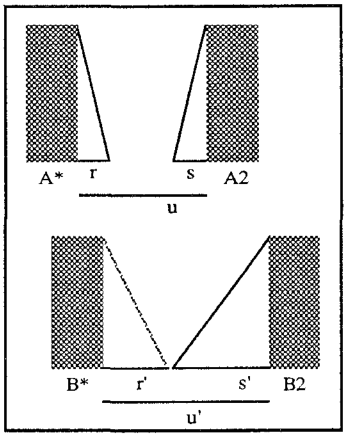

Fig. 2. Section of p2.

Note that the grey regions highlight the (assumed) unknown nature of the rest of the rule base.

The labels r, s indicate the spread of the observation and the rule antecedent and represent their fuzziness.

Clearly A2 is more fuzzy than A*. The labels r', s' indicate a potential spread of the conclusion (drawn in grey as it we have yet to calculate it), and the rule consequent.

By observation, our intuition tells us that B2 is more fuzzy than A2, due to the shallower slope. That is, there would seem to be an increase in the overall fuzziness from the antecedent to the consequent in this rule.

We have not normalised the A*,A2 and the B*,B2 distances, as we prefer to do this explicitly using the values of u, u' as appropriate.

Without using any information from rule 2 other than its core distances, we could calculate a value for r' as shown in Equation (1).

$$r' = r \cdot \frac{u'}{u} \tag{1}$$

This is merely the normalisation of r from the A*,A2 distance metric into the B*,B2 distance. Most likely, the value of r' will be some increase or decrease of the effect of this normalisation. We could similarly calculate a value for the normalisation of s into the B*,B2 distance, called s''.

$$s'' = s \cdot \frac{u'}{u} \tag{2}$$

The relationship between the actual value of s' we can measure from B2, and the calculated value s'' provides an indication of the difference in fuzziness from rule antecedent to consequent. As we have already noted, A2 is clearly fuzzier than B2. This difference can be incorporated into (1) producing:

$$r' = r \cdot \frac{u'}{u} \cdot \left(1 + \frac{s' - s''}{z}\right) \tag{3}$$

Since we are interested in the relative change in fuzziness from rule antecedent to consequent, we divide s' - s'' by z, being either s', or s''. The more conservative choice of divisor is s', which we shall see produces a good compromise between two extremes. With this substitution we produce:

$$r' = r \cdot \frac{u'}{u} \cdot \left(2 - \frac{s}{s'} \cdot \frac{u'}{u}\right) \tag{4}$$

Note that this will not work for crisp rule consequents, so in this case we make the less conservative, and perhaps more obvious, substitution producing Equation 5.

$$r' = r \cdot \frac{s'}{s} \tag{5}$$

Note that r' is no longer dependent on the ratio of the different metrics, and is solely determined by the ratio of rule consequence to antecedent fuzziness. This explains our previous comment regarding Equation (3) being a good compromise, between interpolation solely on the basis of the change in metric, versus solely on the basis of the change in rule fuzziness but ignoring the change in metric.

For completeness, we note that Equation (5) will not work for crisp rule antecendents, hence we resort to Equation (1), which is appropriate as we need to use Equation (1) only in the case where both rule antecedent and conclusion are crisp, where the only information available is the change in metric.

We can now explain the shape of r' in Fig. 1. The larger distance B*,B2 versus A*,A2 would be expected to increase the width of r' versus r, and the shallower slope of B2 versus A2 also indicates a icrease of fuzziness. The steep slope of A* indicates low fuzziness in the observation which effect is combined. The actual values are shown in Fig. 1, the value of r is 5, and the value of r' is 10.

B An example of the Method

Fig. 3 illustrates a situation in which the technique described above provides a natural result, while the original technique provides an unsatisfactory result.

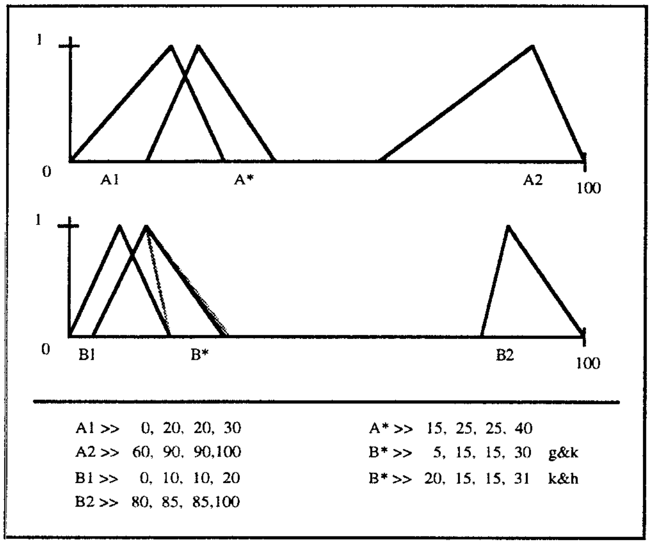

Fig. 3. Rule set p6, methods G&K and K&H.

The distortion of the conclusion is due to the interpolation of the left side support between the rule consequents. Our technique provides a good result because the conclusion supports are only calculated relative to the local context between the appropriate adjacent core points.

III. APPLICATION EXAMPLE

A Background Description

We applied the proposed methods to petroleum reservoir modelling. The source of the data is from holes bored in areas where oil or gas is expected.

These bore holes or wells produce data in two forms. The most reliable is core data, where an actual rock sample is retrieved. Traditionally this took the form of cylindrical rock from the 'core' of the drilling, though it may now involve lowering a device to retrieve rock samples from an already bored well. A cheaper form of data collection is to lower sondes down a bore hole which can measure something which may be useful to predictions of rock properties, and record electrical signals. These signals were traditionally logged on long continuous scrolls of paper. The measurements can be sound, radiation, magnetic etc.

In reservoir modelling, the logs are available in every hole at different depths but retrieving rock samples is an expensive process and hence core data such as permeability and porosity have to be estimated from well logs.

B Sample of Data

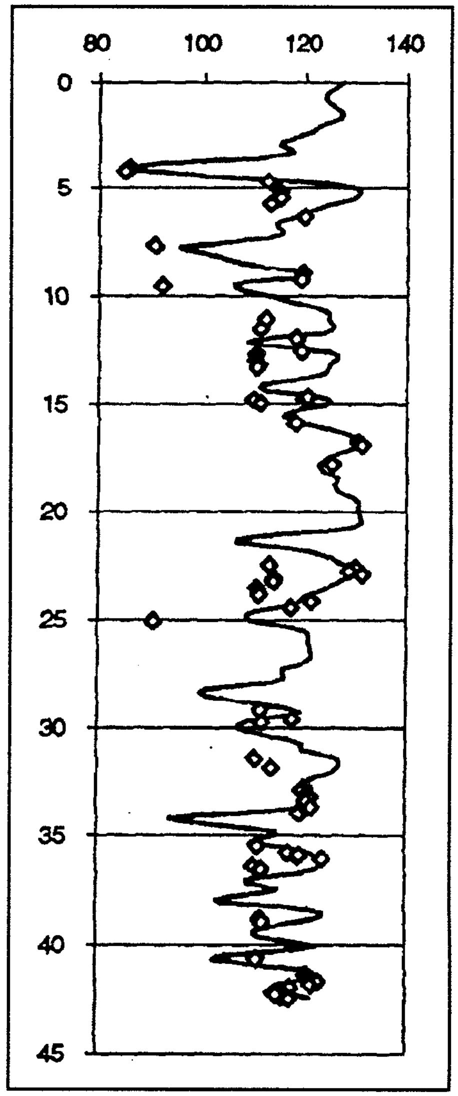

Fig. 4. Sonic log (μsec/ft, top axis) with superimposed core porosity.

C Source of Data

In this sample, we had data from one oil well. The data was obtained from the North West Shelf, Offshore Australia. We only use one log, namely RHOB, and one core data, namely permeability K. A total of 141 data points were recorded at various depths, together with the corresponding permeability measurements.

The input data was normalised in range of 0 and 1, and output data was in range of 0.1 and 0.9. Normalisation or scaling of the input and the output data is not absolutely necessary but is convenient and appropriate for certain applications.

The input data was partitioned into three possible intervals. The linguistic values are small (from 0 to 0.3), medium (from 0.2 to 0.8), and large (from 0.65 to 1). Each interval of input data corresponds to one of fuzzy membership functions which are provided by experts. In addition, the expert tells us two rules. That is,

if RHOB is small, then K is high,

and

if RHOB is large, then K is low.

Also the expert tell us RHOB and K have some correspondence, with linear or polynomial relationship only. Fig. 3 shows three fuzzy membership functions of the input RHOB provided by the expert.

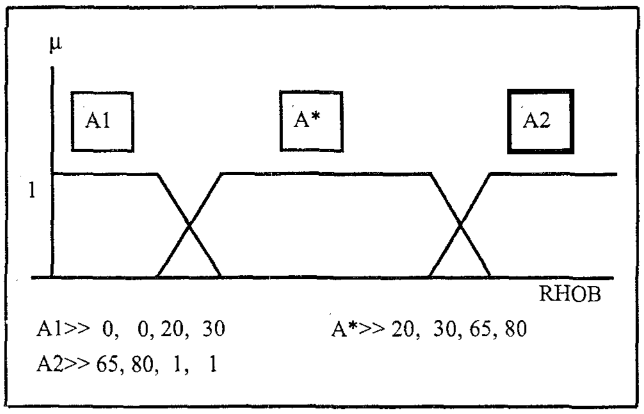

Fig. 5. Input membership functions
(A1=Small; A*=Medium; A2=Large)

We also know the permeability measured in laboratory when RHOB is small and large, but we do not have the values of the permeability when RHOB is medium. Now the problem is how to estimate permeability when RHOB is medium.

D Fuzzy logic to get membership functions

We can get fuzzy membership functions of permeability based on the rules provided by the experts and the fuzzy extension principle. First we use regression methods to get the equations

$$K_{low} = f(RHOB_{large}),$$

and

$$K_{high} = f(RHOB_{small}).$$

Then we use the fuzzy extension principle to get

$$\mu_{f(Ai)} (f(RHOB_j) = \mu_{Ai} (RHOB_j),$$

where i=1,2; and j=small, large.

Fig. 5 shows the correlation between fuzzy sets A and B, crisp sets X and Y, and y = f(x).

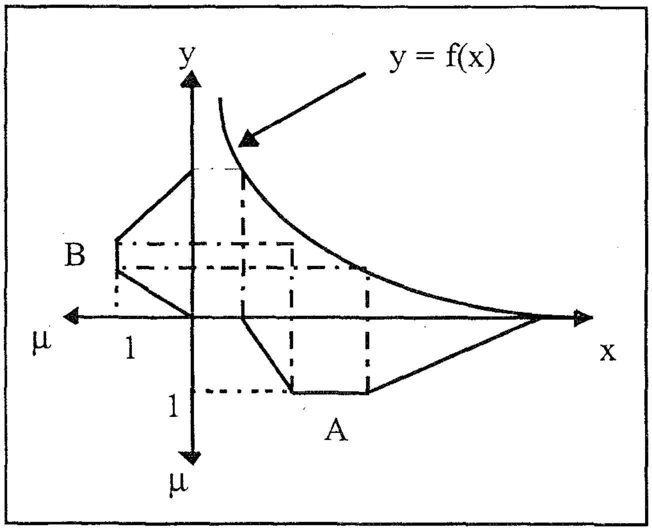

Fig. 6. Fuzzy extension principle

For A1->B1, we get

$$K_{high} = -5^* \ RHOB_{small}{}^* \ RHOB_{small} + 0.85.$$

For A2->B2,

$$K_{low} = -0.8^* \ RHOB_{large} + 1.05.$$

Thus, we can easily produce the membership functions B1 and B2. B* in Fig. 6 is based on G&K's method, and Fig. 7 shows the result using K&H's method.

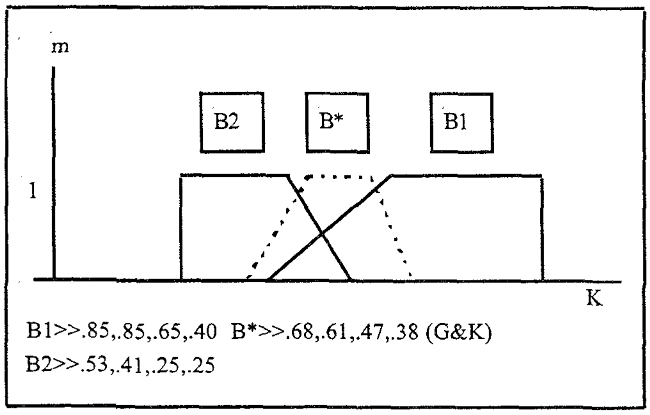

Fig. 7. Output membership function B* (G&K)
(B1=High; B*=Medium; B2=Low)

In Fig. 6, the consequent part for the *medium* rule is quite reasonable. In Fig. 7, however, the consequent is not very useful. The trapezoid for *medium* overlaps noticeably with the trapezoid for *high*, which is counterintuitive, and not particularly meaningful.

This is of course due to the effect of the wide span of the core of B1 as compared with the width of span of the core of B2. This produces a result in this case which is not intuitive.

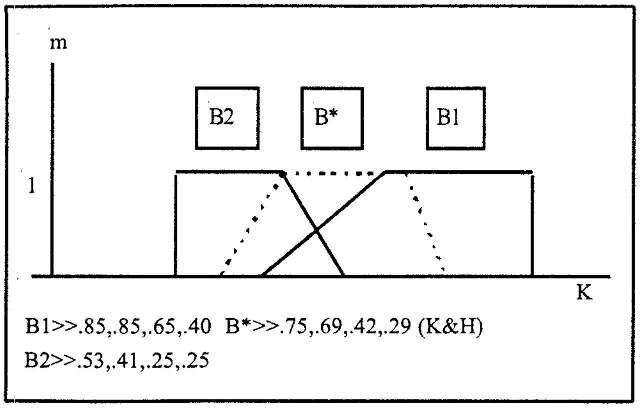

Fig. 8. Output membership function B* (K&H)
(B1=High; B*=Medium; B2=Low)

In the interest of fairness, we must point out that we can find circumstances in which our method produces results which appear less correct than the original Kóczy and Hirota technique. We are investigating these further.

E Prediction of Permeability

We get the following linear and the polynomial equations respectively based on A* and B* using regression methods and fuzzy extension principle, instead of using fuzzy reasoning such as defuzzification of centre of the area which will get the average results.

Linear:

$$K_{medium} = -0.4744 * RHOB_{medium} + 0.7663 \quad (G\&K)$$

$$K_{medium} = -0.7695 * RHOB_{medium} + 0.9126 \quad (K\&H)$$

Polynomial:

$$K_{medium} = -0.033 * RHOB_{medium} * RHOB_{medium} - 0.4416 * RHOBmedium + 0.7601 \quad (G\&K)$$

$$K_{medium} = -0.256 * RHOB_{medium} * RHOB_{medium} - 0.5152 * RHOB_{medium} + 0.865 \quad (K\&H)$$

Table 1 shows the total sum of squares (TSS) error measure of both G&K's and K&H's methods to predict permeability. Clearly, the result of G&K's method is better than that of K&H's.

	G&K	K&H
Linear	3.40	3.47
polynomial	3.43	3.77

Table 1. Total sum of error squares (TSS) for G&K and K&H' methods.

Fig. 9 is the plot of both the linear and polynomial predictions for permeability using our technique G&K.

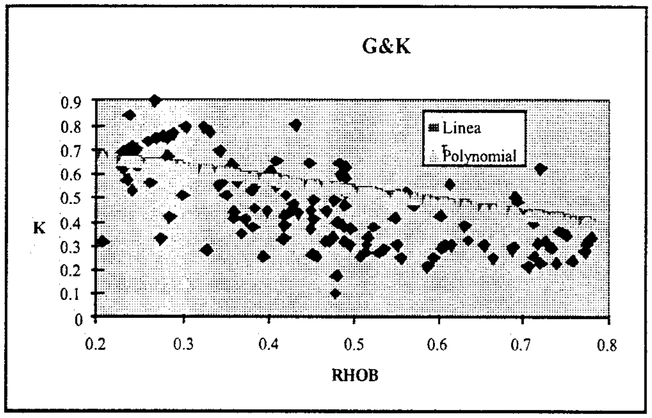

Fig. 9. Plot of linear and polynomial permeability prediction by G&K

The result does not appear to be the line of best fit to the points. This may be a reasonable interpolation from the values of the low and high points. Please note also that at this scale the polynomial function is close to a straight line.

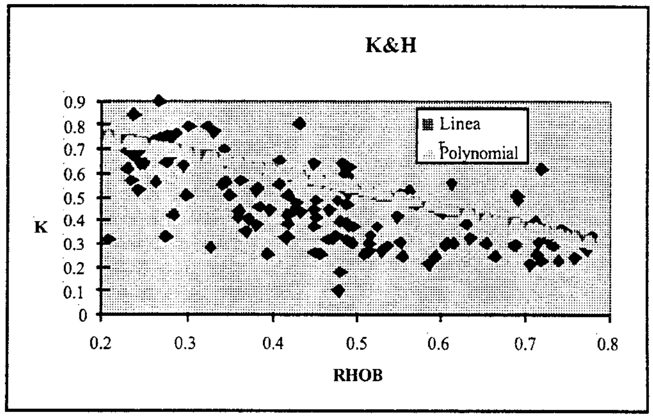

Fig. 10. Plot of linear and polynomial permeability prediction by K&H

In Fig. 10, we see that the result using technique K&H, the line is a worse fit for the points, and produces a 10% worse result in terms of tss (see Table 1) in the polynomial case. In the linear case the results are essentially the same, though we have noted that by qualitative judgement of the Figs. 9 and 10 we can see that the technique G&K is better than the technique K&H.

IV. CONCLUSION

We have demonstrated our technique G&K performs better than the previous technique in an application using petroleum engineering data. That data was processed in a novel fashion to produce an artificially sparse rule base,.

We have also used this technique for interpolaon in hierarchical rule bases [4].

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