Learning Weights from Observations for Hierarchical Fuzzy Signatures

B. Sumudu U. Mendis[1], Tamás D. Gedeon[2], László T. Kóczy[3, 4]

[1]Department of Computer Science
The Australian National University
Canberra, ACT 0200, Australia
{sumudu.mendis[1], tom.gedeon[2]}@anu.edu.au

[3]Department of Telecommunication and Media Informatics, Budapest University of Technology and Economics, Budapest, Hungary.

[4]Institute of Information Technology and Electrical Engineering,
Széchenyi István University, Győr, Hungary
koczy@tmit.bme.hu

Abstract− **Our earlier work proposed and discussed the issues of a method for obtaining weights, which are associated with the weighted relevance aggregation method, for hierarchical fuzzy signatures from real world data. This method also handled the non-differentiability of conventional *max-min* aggregation functions, using a mathematically proved method in the literature. This paper applies the proposed method to extract weights for two real world hierarchical fuzzy signatures structures namely Salary Selection and SARS Patient Classification. Based on the results of the experiments for weights extraction with SARS and non-SARS patients data we show that our weights learning method for hierarchical fuzzy signatures not only performs better in separating SARS and non-SARS patients, but also separating non-SARS data into further significantly distinguished and ordered available categories.**

I. INTRODUCTION

In Kóczy [3] the vector valued fuzzy sets concept [4] has been further generalized to introduce the fuzzy signature concept. Fuzzy signatures model complex structured problems with the help of hierarchically structured vector valued fuzzy sets and a set of aggregation functions. The hierarchically structured vector valued fuzzy sets represent the relationship of the attributes of the object and the set of aggregation functions tunnel the different relationships, between different universes of discourses of attributes, from lower branches to the higher branches of the problem. Thus, fuzzy signatures model problems similar to the nature of human approaches to problem solving. An important advantage of the fuzzy signature concept is that it can be used to compare degree of similarity or dissimilarity of two slightly different objects, which have the same fuzzy signature skeleton. Additionally, fuzzy signatures are capable of dealing with missing input data. Thus, medical and economic diagnoses are the obvious applications of fuzzy signatures.

In [7] we further enhanced the inference in fuzzy signatures, by introducing the weighted relevance aggregation method. The concept behind the weighted relevance aggregation method is that the weights in each branch of the fuzzy signature are the observations of the relevance of that branch to its higher level branches of the hierarchical fuzzy signature structure. Thus, this method introduces additional expert knowledge to the fuzzy signature structure to classify vague data. Fuzzy signatures now have the additional capability to act as sophisticated problem solvers in vague environments similar to some human abilities.

In [6] we proposed a method to learn weighted relevance from observations for hierarchical fuzzy signatures. In this research we use this method to extract the weighted relevancies from two real world problems (SARS and High Salary) discussed in [7]. Also, we observe the improvement of the accuracy of the final results, of the fuzzy signatures of the above two problems, using weights extracted using proposed method compared to the results of the same fuzzy signatures with manually created weights (according to domain experts knowledge) in [7]. In section II, we briefly discuss the methodology of using the gradient descent learning method for learning weights in fuzzy signature structure from observations. In section III we first show the results of experiments with High Salary Selection fuzzy signature and we also discuss the enhancement of the results compared to [7]. Finally, we discuss the results of the second experiment using the SARS Patients Classification fuzzy signature. We also show that our proposed weights learning model for weighted relevance aggregation method for hierarchical fuzzy signatures [7] not only performs better in separating SARS and non-SARS patients, but also in separating non-SARS data into further significantly distinguished and ordered available categories.

II. Issue of Weights Learning from Observations

In [6] we have shown a method which can be used for extracting weights using gradient based method, for weighted relevance aggregation in fuzzy signatures. This section briefly reviews the method of using gradient based leaning for weight extraction for weighted relevance aggregation inference in fuzzy signatures. Also, how to obtain derivative of a function when *max* and/or *min* operators are present in the aggregation function. A detailed discussion of these methods can be found in [6]. Also, in [6] we have shown, using an example, how the proposed method can be used for extracting weights using gradient based method for fuzzy signatures especially when it uses *max*, *min,* and/or *max-avg* aggregation functions for weighted inference.

A. Fuzzy Signatures and Weighted Relevance Aggregation

Fuzzy signatures are vector valued fuzzy sets, where each vector component can be a further vector valued fuzzy set [3]. A fuzzy signature, *s* can be defined as,

$$X \to [a_i]_{i=1}^{k}, \ \ where \ \ a_i = \begin{cases} [0,1]; \ if \ \ leafe \\ [a_{ij}]_{j=1}^{k_i}; \ if \ \ branch \end{cases}$$

Fig.1 shows an example of a fuzzy signature structure with two arbitrary levels *g* and *(g+1)*. Now, aggregation of an arbitrary branch $a_{p...i}$ in level *g* (fig.1) can be written as, $a_{p...i} = @_{p..i}\{a_{p...ij}\}$, where $@_{p..i}$ is an arbitrary aggregation function, *j=1…n*, and $a_{p...ij} \in [0,1]$. These aggregation functions, which are used for inference in fuzzy signatures, can be simple aggregations like minimum (min), average (avg), maximum_average (max-avg), or maximum (max), which were used in [7], or they can be complex aggregation functions as proposed in [3].

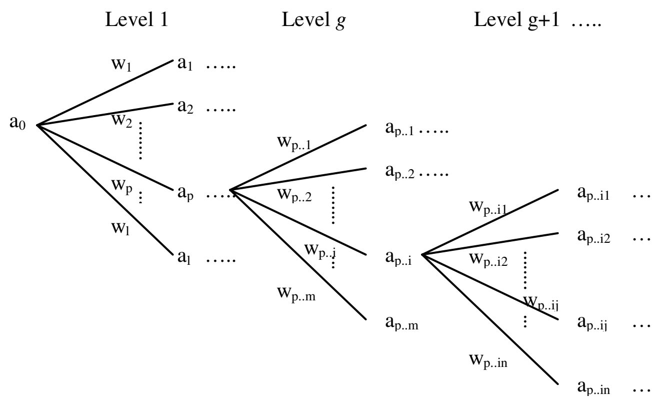

Fig. 1. Fuzzy Signature Structure with two arbitrary levels g and (g+1)

The weighted relevance aggregation concept was proposed in [7] to provide additional expert knowledge to the fuzzy signature structure by introducing the weighted relevance of each branch to its higher branches of the fuzzy signature structure. Thus, the weighted relevance will give an additional ability to fuzzy signatures for decision making in situations where input data are vague and complex.

Weighted relevance aggregation of an arbitrary branch $a_{p...i}$ in a fuzzy signature (fig.1) can be defined as,

$$a_{p...i} = @_{p...i}\{w_{p...ij} \bullet a_{p...ij}\} \tag{1}$$

, where $@_{p..i}$ is an arbitrary aggregation function, *i=1…n*, and $a_{p...ij} \in [0,1]$. The following properties hold for arbitrary weighted relevance w_p,

i) $w_p \in [0,1]$

ii) $\sum_{p=1}^{l} w_p$ is not necessarily equal to 1.

B. Gradient Based Learning in Weights Extraction for Hierarchical Fuzzy Signatures

In [5], learning of OWA operators for conventional fuzzy systems has been discussed. We have proposed a similar method in [6] to learn weights for fuzzy signatures using the steepest gradient descent method.

Now, let us assume that we have *t* number of records (observations) in a training data set. Also, let us denote d_k as the desired value for the training data record $k(\le t)$ and e_k be the squared error between the desired value and the actual value of the k^{th} record. The square error e_k can be written as $e_k = \frac{1}{2}(a_0^k - d_k)^2$, where a_0^k is the final atomic result of the fuzzy signature in (fig.1) for training data record *k*. Further, the generalized version of the a_0^k can be written as follows,

$$a_0^k = @_0[w_i(@_i[w_{ij}(.........@_{ij...p}[w_{ij...pq} \bullet a_{ij...pq}])])] \tag{2}$$

, where $@_0$, $@_i$ and $@_{ij..p}$ are arbitrary aggregation functions, *i=1…l* , *j=1…b, p=1…m, q=1…n* , and $w_i, w_{ij}, w_{ij...pq} \in [0,1]$.

To avoid the constraints on the weighted relevance factor w_i, we replace it by following sigmoid function,

$$w_i = \frac{1}{1+e^{-\lambda_i}}, \tag{3}$$

where $\lambda_i \in \Re$. After the above transformation it becomes clear that for any values of the parameter λ_i the weighted relevance $w_i \in [0,1]$ and $\sum w_i$ is not necessarily equal to 1. Now equation (1) can be modified as,

$$a_{p...i} = @_{p...i}\left\{\left[\frac{1}{1+e^{\lambda_{p...ij}}}\right] \bullet a_{p...ij}\right\} \tag{4}$$

Also, using the equation (4) the equation (2) we can modified as,

$$a_0^k = @_0\left[\frac{1}{1+e^{\lambda_i}}\left(@_i\left[\frac{1}{1+e^{\lambda_{ji}}}\left(.........@_{ij...p}\left[\frac{1}{1+e^{\lambda_{ij...pq}}}\bullet a_{ij...pq}\right]\right)\right]\right)\right]$$

After applying the transformation (3) into the problem, it will become a unconstrained optimization problem and we need to learn λ_i instead of w_i. The appropriate value of arbitrary $w_{ij...pq}$ can be calculated using (3) the optimized parameter $\lambda_{j...pq}$. The steepest gradient descent method has been used to minimize the error e_k. For an arbitrary $\lambda_{ij...pq}$ at arbitrary level *(g+1)* in fig.1, the next update $\lambda_{ij...pq}^{next}$ can be written as,

$$\lambda_{ij...pq}^{next} = \lambda_{ij...pq} - \beta\left(\frac{\partial e_k}{\partial \lambda_{ij...pq}}\right)$$

, where β is the learning rate. In this way we can obtain next updates for each λ in the fuzzy signature structure. These updates can be used to find the new weights for the fuzzy signature using equation (3).

C. Derivatives of *max-min* Functions.

Generally, m*ax-min* functions are not strictly differentiable at all points. Therefore, special methods need to be used to obtain derivatives for such function. A mathematically proved method, which can be used to obtain derivatives of *max-min* functions, has been discussed in [2]. In [6] we have shown that how these theorems can be used to obtain derivative of aggregation functions, which consist of *max*, *min,* and/or *max-avg* [7] [6] operators, in fuzzy signatures.

Now we can write the deravative of e_k with respect to $\lambda_{p...ij}$ (fig. 1) as,

$$\frac{\partial e_k}{\partial \lambda_{p...ij}} = \left(a_0^k - d_k\right)\frac{\partial a_0^k}{\partial \lambda_{p...ij}}$$

The value of $\left(\frac{\partial a_0^k}{\partial \lambda_{p...ij}}\right)$ can be written, using side derderavative of a_0 with respect to $\lambda_{p...ij}$, as follows [6],

$$\frac{\partial a_0}{\partial \lambda_{p...ij}^{\pm}} = \frac{1}{2}\left\{\frac{\partial a_0}{\partial \lambda_{p...ij}^{+}} + \frac{\partial a_0}{\partial \lambda_{p...ij}^{-}}\right\} \quad (5)$$

Let us assume that following aggregation functions $\{\min, avg, \max - avg, \max\}$ are used for the aggregation of fuzzy signature in fig.1. Also, recall the equation (1), which represents the agrregated result of an arbitary branch $a_{p...i} = @_{p...i}\{w_{p...ij} \bullet a_{p...ij}\}$, where $@_{p...i} \in \{\min, avg, \max avg, \max\}$. Now we get four different cases for equation (1) depending on the selection of the $@_{p...i}$. And also, there are two side derivatives (left and right) for each case.

For all cases, Let *g* be an arbitrary level in the fuzzy signature a_0 (fig.1) and *(g+1)* be next consecutive level of the same fuzzy signature, and $\Omega_{p...i} = \{k | k \in [1,n] \ \ and \ \ a_{p...i} = w_{p..qk} \bullet a_{p..ik}\}, i \in [1,m]$, and $p \in [1,l]$.

<u>Case I</u> ($@_{p...i} = \min$):

$$\frac{\partial a_{p...i}}{\partial \lambda_{p...ij}^{+}} = \begin{cases} 0 & ; if \ \ \Omega_{p...i} \neq \{j\} \\ \frac{\partial\left(w_{p...ij} \bullet a_{p...ij}\right)}{\partial \lambda_{p...ij}^{+}} & ; if \ \ \Omega_{p...i} = \{j\} \end{cases} \quad \text{(I1)}$$

$$\frac{\partial a_{p...i}}{\partial \lambda_{p...ij}^{-}} = \begin{cases} 0 & ; if \ \ j \notin \Omega_{p...i} \\ \frac{\partial\left(w_{p...ij} \bullet a_{p...ij}\right)}{\partial \lambda_{p...ij}^{-}} & ; if \ \ j \in \Omega_{p...i} \end{cases} \quad \text{(I2)}$$

<u>Case II</u> ($@_{p...i} = \max$):

$$\frac{\partial a_{p...j}}{\partial \lambda_{p...ij}^{+}} = \begin{cases} 0 & ; if \ \ \Omega_{p...i} \neq \{j\} \\ \frac{\partial\left(w_{p...ij} \bullet a_{p...ij}\right)}{\partial \lambda_{p...ij}^{+}} & ; if \ \ \Omega_{p...i} = \{j\} \end{cases} \quad \text{(II1)}$$

$$\frac{\partial a_{p...i}}{\partial \lambda_{p...ij}^{-}} = \begin{cases} 0 & ; if \ \ \Omega_{p...i} \neq \{j\} \\ \frac{\partial\left(w_{p...ij} \bullet a_{p...ij}\right)}{\partial \lambda_{p...ij}^{-}} & ; if \ \ \Omega_{p...i} = \{j\} \end{cases} \quad \text{(II2)}$$

<u>Case III</u> ($@_{p...i} = avg$):

$$\frac{\partial a_{p...i}}{\partial \lambda_{p...ij}^{+}} = \frac{1}{n}\left\{\frac{\partial\left(w_{p..ij} \bullet a_{p..ij}\right)}{\partial \lambda_{p..ij}^{+}}\right\} \quad \text{(III1)}$$

$$\frac{\partial a_{p...i}}{\partial \lambda_{p...ij}^{-}} = \frac{1}{n}\left\{\frac{\partial\left(w_{p..ij} \bullet a_{p..ij}\right)}{\partial \lambda_{p..ij}^{-}}\right\} \quad \text{(III2)}$$

<u>Case IV</u> ($@_{p...i} = \max - avg$): In this case equation (1) can be modified as follows,

$$a_{p...i} = \frac{1}{2}\left[\max\{a_{p...ik} \times w_{p...ik}\} + avg\{a_{p...ik} \times w_{p...ik}\}\right]$$

$$\frac{\partial a_{p..q}}{\partial \lambda_{p..qr}^{+}} = \begin{cases} \frac{1}{n}\left\{\frac{\partial\left(a_{p..qr} \times w_{p..qr}\right)}{\partial \lambda_{p..qr}^{+}}\right\} & ; if \ \ r \notin \Omega_{p..q} \\ \frac{\partial\left(a_{p..qr} \times w_{p..qr}\right)}{\partial \lambda_{p..qr}^{+}} + \frac{1}{n}\left\{\frac{\partial\left(a_{p..qr} \times w_{p..qr}\right)}{\partial \lambda_{p..qr}^{+}}\right\} & ; if \ \ r \in \Omega_{p..q} \end{cases} \quad \text{(IV3)}$$

$$\frac{\partial a_{p..q}}{\partial \lambda_{p..qr}^{-}} = \begin{cases} \frac{1}{n}\left\{\frac{\partial\left(a_{p..qr} \times w_{p..qr}\right)}{\partial \lambda_{p..qr}^{-}}\right\} & ; if \ \ \Omega_{p..q} \neq \{r\} \\ \frac{\partial\left(a_{p..qr} \times w_{p..qr}\right)}{\partial \lambda_{p..qr}^{-}} + \frac{1}{n}\left\{\frac{\partial\left(a_{p..qr} \times w_{p..qr}\right)}{\partial \lambda_{p..qr}^{-}}\right\} & ; if \ \ \Omega_{p..q} = \{r\} \end{cases} \quad \text{(IV4)}$$

III. EXPERIMENTS

In this section two real world problems, which are SARS Patients Classification problem and High Salary Selection of employees [7], have been used for the experiments. In [7] we have investigated the results of these problems manually, constructed weights for fuzzy signatures using human experts suggestions. The scope of this experiment is to show the proposed weights extraction method works similar or better than compared to the results of the same experiments in [7] but using the manual weights.

A. Experiment 1: High Salary Selection.

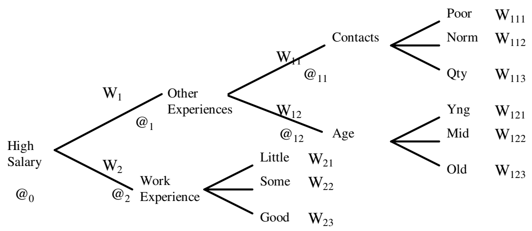

Fig. 2. High Salary Selection Fuzzy Signature

The High Salary selection fuzzy signature structure (fig.2) gives the degree of relevance of having a high salary according to contacts, age, and work experience of particular employee, has been used for the first experiment. The scope of the first experiment is to extract the weighted relevance for the High Salary Selection fuzzy signature structure automatically. Also, it was expected that the result of the High Salary Selection fuzzy signature with learnt weights should perform at least well as the results of (fig 3) High Salary Selection fuzzy signature with manual weights [7].

For this experiment 200 records of test data and 135 records of training data sets were used. The test data set is the original data set used in [7] and the train data set was derived using the same set of rules, which were used to obtain the test data set. Each record of test and train data contains employees following details: contacts, age, and work experience. Those employees actual salary scale falls into 3 soft categories namely low_salary, mid_salary,

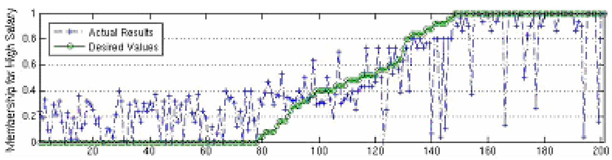

Fig.3. Experiment 1 with manual weights [7]

and high_salary. The *desired values* plot in the fig.3 shows the degree of relevance of each employee's actual salary category to high salary category. For an example, the membership, of employees records from 1-90, to high salary category are less than 0.2. Thus they belong to low_salary category. The *Actual Results* plot (fig.3) shows the results of the High Salary Selection fuzzy signature with manual weights [7].

For the first experiment, the set of combinations of aggregation functions, which are shown in table 1, have been used. This set of aggregation function is also selected according to domain experts' suggestions.

$@_0 = min$	$@_2 = max$	$@_{12} = max$
$@_1 = min$	$@_{11} = max$	

Table 1. Combination of aggregations - SALARY

Fig. 4 shows the test results of the experiment 1 with automatically learnt weights, using the proposed method, at the training phase. According to fig.4 it is clear that the *Actual* data plot in fig.4 is smoother than that of the fig.3. In other words, actual result plot in fig.4 is always follows the desired output pattern better than compared to that of the fig.2.

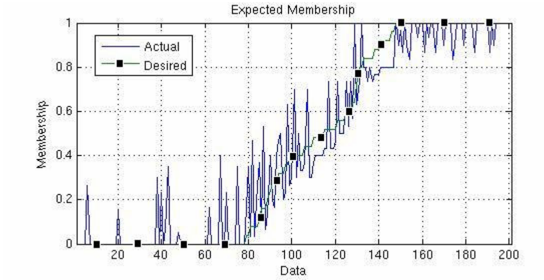

Fig.4. Results of Experiment 1

From the experiment 1, it can be seen that a fuzzy signature with extracted weights can classify the results of salary data into the expected 3 categories more accurately. Therefore, experiment 1 demonstrates the accuracy of our automated weighted relevance extraction method for a hierarchical fuzzy signature structure.

B. Experiment 2: SARS Patients Classification.

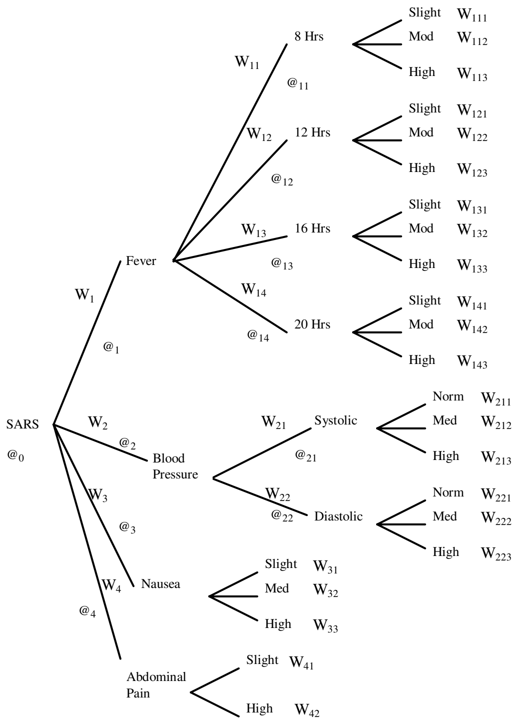

Fig.5. SARS Patients Classification Fuzzy Signature (levels 1 to 3)

The SARS Patients Selection fuzzy signature structure in fig.5 has been used for the second experiment. Same 4,000 records of test patient data, which was used in [7], has been used for testing and another 4000 records of training data set was derived. Each record of test and train data contains 4 different categories of patients namely SARS, normal, pneumonia, and hypertension, have been used for the second experiment. Each of these categories mentioned above has 1,000 records of data, which were derived from real world values using some randomization.

The first experiment with SARS Patients Classification fuzzy signature was carried out to extract the weighted relevance to classify the patients according to input data into two different categories of having SARS or not having SARS, with 1.0 degree of relevance for SARS patient data and 0.0 degree of relevance for non-SARS patient data, were given. The following combination of aggregation functions, which was the best combination in [1], shown in table 2 has been used for the second experiment.

$@_0 = max$	$@_3 = max$	$@_{12} = min$	$@_{21} = min$
$@_1 = max$	$@_4 = max$	$@_{13} = min$	$@_{22} = min$
$@_2 = max$	$@_{11} = min$	$@_{14} = min$	

Table 2. Combination of aggregations - SARS

Fig.6 shows the test results of the experiment. According to this figure all SARS patients data (range 3,000 – 4,000) were kept above 0.5 and all non-SARS patients data (range 1,000 – 3,000) were kept to 0.0 degree of relevance to SARS condition. Thus, we can conclude that our method of extracting weighted relevance for SARS fuzzy signature can configure proper weights for weighted inference in hierarchical fuzzy signature structure for different real world problems.

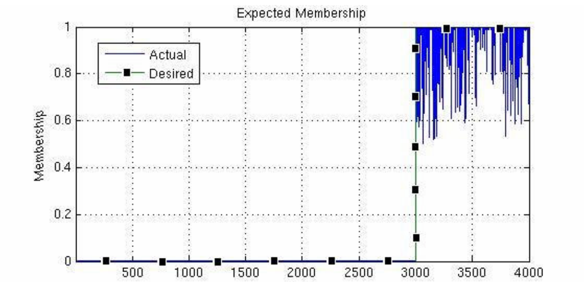

Fig. 6. Results of Experiment 2 – 2 output categories

Further during the second experiment, we were interested to test the capability of the weighted relevance aggregation method, for hierarchical fuzzy signature structures, to separate data into all significantly available different categories based on input data. Also, to observe the ability of our weights learning method to learn well configured weights when output contains more than two categories. Therefore we extended the second experiment to learn weights to classify the 4,000 set of test data into more than two output categories.

For this experiment, the same SARS Patient Selection fuzzy signature in fig.5 with same test data and aggregation combination were selected. First, for the output 3 target categories were given. The 3 target categories are namely SARS patients, non-SARS patients and normal persons. The expected output membership of the fuzzy signature to fall into SARS patients, non-SARS patients and normal persons are 1.0, 0.3, and 0.0 respectively.

As shown in the fig.7 the results of the experiment were not able to classify the test patients data results into 3 output categories, which were expected to be SARS patients (1.0), non-SARS patients (0.3) and normal persons (0.0). Instead, it

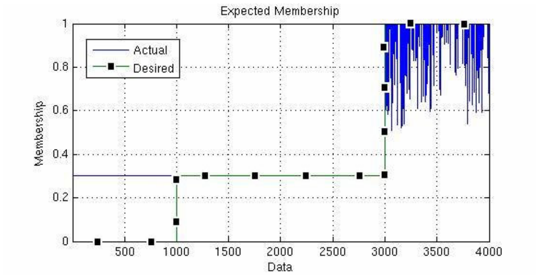

Fig. 7. Results of Experiment 2 - unsuccessful 3 output categories

categorizes the test data into two different categories. That is it kept all SARS patient data above 0.5 degree of relevance and all other data into approximately 0.3 degree of relevance. We repeated the experiment with changing different aggregation combinations as suggested by domain experts'. Good results were obtained after changing the aggregation combination, according to human experts' knowledge, as shown in table 3.

$@_0 = max$	$@_3 = max$	$@_{12} = min$	$@_{21} = min$
$@_1 = max$	$@_4 = max$	$@_{13} = min$	$@_{22} = min$
$@_2 = max$	$@_{11} = min$	$@_{14} = min$	

Tabel 3. New Combination of aggregations - SARS

Fig.8 show the successful results of the experiment with 3 target output results categories. In fig.8, results of normal persons data (range 0-1,000) have been kept at 0.0, results of non-SARS patients data (range 1,000-3,000) have been kept between [0.2, 0.4], and SARS patients data (range 3,000-4,000) have been kept between [0.5, 0.9] degree of relevance to SARS condition. According to the results of this experiment it can be seen that proposed weights extraction method is able to learn weights to classify input data into more than just two categories. Also, we see need of a method to identify the best aggregation combination for the hierarchical fuzzy signatures.

Next, we further divided the non-SARS output category into two categories namely pneumonia and hypertension. Now, the output target is widened and we have given 4 output target categories to the learning algorithm.

For the final experiment, the same SARS Patient Selection fuzzy signature in fig.5 with same test data and new aggregation combination in table 3 were selected. The expected output membership of the fuzzy signature to fall into SARS patients, pneumonia patients, hypertension patients, and normal persons are 1.0, 0.3, 0.2 and 0.0 respectively.

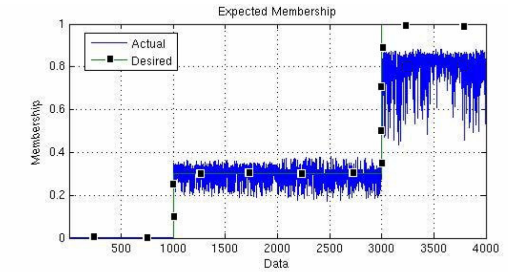

Fig. 8. Results of Experiment 2 – 3 output categories

Fig.9 shows the good results of the final experiment with 4 target output results categories. In fig.9, results of normal persons data (range 0-1,000) have been kept at 0.0, results of pneumonia patients data (range 1,000-2,000) have been kept between (0.0, 0.2), hypertension patients data (range 2,000-3,000) have been kept between (0.2, 0.5] and SARS patients data (range 3,000-4,000) have been kept between [0.4, 0.1) degree of relevance to SARS condition. The results of the final experiment show the ability of the proposed learning method to learn all available and complex output target patterns.

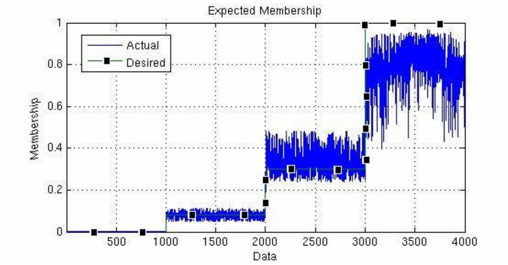

Fig. 9. Results of Experiment 4 – 4 output categories

Thus, the results of the second experiment with SARS Patients Selection fuzzy signature shows that the proposed weights extraction method can learn accurate and effective weights for weighted relevance aggregation method to separate input data into all available categories of output data. As the steepest descent learning method is a local minimizer, future work will be the replacement of this algorithm by a global minimizer or a more improved gradient based learning method. Also, it is clear that our weighted relevance aggregation method for fuzzy signatures is a sophisticated classifier for complex structured real world problems, which normally only humans can handle. During the experiments, a need of a method to identify aggregation combinations for weighted relevance aggregation method from observations has arisen.

IV. CONCLUSION

The requirements of the weighted relevance aggregation method and an algorithm for obtaining weights for the weighted relevance aggregation method for hierarchical fuzzy signatures have been discussed. We have also described how to differentiate *min-max* function for gradient based learning of fuzzy signatures. Two experiments were carried out to find the applicability of the weights extraction method we proposed. The first experiment illustrates the accuracy of the results of our proposed weights extraction method for hierarchical fuzzy signatures. The second experiment concludes not only the accuracy of the proposed method but it also shows the extraction of effective configuration of weights by the proposed weights extraction method for complex classification of input data. Further, all these experiments concluded the ability of weighted relevance aggregation method with hierarchical fuzzy signature as a sophisticated classifier for complex structured real world problems such as medical and economic diagnosis.

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