FUZZY RULE INTERPOLATION USING AN ADDITIVE CONSERVATIVE STRATEGY

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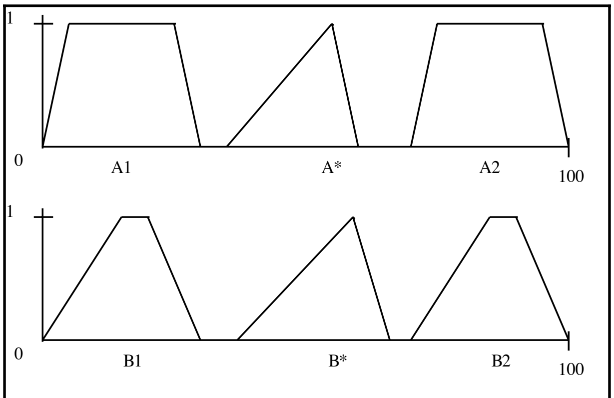
ABSTRACT: Fuzzy rule based systems are useful in application domains where approximate reasoning is required. Where the information is not available to construct a complete, comprehensive rule base either due to availability cost or natural gaps, sparse rule bases are used. Fuzzy rule interpolation is used to provide conclusions for observations for which there may be no overlap with even the supports of existing rules in the rule base. All of the methods are descendants of the Kóczy and Hirota (1990, 1993) method of linear interpolation, and have various advantages and disadvantages. A primary advantage can be in terms of reduced computational cost, while major disadvantages of many techniques are the distorted and abnormal fuzzy rule produced under some circumstances. In this paper we introduce our method which always produces acceptably formed rules, and is additively conservative (Gedeon and Kóczy, 1996) with respect to the degree of local fuzziness in the rule base, as measured from the nearby rules in the fuzzy rule base.

1 INTRODUCTION

In this section we discuss the philosophical basis of our approach. Figure 1 illustrates a pair of rules:



A1 >>	0, 5, 25, 30	A* >>	35, 55, 55, 60
A2 >>	70, 75, 95,100	B* >>	37, 59, 59, 66 tg+
B1 >>	0, 15, 20, 30		
B2 >>	70, 85, 90,100		

Figure 1. Rule set p2.

The two rules are *if x is A1 then y is B1*, and *if x is A2 then y is B2*. The observation A* does not overlap with the antecedents of either of the flanking two rules. An interpolated conclusion B* is shown, between the flanking consequents of the two rules. For simplicity of presentation, both the antecedent and consequent universes of discourse have been scaled to the 0 to 100 range, and rules are represented by 2 support and 2 core points (to allow trapezoidal forms).

Our initial starting point is that we assume as little homogeneity in the rule base as possible. Thus, we treat the nearest core points of rules as all that is visible. Thus, A* is in a valley between A1 and A2, and whether A1 or A2 are actually plateaux is not visible, and is not used. The core points of B* are derived by simple linear interpolation between the nearest core points of A1, A2 and B1, B2.

Once the core points of B* are determined, we reduce further the assumption of homogeneity in the rule base. That is, the interpolation of the right of B* is only between the rightmost core point of B* and the leftmost core point of B2. This is consistent with the premise that A* need not be symmetrical, and with the notion that determining the right side of B* should be based on 'nearby' information such as the right side of A* and the left sides of A2 and B2. This is in contrast to Kóczy and Hirota's initial method which derived the right side of B* from the right sides of A1, B1, A2, B2 and A*. All of these except the latter are further away from the right side of A* or B* than the 'nearby' information we propose to use.

The use of such 'nearby' information has real world plausibility, while there are few domains where it could be readily proven that there is justification for the assumption of handedness of the rule shapes. In view of the claims to intuitive acceptability we make, we will describe and derive our formulation geometrically below. We also briefly review our previous conservative interpolation technique (Gedeon and Kóczy, 1996).

GEOMETRIC DESCRIPTION OF METHOD

Given the subsection of the previous figure shown in Figure 2.

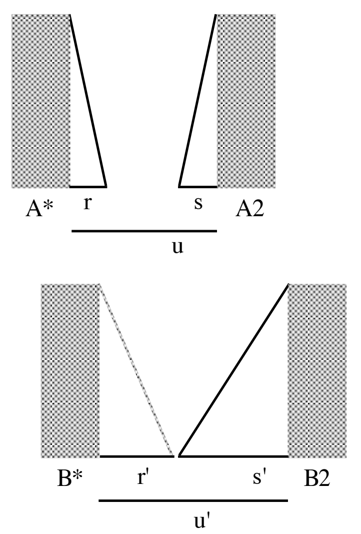


Figure 2. Section of p2.

Note that the grey regions highlight the (assumed) unknown nature of the rest of the rule base. The labels r, s indicate the spread of the observation and the rule antecedent and represent their fuzziness. The labels r', s' indicate the spread of the conclusion (drawn in grey as it is yet to be calculated) and the rule consequent. Our intuition tells us that B2 is more fuzzy than A2. We have not normalised the A*,A2 and the B*,B2 distances, as we will do this explicitly using the values of u, u' as appropriate.

Without using any information from rule 2 other than its core distances, we could calculate a value for r' as shown in Equation (1).

$$r' = r \cdot \frac{u'}{u} \qquad (1)$$

This is merely the normalisation of r from the A*,A2 distance metric into the B*,B2 distance. Most likely, the value of r' will be some increase or decrease of the effect of this normalisation. We could similarly calculate a value for the normalisation of s into the B*,B2 distance, called s''.

$$s'' = s \cdot \frac{u'}{u} \qquad (2)$$

The relationship between the actual value of s' we can measure from B2, and the calculated value s'' provides an indication of the difference in fuzziness from rule antecedent to consequent. As we have already noted, A2 is clearly fuzzier than B2. This difference can be incorporated into (1) producing:

$$r' = r \cdot \frac{u'}{u} \cdot \left(1 + \frac{s' - s''}{z}\right) \qquad (3)$$

Since we are interested in the relative change in fuzziness from rule antecedent to consequent, we divide s' - s'' by z, being either s', or s'' (Gedeon and Kóczy, 1996).

The use of either term produces a formula which has a discontinuity when either s, or s' is zero. That is, when the antecedent or consequent respectively is crisp. We note that the use of s'' as the divisor provides a particularly elegant result.

The method described in this paper avoids the problems of crisp antecedents and consequents. Further, the notion of local fuzziness is allowed to increase only from rule antecedent to consequent. That is, where the slope of the consequent is steeper than that of the antecedent, this is not propagated as this would imply that knowledge in the (sparse) rule base was sufficient to take an observation with high fuzziness and return a less fuzzy conclusion. This appears counter-intuitive to us.

The formula we use here is:

$$r' = r \, . \left(1 + \text{pos}\left(\frac{s'}{u'} - \frac{s}{u}\right)\right) \qquad (4)$$

Note that r' is no longer dependent on the ratio of the different metrics, and is determined by positive increase in the ratio of rule antecedent/consequent fuzziness respectively to the distance between nearby core points. This is the method labelled *tg+* on diagrams in this paper.

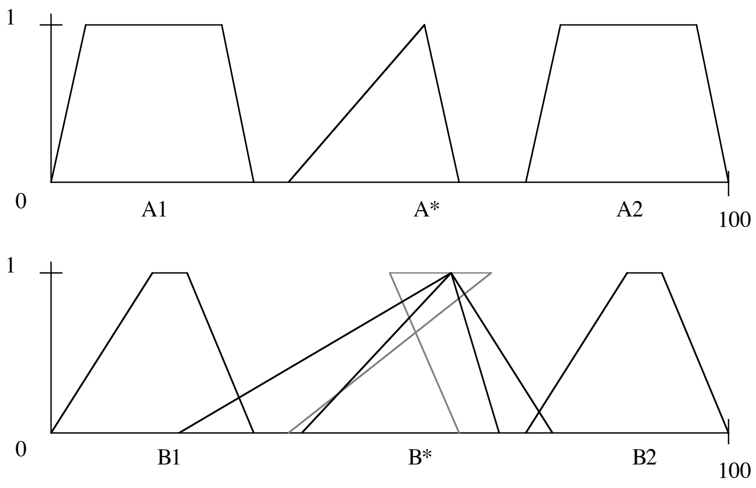
For comparisons in the rest of this paper, method *g&k* (Gedeon and Kóczy, 1996), we note that Equation (3) with $z = s''$ will not work for crisp rule antecedents, hence we resort to z = s', except where the consequent is also crisp, where we resort to Equation (1).

We can now explain the shape of r' in Figure 2. The shallower slope of B2 versus A2 indicates an increase of fuzziness which is captured by our method. The steep slope of A* indicates low fuzziness in the observation which effect is maintained by our method. The actual values are shown in Figure 1, the value of r is 5, and the value of r' is 7.

RESULTS

In this section we show a number of examples of rule interpolation using our additive fuzziness conservation rule interpolation method. The diagrams show as comparison the initial Kóczy and Hirota (1990, 1993) linear interpolation method results, and the Gedeon and Kóczy (1996) results, with impossible fuzzy rules produced being highlighted in grey. Note that most of the following figures were chosen to highlight the occasionally abnormal results of the linear interpolation method.

p2



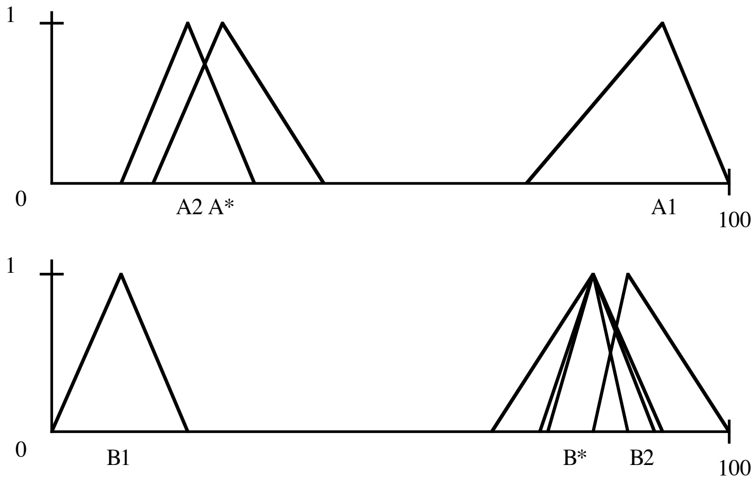
A1 >>	0, 5, 25, 30	A* >>	35, 55, 55, 60
A2 >>	70, 75, 95,100	B* >>	35, 65, 50, 60 k&h
B1 >>	0, 15, 20, 30	B* >>	19, 59, 59, 74 g&k
B2 >>	70, 85, 90,100	B* >>	37, 59, 59, 66 tg+

Figure 3. Rule set p2, methods K&H, G&K and this one.

Figures 3 illustrates the incorrect conclusions that are possible using the original rule interpolation technique. The local technique using the adjacent core points only solves this problem for both of our techniques.

In Figure 3, both the left and right fuzzy rules show a relative increase in fuzziness from antecedent to consequent. The very wide conclusion produced from the quite fuzzy observation using the *G&K* method is perhaps counter-intuitive, we prefer the less extravagant conclusion produced by our method.

p7



A1 >> 70, 90, 90,100	A* >> 15, 25, 25, 40		
A2 >> 10, 20, 20, 30	B* >> 73, 80, 80, 89	k&h	
B1 >> 0, 10, 10, 20	B* >> 72, 80, 80, 85	g&k	
B2 >> 80, 85, 85,100	B* >> 65, 80, 80, 90	tg+	

Figure 4. Rule set p7, methods K&H, G&K and this one.

Figure 4 illustrates the converse situation, where both rules are less fuzzy on the consequent. Here, our method produces a wider conclusion, which accords with our belief that a rule base can not reduce the effective fuzziness of observations.

DISCUSSION AND CONCLUSION

The interpolated rules we produce are intuitive in the sense of maintaining the local change in fuzziness in the rule base. This has significant benefit for practical application, as the degree of fuzziness is conserved. This is significant particularly in cases where the existing rules have less fuzzy consequents than antecedents. Even if we accept that this represents an increase in certainty of conclusions made using the rule, we should be treat conservatively the notion that this effect truly hold in a sparse rule base for points which are 'between' existing rules.

The difference between the geometric interpretation of a rule interpolation transform and the conceptual meaning of the operations implemented by the geometric transform is the root of the occasional problem with abnormality of results using Kóczy and Hirota's rule interpolation technique. This is the same problem with our previous technique.

From an alternative viewpoint, however, we can view the situations in which a rule interpolation technique produces abnormal results as indicating something about the interaction of the application domain, and the rule interpolation method chosen. We intend to examine these implications in another paper.

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