Hierarchical Fuzzy Signature Structure for Complex Structured Data

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Abstract—This paper presents an algorithm for constructing a hierarchical fuzzy signature structure by using concepts from interclass separability and hierarchical fuzzy systems. Fuzzy signature is introduced to handle complex structured data and interdependent features problems. Fuzzy signature can also be used in cases where data is missing. However, when dealing with a very large data set, it is possible that they hide hierarchical structure that appears in the sub-variable structures. The proposed hierarchical fuzzy signature structure will be used in problems that fall into this category. In the end, reduced hierarchical fuzzy signature structures can assist human experts better by removing unnecessary information when making decisions.

Index Terms—Fuzzy signature structure, hierarchical, complex structured data, and large databases.

I. INTRODUCTION

Fuzzy modelling has become very popular because of the main feature of its ability to assign meaningful linguistic labels to the fuzzy sets [1] in the rule base [2,3]. However, a serious problem is caused by the high computational time and space complexity of rule bases describing systems with multiple inputs with proper accuracy. The complexity allows little general systems application (or real time control application) of classical fuzzy algorithms, where the inputs exceed about 6 to 10. These traditional fuzzy systems deal with very simple structured data, where the number of inputs is well defined, and values for each input occur for most or all data items. This further reduces their general applicability.

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Basically, fuzzy rule bases suffer from rule explosion. The number of possible rules necessary is \(O(T^K)\) where \(K\) is the number of dimensions and \(T\) is the number of terms per input. In order to increase the problems solvable by fuzzy rule-base systems, it is essential to reduce \(T, k\), or both. Decreasing \(T\) leads to sparse fuzzy systems, i.e. fuzzy rule-bases with “gaps” between the rules [4]. On the other hand, decreasing \(K\) leads to hierarchical fuzzy systems [5, 6]. The hierarchical fuzzy system is based on the following idea. Often, the multi-dimensional input space \(X = X_1 \times X_2 \times \ldots \times X_k\) can be decomposed into some subspaces, e.g. \(Z_0 = X_1 \times X_2 \times \ldots \times X_k(< k)\), so that in \(Z_0\) a partition \(\Pi = \{D_1, D_2, D_3\}\) can be determined. In each \(D_i\), a sub-rule base \(R_i\) can be constructed with local validity. The hierarchical rule base structure becomes:

\[
R_0: \text{if } Z_0 = D_1 \text{ then use } R_1 \\
\quad \text{if } Z_0 = D_2 \text{ then use } R_2 \\
\quad \ldots \\
R_k: \text{if } Z_0 = D_k \text{ then use } R_k
\]

The fuzzy rules in rule base \(R_0\) are termed meta-rules since the consequences of the rules are pointers to other sub-rule bases instead of fuzzy sets. The complexity of fuzzy systems can be reduced when the suitable \(Z_0\) and \(\Pi\) are found such that in each sub-rule base \(R_i\) the input space \(X_i\) is a subspace of \(X / Z_0 = X_{D_1} \times X_{D_2} \times \ldots \times X_k\) [6]. The main difficulty in the automatic construction of such system is largely in finding a suitable subspace \(Z_0\) and \(\Pi\).

Fuzzy signatures which structure data into vectors of fuzzy values, each of which can be a further vector, are introduced to handle complex structured data [7, 8, 9]. This will widen the application of fuzzy theory to many areas where objects are complex and sometimes interdependent features are to be classified and similarities / dissimilarities evaluated. Often, human experts can and must make decisions based on comparisons of cases with different numbers of data components, with even some components missing. Fuzzy signature is created with this objective in mind. This tree structure is a generalisation of fuzzy sets and vector valued...
fuzzy sets in a way modelling the human approach to complex problems.

However, when dealing with a very large data set, it is possible that they hide hierarchical structure that appears in the sub-variable structures. This paper is used to address problems having this characteristic, and the possible use of hierarchical structure in creating the fuzzy signature.

II. FUZZY SIGNATURE

The original definition of fuzzy sets was $A: X \rightarrow [0,1]$, and was soon extended to $L$-fuzzy sets by Goguen [10],

$$A_L : X \rightarrow \mathbb{L}^k$$

$A_L : X \rightarrow \mathbb{L}, \mathbb{L}$ being an arbitrary algebraic lattice. A practical special case, Vector Valued Fuzzy Sets was introduced by Kóczy [11], where $A_{v,k} : X \rightarrow [0,1]^k$, and the range of membership values was the lattice of $k$-dimensional vectors with components in the unit interval. A further generalisation of this concept is the introduction of fuzzy signature and signature sets, where each vector component is possibly another nested vector (right).

Fuzzy signature can be considered as special multi-dimensional fuzzy data. Some of the dimensions are inter-related in the sense that they form sub-group of variables, which jointly determine some feature on a higher level. Let us consider an example. Figure 1 shows a fuzzy signature structure.

The fuzzy signature structure shown in Figure 1 can be represented in vector form as follow:

$$x = \begin{bmatrix} x_{11} \\ x_{12} \\ x_{21} \\ x_{221} \\ x_{222} \\ x_{223} \\ x_{23} \\ x_{31} \\ x_{32} \end{bmatrix}^T$$

$$A_L : X \rightarrow \mathbb{L}, \mathbb{L}$$

The relationship between higher and lower level is governed by a set of fuzzy aggregations. The results of the parent signature at each level are computed from their branches with appropriate aggregation of their child signature. Let $a_1$ be the aggregation associating $x_{11}$ and $x_{12}$ used to derive $x_1$, thus $x_1 = x_{11} \cdot a_1 \cdot x_{12}$. By referring to Figure 1, the aggregations for the whole signature structure would be $a_1, a_2, a_{22},$ and $a_3$. The aggregations $a_1, a_2, a_{22}$, and $a_3$ are not necessarily identical or different. The simplest case for $a_{22}$ might be the min operation, the most well known t-norm. Let all aggregations be $\min$ except $a_{22}$ be the averaging aggregation. We will show the operation based on the following fuzzy signature values for the structure in the example.

$$x = \begin{bmatrix} 0.3 \\ 0.4 \\ 0.2 \\ 0.6 \\ 0.8 \\ 0.1 \\ 0.9 \\ 0.1 \\ 0.7 \end{bmatrix}^T$$

After the aggregation operation is perform to the lowest
branch of the structure, it will be described on higher level as:

\[
x = \begin{bmatrix} 0.3 \\ 0.2 \\ 0.5 \\ 0.9 \\ 0.1 \end{bmatrix}
\]

Finally, the fuzzy signature structure will be:

\[
x = \begin{bmatrix} 0.3 \\ 0.2 \\ 0.1 \end{bmatrix}
\]

Each of these signatures contains information relevant to the particular data point \( x_0 \); by going higher in the signature structure, less information will be kept. In some operations it is necessary to reduce and aggregate information to become compatible with information obtained from another source (some detail variables missing or simply being locally omitted). Such is when interpolation within a fuzzy signature rule base is done, where the fuzzy signature flanking an observation are not exactly of the same structure. In this case the maximal common sub-tree must be determined and all signature must be reduced to that level in order to be able to interpolate between the corresponding branches or roots in some cases.

III. HIERARCHICAL DATA STRUCTURE

When dealing with a very large data set, it is possible that they hide hierarchical structure that appears in the sub-variable structures. In this case, the normal fuzzy signature structure as described in the previous section will be huge. In this case, it is necessary to identify the hierarchical structure hidden in the large data set. This section discusses the method used to generate hierarchical fuzzy system from data with hierarchical nature. In the next section, the method is extended to generate hierarchical fuzzy signature.

The main requirement of a reasonable \( \Pi \) is that each of its elements \( D_i \) can be modelled by a rule base with local validity. In this case, it is reasonable to expect \( D_i \) to contain homogeneous data. The problem of finding \( \Pi \) can thus be reduced to finding homogeneous structures within the data. This can be achieved by clustering algorithms.

The subspace \( Z_0 \) is used by meta-rules to select the most appropriate sub-rule base to infer the output for a given observation (i.e. system input). In general, the more separable the elements in \( \Pi \), the easier the sub-rule base selection becomes. Therefore, the proper subspace can be determined by means of some criterion that ranks the importance of different subspaces (combination of variables) based on their capability in separating the components \( D_i \).

Unfortunately, the problems of finding \( \Pi \) and \( Z_0 \) are not independent of each other. Consider the following helicopter control fuzzy rules extracted from the hierarchical fuzzy system in [5]:

\[
\text{If distance (from obstacle) is small then Hover} \\
\text{Hover: if (helicopter) body rolls right then} \\
\text{move lateral stick leftward} \\
\text{if (helicopter) body pitches forward then} \\
\text{move longitudinal stick backward}
\]

Geometrically, an example of the data corresponding to the fuzzy rules is illustrated in Figure 2. From the figure, three clusters can be clearly observed in \( x_1 \). However, from \( x_2 \) and \( x_3 \), the data points are evenly distributed across the entire domain. Therefore, finding a ‘good’ subspace \( Z_0 \) where the clusters are clearly separable is crucial for the success of the clustering algorithm.

On one hand, we need to rely on the feature selection technique to find a ‘good’ subspace \( Z_0 \) for the clustering algorithm to be effective. However, most of the feature selection criteria assume the existence of the clusters beforehand.

![Figure 2: Geometric representation data in a hierarchical data structure](image)

One way to tackle this problem is to adopt a projection-based approach. The data is first projected to each individual dimension and fuzzy clustering is performed on the individual dimensions. The number of clusters \( C \) for clustering can be chosen arbitrarily large. With the set of one-dimensional fuzzy clusters obtained, the separability (importance) of each input dimension can be determined separately. With the ranking, we can then select the \( n \) most important features to form the \( n \)-dimensional subspace \( Z_0 \).

In this case here, a hierarchical fuzzy system can be obtained using the following steps:
1. Perform fuzzy c-means clustering on the data along the subspace $Z_0$. The optimal number of clusters, $C$, within the set of data is determined by means of the FS index [12].

2. The previous step results in a fuzzy partition $\Pi = \{D_1, \ldots, D_C\}$. For each component in the partition, a meta rule is formed as:

   \[ \text{If } Z_0 \text{ is } D_i \text{ then use } R_i \]

3. From the fuzzy partition $\Pi$, a crisp partition of the data points is constructed. That is, for each fuzzy cluster $D_i$, the corresponding crisp cluster of points is determined as

   \[ P_i = \{p | \mu_i(p) > \mu_j(p) \forall j \neq i\} . \]

4. Construct the sub-rule bases. For each crisp partition $P_i$, apply a feature extraction algorithm to eliminate unimportant features [13]. The essence of this feature selection is to look for cylindricity in the input variables. The remaining features (know as true inputs) are then used by a fuzzy rule extraction algorithm to create the fuzzy rule base $R_i$. Here, the use of the projection-based fuzzy modeling approach could be used [14].

IV. HIERARCHICAL FUZZY SIGNATURE STRUCTURE

In the previous section, the concept of finding interclass separability and the algorithm of constructing the hierarchical fuzzy rules have been discussed. In this section, we will take a look at how those concepts can be used to construct the hierarchical fuzzy signature structure. Normally in a large database, it consists of different hierarchical structure and sub-variables for different purposes. It is therefore necessary to reduce the structure and remove any redundant information when presenting the final fuzzy signature structure to human. The main objective of this research is to deal with large data set, hopefully to generate a reduced fuzzy signature structure that omits unnecessary information. Each item of the data has their own fuzzy signature structure, i.e. a set of treelike structured features.

The propose algorithm for constructing the hierarchical fuzzy signature structure technique is as follows:

1. Rank the importance of input variables, using the interclass separability criterion described in the previous section.

2. For each cluster $C_i$, eliminate the variable that cylindricity is found.

3. If there is more level in that variable, move one level down and perform sub cluster identification, i.e. repeat steps 1 and 2.

4. Step 3 is repeated until no more variable can be removed.

5. The reduced fuzzy signature structure is considered a hierarchical fuzzy signature structure.

6. Repeat step 2 for the next identified cluster $C_i$, until all clusters have been processed.

   By using the example in Figure 1. Let us assume that a large database is given: $S = \{x_i\}$. A hierarchical fuzzy signature structure based on $S$ can be constructed as follow:

   1. Reduce the data to the highest level (to vectorial fuzzy values), which represents the least details model of the data set, i.e.

   \[ x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}^T \]

   2. By applying the interclass separability technique, where potential cylindric clusters $C_i$ can be determined, i.e. where data points are evenly distributed along the dimension.

   3. For each cluster $C_i$, identification of the cylindricity is performed. Suppose, $C_1$ and $C_2$ were found in $x_1$, where $C_1$ is cylindric in $x_2$ and $C_2$ is cylindric in $x_3$.

   4. First, investigate $C_1$, and within $C_1$, $x_3$ can be moved one level down to $x_{31}$ and $x_{32}$.

   5. The interclass separability is performed at this local area to see whether a local reduction can be done.

   6. Similarly for $C_2$, the model is split into $x_{21}$, $x_{22}$, and $x_{23}$ by going down one level.

   7. Again a local interclass separability is performed. If $x_{23}$ was found most separating and it gives $C_{21}$ and $C_{22}$, further in $C_{21}$, $x_{21}$ is eliminated, while in $C_{22}$, $x_{22}$ is eliminated.

   8. For $C_{22}$, it can be kept as such, however for $C_{21}$, it can be split further more to $x_{221}$, $x_{222}$, and $x_{223}$.

   9. Until this point without going further into $x_{221}$, $x_{222}$, and $x_{223}$, we already know $x_2$ and $x_1$ can be eliminated. Thus, the reduced signature structure for $C_2$ at this point is:
If further processing is continued for the above case, the interclass separability check can be performed on $x_{221}$, $x_{222}$, and $x_{223}$ and may be further sub-clusters $C_{22i}$ can be obtained. Thus possible further reduction of the three may be performed locally. The steps are repeated until all lower levels have been visited to obtain a full hierarchical fuzzy signature structure.

V. CONCLUSIONS

We have described a technique for fuzzy signature structure for dealing with large database. The hierarchical fuzzy signature structure presented can be constructed by feature selection and interclass separability to arrive at a reduced fuzzy signature structure which best describes the complex and interdependent features. This hierarchical fuzzy signature structure can also handle missing data and remove any redundant information to solve a problem. This will greatly assist human expert in understanding the problem and making decision. The future work for this hierarchical fuzzy signature structure will be to examine the use of fuzzy rules interpolation concept to reduce rules in cases where cylindricity cannot be found.

REFERENCES