Paper:

Fuzzy Rule Interpolation by the Conservation of Relative Fuzziness

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If the number of variables is growing the size of fuzzy rule bases increase exponentially. To reduce size and inference/control time, it is often necessary to deal with *sparse* rule bases. In such bases, classic algorithms such as the CRI of Zadeh and the Mamdani-method do not function. In such rule bases, rule interpolation techniques are necessary. The linear rule interpolation (KH-interpolation) based on the *Fundamental Equation of Interpolation* introduced by Koczy and Hirota is suitable for dealing with sparse bases, but this method often results in conclusions which are not directly interpretable, and need some further transformations. One of the possible ways to avoid this problem is the interpolation method based on the conservation of fuzziness, proposed recently by Gedeon and Koczy for trapezoidal fuzzy sets. In this paper, a refined version of that method will be presented that is fully in accordance with the Fundamental Equation, with extensions to multiple dimensions, and then to arbitrarily shaped membership functions. Several possibilities for the latter will be shown.

Keywords: Fuzzy inference, Interpolation

1. Introduction - Fuzzy Rule Interpolation

If a fuzzy model contains k variables and maximum T linguistic (or other fuzzy) terms in each dimension, the order of the number of necessary rules is $O(T^k)$. This expression can be decreased by decreasing either T or k or both. The first method leads to sparse rule bases and rule interpolation that was first introduced by Kóczy and Hirota[6,7]. This method was based on the *Fundamental Equation of Rule Interpolation*:

$$d(A^*,A_1) : d(A^*,A_2) = d(B^*,B_1) : d(B^*,B_1) \quad \ldots\ldots (1)$$

where $A_1 \rightarrow B_1$ and $A_2 \rightarrow B_2$ are rules, A^* *is* an observation, and B^* is the conclusion searched for, and d denotes some kind of distance or degree of dissimilarity. In the first interpolation algorithms, this distance was introduced as the set of lower and upper α-cut distances,[5] describing the relative position of two comparable convex and normal fuzzy sets (CNF-sets) unambiguously. The interpretability of these distances assumes the existence of a partial ordering and a distance in the input and output universes of discourse X and Y, that is fully in accordance with the concept of gradual rules.[2] This family of interpolation techniques has various advantageous properties, but the conclusion generated by the solution of the Fundamental equation

$$\inf/\sup\{B^*_\alpha\} = \frac{\dfrac{1}{d_{\alpha L/U}(A_{1\alpha},A^*_\alpha)}\inf/\sup\{B_{1\alpha}\} + \dfrac{1}{d_{\alpha L/U}(A_{2\alpha},A^*_\alpha)}\inf/\sup\{B_{2\alpha}\}}{\dfrac{1}{d_{\alpha L/U}(A_{1\alpha},A^*_\alpha)} + \dfrac{1}{d_{\alpha L/U}(A_{2\alpha},A^*_\alpha)}} \quad \ldots\ldots (2)$$

does not conserve piecewise linearity of rules and observation. Further, it often results in all abnormal membership function for B^* that needs further transformation for obtaining a regular fuzzy set, which will in these cases not be normal, and e.g. multiple step inference by this KH-interpolation is possible only by repeated transformations of results. Kóczy and Kovacs,[9] Shi and Mizumoto,[10] Kawase and Chen[4] have determined various conditions when the conclusion of KH-interpolation is always directly interpretable. The application of these conditions leads to restriction of the shape of rules and of observations that might be an obstacle to practical applications in some contexts. Some alternative or modified interpolation algorithms avoiding the problem of conclusion have been proposed by Wu et al.[11] and Baranyi et al.[1] however, in these approaches the Fundamental Equation cannot be observed unconditionally.

Gedeon and Kóczy[3] have introduced an alternative method that always guarantees the direct interpretability of the conclusion *and* conserves the basic idea of the starting equation. However, that approach is not applicable for certain crisp special cases. In the next, we will present the basic idea of this approach, and then we will propose modified formulas that are fully applicable and observe the Fundamental Equation.

2. Interpolation by the Conservation of Fuzziness

The method introduced[3] is based on the following idea. While KH-interpolation and other related methods consider the entire shape of rule antecedents and consequence flanking observation (in a more general case: in the whole rule base), often the size of these fuzzy sets is not comparable with the observation. A typical example is when antecedents cover *regions* of input space, such as in the case of having a structured hierarchical rule base with *sparse partition*[8], each element of the partition having usually a much larger extent than the observation (which latter might even be crisp). In this example, the "partition" is a set of disjoint fuzzy subsets in one of subspaces of the entire input space (in the sense that subspace is one of the projections of original space). Suppose that the input universe is $X = X_1 \times X_2$ where each X_i might be multi-dimensional in general, then the partition Π is in the upper, "meta-rule" level, so

$$\Pi = \{A_{01}, A_{02} \dots, A_{0r_0}\},$$

and Π is sparse if

$$supp(\bigcup_{i=1}^{r_0} A_{0i}) \neq X_1,$$

the support of the union of the elements of the partition is a proper subset of subspace.

Such a hierarchical structuring of the rule base and the introduction of a fuzzy partition is not reasonable if the sets A_{0i} are not rather large, so the sub-rule bases belonging to each of them are valid in a relatively large region of X_1. The observation is, however, typically rather "narrow", otherwise it would not be informative. The interpolation problem appears when the observation does not fire any of A_{0i} but hits in the gap among them. This situation will make it necessary that several (at least two) sub-rule bases are interpolated. In such a situation the significant feature of observation is how close it is to the nearest flanks of the neighboring regions, and its distance from their farther sides is irrelevant!

To be able to interpolate the conclusion only by evaluating the closer flanks in each dimension, we have introduced the idea of *interpolation by the conservation of fuzziness.* The method was proposed for the case of *trapezoidal* rules and observations (including the special cases of triangular or CVCII crisp sets).[3] Like in the case of KH-interpolation, it is necessary that the notions of ordering and distance exist in X_i [5]. Let A_1 and A^* be both trapezoidal and one-dimensional. The distance of the *cores* of these two is denoted by $d_1(A_1, A^*)$. (In this case there is neither U nor L indicated in the subscript of d, as this distance is neither one, it is between the U-point of A_1 and the L-point of A^*.) If both the antecedent and observation are multi-dimensional, the same calculation should be done in each dimension separately, and the resulting distances are obtained by the Euclidean Sum, analogously[5].

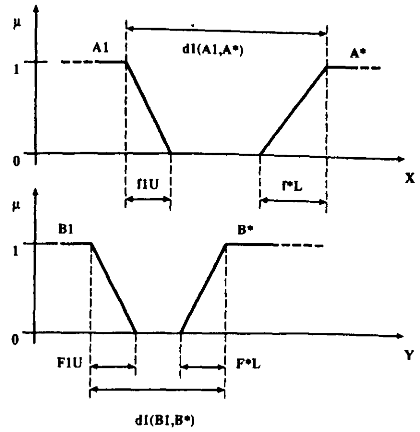


Fig. 1. Denotations of the fuzziness values of facing trapezoidal rules.

The difference between the support and the core, which we shall call here the *fuzziness* of the set (obviously 0 if the flank is vertical, i.e., the set is crisp from the side in question) will be denoted by

$$f_{1U} = \sup\{supp(A_1)\} - \sup\{core(A_1)\},$$

and

$$f_L^* = \inf\{core(A^*)\} - \inf\{supp(A^*)\}.$$

Similarly, the fuzziness of the other side of observation (f_U^*) of the other antecedent (f_{2L}), and of the consequence can be defined, the left one being the following (assumed that the ordering in Y is defined so $B_1 \prec B_2$):

$$F_{1U} = \sup\{supp(B_1)\} - \sup\{core(B_1)\}.$$

Finally, the core distance of the two consequence should be $d_1(B_1, B^*)$ (although it is not yet known in the moment).

With the above mentioned denotations (see also **Fig. 1**), the first rough approximation for the fuzziness of the conclusion B^* might be the following:

$$F'^* = f^* \frac{d_1(B_1, B^*)}{d_1(A_1, A^*)},$$

stating that the ratio of the fuzziness of the conclusion and of observation should be determined by the ratio of the core distances of the antecedent observation, and the consequent conclusion pairs. The disadvantage of this formula is that it does not take into account the fuzziness of membership functions in the rule. We introduced a normalized fictive value of F_{1U}:

$$F'_{1U} = f_{1U} \frac{d_1(B_1, B^*)}{d_1(A_1, A^*)},$$

(which is usually different from the real F_{1U}) and the expression of F'^* can be modified such as this:

$$F_L''^* = F_L'^*(1 + \frac{F_{1U} - F'_{1U}}{z})$$

$$= f_L^* \frac{d_1(B_1, B^*)}{d_1(A_1, A^*)}(1 + \frac{F_{1U} - F'_{1U}}{z}),$$

where z stands for either F_{1U} or F'_{1U}. In [3] the first possibility was chosen and the final formula for the fuzziness of the conclusion was

$$F_L''^* = f_L^* \frac{d_1(B_1,B^*)}{d_1(A_1,A^*)}\left(2 - \frac{f_{1U}}{F_{1Uz}} \frac{d_1(B_1,B^*)}{d_1(A_1,A^*)}\right) \quad \ldots\ldots (3)$$

Analogous formulas can be found for the other flank of A^*.

As it was mentioned in the paper, this choice induced a problem when B_i was crisp, so in that case we proposed

$$F_L''^* = f_L^* \frac{F_{1U}}{f_{1U}} .$$

In the next, the method described above will be modified.

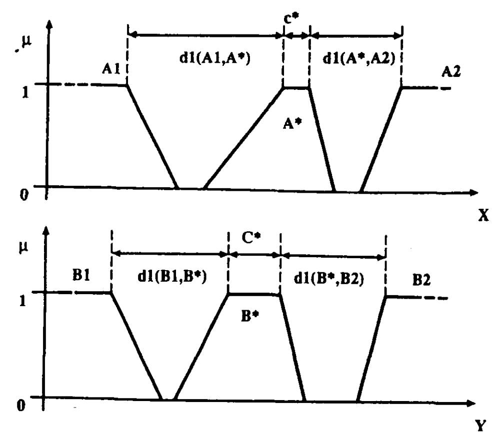


Fig. 2. Denotations with interpolated trapezoidal rules.

3. Conservation of Fuzziness and the Fundamental Equation

The method summarized in the previous section has no direct connection with the Fundamental Equation of interpolation, although the way of getting the position of the core of the conclusion is in full accordance with it. Let us show formulas for that. Let the core length of observation be denoted by c^*, and the core length of the conclusion C^*. (For the denotations used, see also **Figure 2**). The latter can be calculated by

$$\frac{C^*}{c^*} = \frac{d_1(B_1,B_2)}{d_1(A_1,A_2)} ,$$

where

$$d_1(B_1,B_2) = d_1(B_1,B^*) + C^* + d_1(B^*,B_2) ,$$

(where none of the members on the right side are known beforehand) and

$$d_1(A_1,A_2) = d_1(A_1,A^*) + c^* + d_1(A^*,A_2) .$$

The former equation expresses the fact that the relative core lengths of the conclusion and observation (normalized by the distances of the cores of the two flanking rules) should be in the same ratio as the relative core distances between consequence and antecedents (normalized by the same factors).

From there we obtain:

$$C'^* = c^* \frac{d_1(B_1,B_2)}{d_1(A_1,A_2)} . \quad \ldots\ldots (4)$$

The position of $core(B^*)$ between B_1 and B_2 can be determined directly by the Fundamental Equation just by changing the definition of distance (dissimilarity) slightly:

$$\frac{d_1(A_1,A^*)}{d_1(A^*,A_2)} = \frac{d_1(B_1,B^*)}{d_1(B^*,B_2)} ,$$

from which, taking also the equation for C^* into account:

$$d_1(B_1,B^*) = d_1(A_1,A^*) \frac{d_1(B_1,B_2)}{d_1(A_1,A_2)} , \quad \ldots\ldots (5)$$

and similarly

$$d_1(B^*,B_2) = d_1(A^*,A_2) \frac{d_1(B_1,B_2)}{d_1(A_1,A_2)} \quad \ldots\ldots (6)$$

(although this latter one can be also obtained from the previous results as $d_1(B^*,B_2) = d_1(B_1,B_2) - d_1(B_1,B^*) - C^*$ always holds) and all the quantities on the right side are known from rules and observation.

The critical question is how the principle of the conservation of fuzziness can be brought into accordance with the Fundamental Equation. In this problem, not both rules flanking observation are considered, but only one. However, the starting point for calculating the unknown fuzziness of B^* *is* that its dissimilarity to the core distance between B_1 and B^* should be in the same ratio to the dissimilarity of the consequent B_1, as the dissimilarity of the fuzziness of A^* to the core distance of A_1 and A^* *is* to the dissimilarity of the fuzziness of A_1 to the same core distance. We intentionally use the term dissimilarity instead of distance, as in the same context we use distance for the geometric distance along X. If we describe the dissimilarity between two lengths in X by their ratio, we get (omitting the subscripts L and U for simplicity in the next few lines):

$$\delta(F^*,d(B_1,B^*)) = \frac{F^*}{d(B_1,B^*)} = \text{, etc.}$$

From here, the Fundamental Equation for dissimilarities δ yields

$$\delta(F^*,d(B_1,B^*)) : \delta(F_1,d(B_1,B^*)) = \delta(f^*,d(A_1,A^*)) : \delta(f_1,d(A_1,A^*)) ,$$

$$\frac{\dfrac{F^*}{d(B_1,B^*)}}{\dfrac{F_1}{d(B_1,B^*)}} = \frac{\dfrac{f^*}{d(A_1,A^*)}}{\dfrac{f_1}{d(A_1,A^*)}} ,$$

from which we get

$$\frac{F^*}{F_1} = \frac{f^*}{f_1} ,$$

and finally

$$F_L^* = f_L^* \frac{F_{1U}}{f_{1U}} \quad \ldots\ldots (7)$$

It is interesting to compare this with the formula proposed[3].

$$F_L^* = = f_L{}^* \frac{d_1(B_1,B^*)}{d_1(A_1,A^*)}(1 + \frac{F_{1U} - F'_{1U}}{z})$$

Let us namely choose $z = F'_{1U} = f_{1U}\frac{d_1(B_1,B^*)}{d_1(A_1,A^*)}$ instead of F_{1U}. So we obtain

$$F_L^* = f_L{}^* \frac{d_1(B_1,B^*)}{d_1(A_1,A^*)}(1 + \frac{F_{1U} - F'_{1U}}{f_{1U}\frac{d_1(B_1,B^*)}{d_1(A_1,A^*)}})$$

$$= f_L{}^* \frac{d_1(B_1,B^*)}{d_1(A_1,A^*)}(1 + \frac{F_{1U} - F'_{1U}}{f_{1U}\frac{d_1(B_1,B^*)}{d_1(A_1,A^*)}} - 1) = f_L{}^* \frac{F_{1U}}{f_{1U}},$$

which is identical with the former result obtained by using the Fundamental Equation. In this paper we propose this modified formula for the calculation of the fuzziness of the conclusion.

Similarly, the upper fuzziness is obtained by

$$F_U^* = f_U{}^* \frac{F_{2L}}{f_{2L}}. \quad (8)$$

It should be investigated under what conditions formulas obtained here do not work.

If in F_L^*, $f_{1U} = 0$ (A_1 is crisp on the upper side), it is reasonable to differentiate the cases when $F_{1U} = 0$ (B_1 is also crisp on the upper side) and when $F_{1U} \neq 0$ (it is not crisp). In the former case, it is reasonable to assume that $\lim_{f_{1U},F_{1U} \to 0} \frac{f_{1U}}{F_{1U}} = \frac{d_1(B_1,B^*)}{d_1(A_1,A^*)}$, from which we obtain

$$F_{L(f_{1U},F_{1U} \to 0)}^* = f_L{}^* \frac{d_1(B_1,B^*)}{d_1(A_1,A^*)}.$$

In the latter case, however, the ratio of the fuzziness of observation and the antecedent is infinity, and having a non-crisp consequent, it is impossible to conserve the relative fuzziness in such a case here, it is not possible to determine B^* as the starting knowledge is semantically inconsistent. Similar considerations can be done for the upper fuzziness of B^* as well.

Of course, if $f^* = 0$ itself (A^* *is* crisp on the upper side), the formula will result automatically into $F^* = 0$, meaning that B^* should be crisp, too.

The formulas that were obtained for C^* *and* $d_1(B_1,B^*)$ will always be applicable, as $d_1(A_1,A_2) > 0$ is always true, otherwise there is no sense in interpolating at all.
Results of the interpolation by the conservation of fuzziness (KHG) compared to KH (linear) interpolation and the first version of this method (GK).

At the end of this section, let us illustrate the new interpolation method by a simple numerical example (see **Fig.3**). The data are taken from (3). Let the two trapezoidal rules be determined by the following characteristic points in the universes $X = [0,100]$ and $Y = [0,100]$:

A_1: 0, 5, 25, 30
A_2: 70, 75, 95, 100
B_1: 0, 15, 20, 30
B_2: 70, 85, 90, 100

The observation is

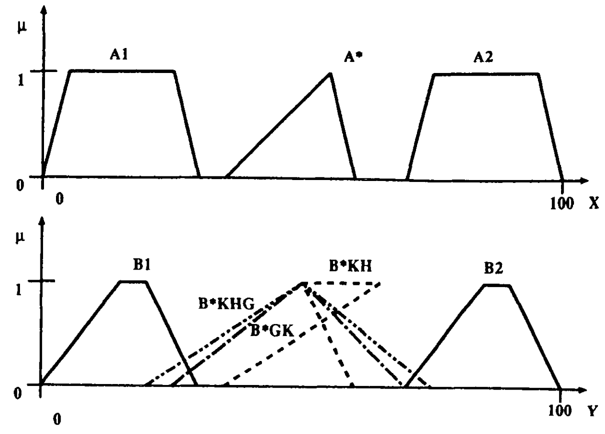


Fig. 3. Results of the interpolation by the conservation of fuzziness (KHG) compared to the KH (linear) interpolation and the first version of this method (GK).

A^*: 35, 55, 55, 60.

Let us calculate first what would be the conclusion if KH-interpolation method were used on this rule pair and observation.

The immediate conclusion will be in this case

B_{KH}^*: 35, 65, 50, 60.

Clearly, this needs further treatment as 65 > 50 indicates a "loop" in the resulting membership function.

If, however, the method suggested in (3) is applied, we obtain

B_{GK}^*: 24, 59, 59, 69,

which is a directly acceptable membership function.

Let us calculate now the result with help of formulas obtained in this section.

Core Length

$$C^* = c^* \frac{d_1(B_1,B_2)}{d_1(A_1,A_2)} = 0 \times \frac{65}{50} = 0,$$

Core Position

$$d_1(B_1,B^*) = d_1(A_1,A^*)\frac{d_1(B_1,B_2)}{d_1(A_1,A_2)} = 30 \times \frac{65}{50} = 39,$$

and

$$d_1(B^*,B_2) = d_1(A^*,A_2)\frac{d_1(B_1,B_2)}{d_1(A_1,A_2)} = 20 \times \frac{65}{50} = 26,$$

which can be also obtained by
$d_1(B^*,B_2) = d_1(B_1,B_2) - d_1(B_1,B^*) - C^* = 65 - 39 - 0 = 26$.
From here, the peak of the singleton core conclusion is at 20 + 39 = 59, which is identical with the former value (of B_{GK}^*).

Values of fuzziness

$$F_L^* = f_L^* \frac{F_{1U}}{f_{1U}} = 20 \times \frac{10}{5} = 40,$$

and

$$F_U^* = f_U^* \frac{F_{2L}}{f_{2L}} = 5 \times \frac{15}{5} = 15,$$

The resulting conclusion is

B^*_{KHG}: 19, 59, 59, 74 ,

which is different from B^*_{GK} and is fully in accordance with the Fundamental Equation.

Another example from (3) is the following:

A_1: 0, 20, 30, 40
A_2: 70, 80, 90, 100
B_1: 0, 30, 35, 40
B_2: 80, 85, 95, 100

The observation is crisp:

A^*: 45, 45, 65, 65 ,

The result of KH-interpolation is again non-convex:

B^*_{KH}: 51, 53, 65, 60 ,

where the right side of the set has to be normalized as 65 > 60. The result by Gedeon and Kóczy[3] is

B^*_{GK}: 50, 50, 65, 65 ,

which is crisp such as observation.

The present method will give the following results:

$$C^* = c^* \frac{d_1(B_1,B_2)}{d_1(A_1,A_2)} = 15 \times \frac{50}{50} = 15 ,$$

$$d_1(B_1,B^*) = d_1(A_1,A^*) \frac{d_1(B_1,B_2)}{d_1(A_1,A_2)} = 15 \times \frac{50}{50} = 15 ,$$

$$F^*_L = f^*_L \frac{F_{1U}}{f_{1U}} = 0 \times \frac{5}{10} = 0 ,$$

and

$$F^*_{UL} = f^*_U \frac{F_{2L}}{f_{2L}} = 0 \times \frac{5}{10} = 0 .$$

From this data, the conclusion will be

B^*_{KHG}: 50, 50, 65, 65 ,

which is identical in this case with the previous result, as here both fuzziness values are 0.

4. Multi-dimensional Case

In the previous section the basic method was discussed and formulas calculated for a single input variable. Of course, interpolation is really useful only if the number of dimensions is large, and the formula $r = T^k$ brings in intractability.

There are various possibilities to extend rule interpolation into multiple dimensions. The major concern is to keep the computational complexity low, so to apply simple formulas for calculating the resulting distance.

In (5, 6, 7) the use of Euclidean metrics and the Pythagorean sum of the normalized component distances was proposed, and in many experiments that proved to be a good choice from the practical point of view (even though the use of the Hausdorff-distance, e.g., might also have some advantages). So we propose for the summation of real metric dissimilarities the same approach. As a result, we obtain the following expressions:

$$C^* = \sqrt{\frac{c_1^{*2}}{|X_1|^2} + \frac{c_2^{*2}}{|X_2|^2} + \dots + \frac{c_k^{*2}}{|X_k|^2}}\, d_1(B_1,B_2) \times \sqrt{\frac{d_{11}^2(A_1,A_2)}{|X_1|^2} + \frac{d_{12}^2(A_1,A_2)}{|X_2|^2} + \dots + \frac{d_{1k}^2(A_1,A_2)}{|X_k|^2}} \quad (9)$$

$$d_1(B_1,B^*) = \times \sqrt{\frac{d_{11}^2(A_1,A^*)}{|X_1|^2} + \frac{d_{12}^2(A_1,A^*)}{|X_2|^2} + \dots + \frac{d_{1k}^2(A_1,A^*)}{|X_k|^2}} \times \frac{d_1(B_1,B_2)}{\sqrt{\frac{d_{11}^2(A_1,A_2)}{|X_1|^2} + \frac{d_{12}^2(A_1,A_2)}{|X_2|^2} + \dots + \frac{d_{1k}^2(A_1,A_2)}{|X_k|^2}}} \quad (10)$$

and a similar expression for $d_1(B^*,B_2)$. (Here, k is the number of input dimensions.)

It is more interesting how the fuzziness of the conclusion can be calculated as that value is based on the dissimilarity of the relative fuzziness, certainly not a metric concept. To find a reasonable solution first the semantic interpretation of the use of the Euclidean distance must be discussed. In the expressions where the resulting distances of pairs of rules or even resulting core sizes are calculated, there are always two expressions containing the same structure of Pythagorean sum, one in the numerator and one in the denominator of the fraction. In both cases, the number of elements added is identical, it is k, the number of input dimensions. If the expressions are multiplied now by $k^2/k^2 = 1$, and k^2 is brought under the square root, we find expressions of the structure $\sqrt{\frac{\sum_{i=1}^{k} m_i^2}{k}}$, which is the quadratic *mean* of the members m_i.

Similarly, if we apply the general Minkowski-distance with exponent m instead of the Euclidean one (as proposed[5]), the expressions can be interpreted as means of the power m in both the numerator and the denominator. In the case of dissimilarities of the relative fuzziness, we must apply a corresponding type of mean, as the degree of dissimilarity is calculated here by the ratio, i.e., a multiplicative operation, it seems to be reasonable to apply a multiplicative averaging operator, as the simplest possibility, the *geometric mean.* Consequently the resulting degrees of fuzziness will be calculated by

$$F_l^* = \sqrt[k]{f_{L1}^* \times f_{L2}^* \times \dots \times f_{Lk}^*}\, \frac{F_{1U}}{\sqrt[k]{f_{1U1} \times f_{1U2} \times \dots \times f_{1Uk}}} \quad . (11)$$

A similar formula is applicable for F_u*

5. General Shape Membership Functions

In the previous sections, the interpolation method based on thc conservation of fuzziness and the Fundamental Equation of Fuzzy Rule Interpolation has been presented for trapezoidal shape membership functions. Of course, these include also the special cases of triangular, crisp, and crisp singleton sets. In most real applications, the sets applied in rules and observation belong to one of these classes. From the practical application point, these cases are important, as more general membership function shapes are rarely if ever applied in commercial or industrial fuzzy controllers and expert systems. However, it is interesting to discuss how this approach can be extended to general shape membership functions. As a condition, we will assume[5] that membership functions are normal and convex. (Even though interpolation methods for arbitrary shaped membership functions have been proposed also[1].)

Obviously, (4), (5) and (6) do not depend on the shape of the membership function (as long as it is convex and normal, and consequently it has a connected core). The crucial question is how (7) and (8), formulas defining support points of B^* and by these, defining the whole shape of the conclusion fuzzy set, can be extended to other than trapezoidal cases.

It should be noticed that in the calculation of the ''fuzziness'' values, i.e., the widths of the non-crisp regions in each dimension, the difference of the x_i coordinate of the core and support point is calculated. Consequently, for arbitrary membership function shape, for any α, the position of the α-cut point will determine the (local) value of "α–fuzziness" either in relation to the core, or in relation to the support.

The following formula will define the location of the α-cut point of B^* (upper or lower) in these two senses, depending on the way of defining F and f:

$$F^*_{L/U\alpha} = f^*_{L/U\alpha}\frac{F_{1U/2L\alpha}}{f_{1U/2L\alpha}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (12)$$

so

$$F^*_{L/U\alpha}(c) = |B^*_{L/U\alpha} - B^*_{L/U\alpha=1}|$$

$$f^*_{L/U\alpha}(c) = |A^*_{L/U\alpha} - A^*_{L/U\alpha=1}|$$

$$F_{1U/2L\alpha}(c) = |B_{1U/2L\alpha} - B_{1U/2L\alpha=1}|$$

$$f_{1U/2L\alpha}(c) = |A_{1U/2L\alpha} - A_{1U/2L\alpha=1}|$$

describe the core relative definitions of F and f, and

$$F^*_{L/U\alpha}(s) = |B^*_{L/U\alpha} - B^*_{L/U\alpha=0}|$$

$$f^*_{L/U\alpha}(s) = |A^*_{L/U\alpha} - A^*_{L/U\alpha=0}|$$

$$F_{1U/2L\alpha}(s) = |B_{1U/2L\alpha} - B_{1U/2L\alpha=0}|$$

$$f_{1U/2L\alpha}(s) = |A_{1U/2L\alpha} - A_{1U/2L\alpha=0}|$$

are the definitions for the support relative case.

Figure 4 depicts a general case in X with denotations of some definitions for f.

It is hard to decide whether α=1 or α=0 is more suitable in a general case, it should be decided depending on the application context. It is also possible to take the average of the two, by applying any mean, e.g., in the simplest case

$$F^*_{L/U\alpha}\left(\frac{c+s}{2}\right) = \frac{|B^*_{L/U\alpha} - B^*_{L/U\alpha=1}| + |B^*_{L/U\alpha} - B^*_{L/U\alpha=0}|}{2}$$

and similarly $f^*L/U\alpha(\frac{c+s}{2})$, etc.

Finally, we mention another possibility, if membership functions are continuously differentiable, for each α the derivatives $\frac{dB^*(\alpha)}{dy}$, etc. can be determined and the fictive core

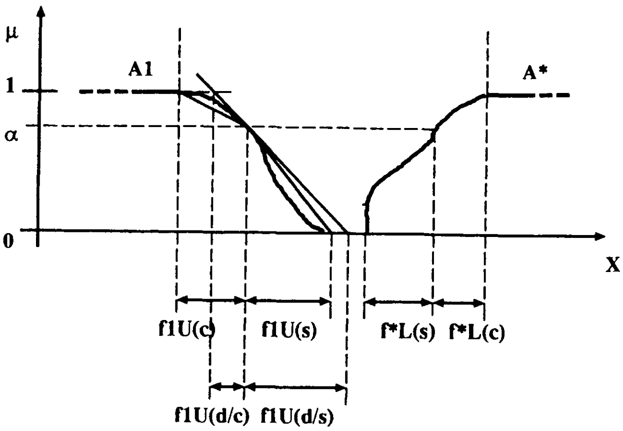


Fig. 4. Various denotation of fuzziness values for general shape membership functions.

be determined by substituting $B^*(y)$=1 or = 0, etc., and the derivative-based core relative and support relative fuzziness values $F^*_{L/U\alpha}(d/s)$, $f^*_{L/U\alpha}(d/s)$ etc. can be calculated.

The extension of these formulas for the multi-dimensional case is done in the same way as it is done for the trapezoidal case.

6. Conclusions

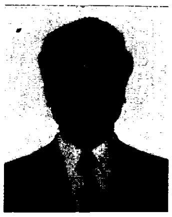
In this paper we investigated the question whether interpolation of fuzzy rules can be done just by the information available from the closer of the two flanks of rules flanking the observation. The method we suggested was based on the idea of conserving the relative fuzziness values, proposed in a former paper. However, here the Fundamental Equation of fuzzy rule interpolation was considered as the starting point, and some different formulas were obtained. The one dimensional case was extended to multiple input variables, by using the Euclidean distance for calculating the core lengths and core distances, and the geometric mean for the fuzziness values. In the last section results valid for trapezoidal rules were extended to general shape membership functions.

The advantage of this method in comparison with the former introduced KH and related interpolations is that here no information on the farther flanks of the neighboring rules is necessary, moreover, those flanks do not influence the shape of the conclusion. This latter being always immediately interpretable, obtaining abnormal shape or subnormal membership functions is guaranteed. One of the possible application fields of this method is the interpolation on the meta levels of structured hierarchical rule bases where the interpolated membership functions are elements of the sparse fuzzy partition of a subspace of input space, and have necessarily very different (larger) support etc. size than observations.

References

1) P. Baranyi, T. D. Gedeon and L. T. Koczy, "A general interpolation technique in fuzzy rule bases with arbitrary membership functions", IEEE International Conf. on Systems, Man and Cybernetics, Bejing, 1996, pp.510-515.

2) D. Dubois and H. Prade, "Gradual rules in approximate reasoning", Information Science 61 (1992), pp.103-122.
3) T. D. Gedeon and L. T. Kóczy, "Conservation of fuzzyness in rule interpolation", Symp. on New trends in the Control of Large Scale Systems, Vol.I., Herlany, Slovakia, 1996, pp.13-19.
4) S. Kawase and Q. Chen, "On fuzzy reasoning by Koczy's linear Rule Interpolation", Tech. Report, Teikyo Heisei University, Ichihara, Chiba, Japan, 1996, 9p.
5) L. T. Kóczy and K. Hirota, "Odering, distance and closeness of fuzzy sets", Fuzzy Sets and Systems 59 (1993), pp.281-293.
6) L. T. Kóczy and K. Hirota, "Approximate reasoning by linear rule interpolation and general approximation", Internat. J. of Approximate Reasoning 9 (1993), pp.179-225.
7) L. T. Kóczy and K. Hirota, "Interpolative reasoning with insufficient evidence in sparse fuzzy rule bases", Information Science 71 (1993), pp.169-201.
8) L. T. Kóczy and K. Hirota, "Modular rule bases in fuzzy control", EUFIT'93, Aachen, 1993, pp.606-610.
9) S. Kovacs and L. T. Kóczy, "Shape of fuzzy conclusion generated by linear interpolation of trapezoidal If...then rules", Tatra Mountains Mathematical Publications 6 (1995), pp.83-93.
10) Y. Shi and M. Mizumoto, "Some considerations on Koczy's Linear Interpolative Reasoning Method (In Japanese, with English Abstact), Journal of SOFT 8 (1996), pp.147-15.
11) Z. Q. Wu, M. Mizumoto and Y. Shi, "An improvement to Koczy and Hirota's Interpolative Reasoning in sparse fuzzy rule bases", International J. of Approximate Reasoning 15 (1996), pp.185-201.



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- K. Kaoru and T. Fukuda, "Soft Computing in Mechatronics", Physica-Verlag (A Springer-Verlag Company), B5, 186 pages, (1999).

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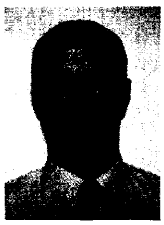

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- Gedeon, TD., "Data Mining of Inputs: Analysising Magnitude and Functional Mesears", International Journal of Neural Systems, vol.8 no.2, pp.209-218, 1997.
- Gedeon, TD., "Stochastic bidirectional training", IEEE International Conference on System Man and Cybernetics, San Diego, pp. 1968-1971, 1998.
- Gedeon, TD., "Clustering Significant Words using their Co-occurrence in Document Sub-Collections", Proceedings European Congress of Intelligent Tachniques & Soft Computing, pp. 302-306, 1999.

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