DEVELOPMENT OF A HIERARCHICAL FUZZY RULE BASE FOR PETROLUEM DATA

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ABSTRACT

The hierarchical fuzzy rule base was investigated in this paper. A readily implementable algorithm was proposed for the rule base inference. The hierarchical fuzzy rule base has several advantages over conventional rule bases, namely the reduced complexity and high interpretability. To validate the practicality of the model, a hierarchical fuzzy rule base was developed for a set of real world petroleum data. The resulting model is presented and discussed. It has been shown that the model achieves promising accuracy with fuzzy rules that are relatively easy to understand.

1. INTRODUCTION

Fuzzy rule bases are widely used for process simulation and control. Given the relevant information, fuzzy rule bases can be used to model a problem domain using fuzzy rules. They distinguish themselves from neural networks with their capability to explain their inference results using a set of fuzzy rules. Knowledge elicited from problem domain experts can also be easily encoded into fuzzy rules to form fuzzy rule bases.

When dealing with complex systems whose number of input variables is large, two problems arise. Firstly, fuzzy rule bases suffer from rule explosion. The number of possible rules necessary is $O(T^k)$ where K is the number of dimensions and T is the number of terms per input. In other words, the numbers of rules grows exponentially as the number of input variables increases. The second problem is the loss of interpretability of fuzzy rules.

Hierarchical fuzzy systems [1-3] alleviate the problem of rule explosion but overlooked the interpretability issue to some extent. Most of the hierarchical fuzzy systems are based on the idea of decomposing the rule base into multiple cooperative rule bases, so that each rule base deals with only a limited or fixed number of input variables. The output of a rule base becomes the input to other rule bases. Figure 1 shows two typical hierarchical models found in the literature. Such decompositions can effectively reduce the complexity of the system (i.e. the number of rules). However, the input to system has to pass through multiple levels of fuzzy systems, where each system modifies the result based on some fuzzy rules. In this case, the transformation of the input to the output becomes hardly traceable.

Perhaps the most practical hierarchical fuzzy system is proposed by Sugeno et al. [4] in their attempt to build the unmanned helicopter. The hierarchical fuzzy system was built on the knowledge elicited from experienced helicopter operators. Using the model, Sugeno was able to encode the operators' knowledge directly (and naturally) into fuzzy rules. Koczy et al. [5] later presented a formal definition of the hierarchical fuzzy model used in [4] (more details in section 2). The research of the model is still at its early stage and the discussion in [5] is solely on a theoretical level.

In this paper, we present a readily implementable algorithm for the hierarchical system proposed in [5]. We also demonstrate the feasibility and effectiveness of the hierarchical system in the real world by applying it to a set of petroleum data. The development of the hierarchical systems will also be discussed.

The organization of this paper is as follows. Section 2 discusses a hierarchical fuzzy rule base. Section 3 presents the process of developing a hierarchical fuzzy rule base for a set of real world petroleum data. Section 4 discusses the parameter identification process designed to improve rule base's accuracy. The conclusion is presented in section 6. The discussion of the completed hierarchical fuzzy rule base is presented in section 5.

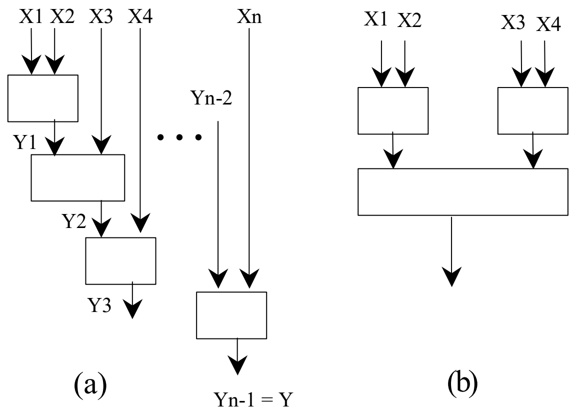

Figure 1: Typical structures of hierarchical fuzzy rule bases found in the literature

2. HIERARCHICAL FUZZY RULE BASE

In this section, the general idea of the hierarchical fuzzy system [5] is presented. Often, the multi-dimensional input space $X = X_1 \times X_2 \times \ldots \times X_k$ can be decomposed into some subspaces, e.g. $Z_0 = X_1 \times X_2 \times \ldots \times X_{k0}$ ($k_o < k$), so that in Z_0 a partition $\Pi = \{D_1, D_2, \ldots, D_n\}$ can be determined. In each D_i, a sub-rule base R_i can be constructed with local validity. The hierarchical rule base structure becomes:

R_0: *if* z_0 *is* D_1 *then use* R_1
if z_0 *is* D_2 *then use* R_2
⋮
if z_0 *is* D_n *then use* R_n

R_1: *if* z_1 *is* A_{11} *then* y *is* B_{11}
if z_1 *is* A_{12} *then* y *is* B_{12}
⋮
if z_1 *is* A_{1m1} *then* t *is* B_{1m1}

R_n: *if* z_n *is* A_{n1} *then* y *is* B_{n1}
if z_n *is* A_{n2} *then* y *is* B_{n1}
⋮
if z_n *is* A_{nmn} *then* y *is* B_{nmn}

The complexity of the system is reduced if the proper Z_0 and Π can be found. The fuzzy rules in sub rule base R_0 are termed meta rules since the consequences of the rules are pointers to other sub rule bases instead of fuzzy sets.

For implementation, we propose the following recursive algorithm, modified from the original Mamdani fuzzy rule base algorithm [6].

Procedure Evaluate(f, X)
f: hierarchical fuzzy system
X: $[x_1, x_2, \ldots, x_n]$ n-dimensional input to the system

for each rule$_i$ in f
Calculate the degree of match w_i in the rule premise:
$W_i = A_{i1}(x_1) \times A_{i2}(x_2) \times \ldots \times A_{in}(x_n)$
where A_{ij} is the j^{th} term of rule i.

$$\text{Let } B_i = \begin{cases} \textit{fuzzy set in the rule consequent} & \textit{if } rule_i = normal\ rule \\ evaluate\ (R_j, X) & \textit{if } rule_i = meta\ rule \end{cases}$$

where R_i is the sub rule base pointed to by meta rule$_i$.

return $\max_i (w_i B_i)$
end for

The output of the procedure is a fuzzy set that can be defuzzified by taking the center of gravity. The hierarchical fuzzy rule base has the following advantages. The inference process of the rule base can be more efficient. Unlike conventional fuzzy systems, not all fuzzy rules have to be processed. Some fuzzy rules in the sub rule bases will not be processed when unnecessary. The resulting fuzzy model can be more interpretable due to its similarity with the human reasoning process. This can be justified by the fact that Sugeno has successfully encoded the knowledge of the helicopter operators directly into fuzzy rules using a similar hierarchical model [4].

3. DEVELOPMENT OF THE HIERARCHICAL FUZZY RULE BASE

In this section, the development of the hierarchical fuzzy model is discussed. To show the practicality of the model, a set of real world petroleum data is used. The data has 8 input dimensions. In terms of fuzzy rule interpretability, the use of the hierarchical fuzzy rule base can be more beneficial than conventional fuzzy rule bases. The petroleum data is briefly explained in sub section 3.1. This is followed by the discussion of the development process in section 3.2.

3.1. Petroleum Data

The data used is a set of benchmark data in reservoir characterisation. The data set is obtained from a real reservoir. The objective is to develop an estimator to predict porosity (PHI) from well logs.

The well logs available are: GR (Gamma Ray), RDEV (Deep Resistivity), RMEV (Shallow Resistivity), RXO (Flushed Zone Resistivity), RHOB (Bulk Density), NPHI (Neutron Porosity), PEF (Photoelectric Factor) and DT (Sonic Travel Time). Normalised data (scaled between 0 and 1) is used.

There are altogether 633 rows of data. Since accuracy and the generalization ability are not the main concern of this study, the same set of data is used for training as well as testing. The goal of the experiment is to verify the practicality of the hierarchical model by constructing a hierarchical fuzzy system with reasonable accuracy and good interpretability out of real world data.

3.2. Hierarchical Fuzzy Modeling

Due to the lack of rule extraction technique designed for the hierarchical fuzzy rule base generation, one of the most convenient approaches is to first develop a conventional ('flat') fuzzy system, and then convert it to a hierarchical model by fuzzy sets merging (more details later).

3.2.1. Fuzzy Rule base Generation

For the fuzzy system generation, the fuzzy modeling methodology proposed in [6] is used. The rule extraction process starts with the determinition of the partition of the output space. This is done by using fuzzy c-means clustering [7]. The optimal number of clusters are determined by means of a criterion [6]. For each output fuzzy cluster B_i resulting from the fuzzy c-means clustering, a cluster in the input space A_i can be induced. The input cluster can be projected onto the various input dimensions to produce rules of the form:

If x1 is A_{i1} and x2 is A_{i2} and … x_n is A_{in} then y is B_i

However, it is remarked in the paper [6] that there can be more than one fuzzy cluster in the input space which corresponds to the same fuzzy cluster B_i. In this case more than one rule is formed with the same consequent. Suppose that two input clusters (A_i and A_j) are induced from the output cluster B_i, we obtain the following two rules:

If x1 is A_{i1} and x2 is A_{i2} and … x_n is A_{in} then y is B_i
If x1 is A_{j1} and x2 is A_{j2} and … x_n is A_{jn} then y is B_i

No concrete procedures for determining the number of input clusters to be induced from an output cluster is discussed in the paper [6]. In [8], we proposed a projection-based approach to deal with the problem. The completed model then go through the parameter identification process to improve its accuracy. The process is described in section 4.

3.2.2. Conversion to Hierarchical Fuzzy Rule base

Prior to this stage, a conventional fuzzy rule base has already been generated. Two or more fuzzy rules are merged to form hierarchical fuzzy rules. For example, the two rules:

If x_1 is A_{11} and x_2 is A_{12} then y is B1
If x_2 is A_{21} and x_2 is A_{22} then y is B2

can be merged to form:

If x_1 is ($A_{11} \cup A_{21}$) then use R_1
R_1: if x_2 is A_{12} then y is B_1
if x_2 is A_{22} then y is B_2

We remark that although the produced hierarchical version has more rules (1 meta rule + 2 rules) than the original version (2 rules), the inference process is actually more efficient in the hierarchical version. This is because the total number of terms in rule antecedents for the hierarchical version (3 terms) is less than the original version (4 terms).

To maintain the fuzzy rule base accuracy, it is essential that A_{11} and A_{21} coincide as much as possible. Subjective evaluation is used in this study to identify candidates for the merging process although better ideas can be adapted from [9, 10].

4. PARAMETER IDENTIFICATION

Parameter identification is a process to tune the parameters of membership functions in the rule antecedents. The technique described in [6] is designed for trapezoidal fuzzy sets. The algorithm is as follows:

1. Set the value *f* for adjustment.
2. Let p^k_j be the k^{th} parameter of the j^{th} fuzzy sets.
3. Calculate $p^{k+}_j = p^k_j$ + f and $p^{k-}_j = p^k_j$ – f.
   If k = 2, 3, 4, and $p^k_j + f > p^{k+1}_j$, then $p^{k+}_j = p^{k-1}$
   If k = 1, 2, 3, and $p^k_j - f < p^{k-1}_j$, then $p^{k-}_j = p^{k+1}$
4. Choose the parameter which shows the best performance among $\{p^{k+}_j, p^k_j, p^{k-}_j\}$ and replace p^k_j with it.
5. Go to step 2 while unadjusted parameter exist.
6. Repeat step 2 until we are satisfied with the performance.

In [6], *f* = 5% of the width of the universe of discourse is used. Figure 2 shows the parameter adjustment process.

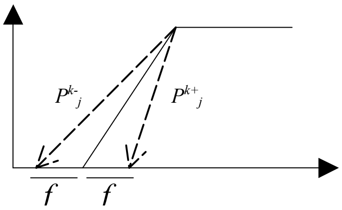

Figure 2: Parameter Adjustment

5. HIERARCHICAL FUZZY MODEL

In this section, the fuzzy rule bases developed using the methodology discussed in the previous sections are presented and analyzed.

Using the method discussed in section 3.2.1, a conventional fuzzy rule base have been constructed. The rule base has 6 rules. Trapezoidal membership functions are used in both the rule antecedents and consequences. The rules, together with the trapezoidal membership function represented by their 4 characteristic points are shown below:

1) if GR is [0.06 0.54 0.66 0.99] and
RDEV is [0.02 0.36 0.48 0.87] and
RMEV is [0.01 0.41 0.53 0.84] and
RXO is [0.03 0.31 0.42 1.00] and
RHOB is [0.31 0.54 0.62 1.00] and
NPHI is [0.19 0.45 0.54 0.93] and
PEF is [0.02 0.12 0.16 0.30] and
DT is [0.17 0.46 0.55 0.80] Then y is y1

2) if GR is [0.07 0.50 0.62 1.00] and
RDEV is [0.07 0.41 0.53 0.96] and
RMEV is [0.06 0.40 0.51 0.87] and
RXO is [0.02 0.26 0.35 0.91] and
RHOB is [0.17 0.51 0.62 0.94] and
NPHI is [0.16 0.43 0.53 0.95] and
PEF is [0.02 0.13 0.17 0.41] and
DT is [0.04 0.48 0.59 0.87] Then y is y2

3) if GR is [0.12 0.43 0.53 0.84] and
RDEV is [0.09 0.36 0.45 0.73] and
RMEV is [0.10 0.36 0.44 0.70] and
RXO is [0.00 0.20 0.27 0.62] and
RHOB is [0.11 0.43 0.55 0.96] and
NPHI is [0.11 0.36 0.46 1.00] and
PEF is [0.01 0.16 0.20 1.00] and
DT is [0.29 0.57 0.65 0.87] Then y is y3

4) if GR is [0.08 0.41 0.53 0.96] and
RDEV is [0.01 0.35 0.48 0.94] and
RMEV is [0.01 0.34 0.46 0.96] and
RXO is [0.01 0.19 0.26 0.67] and
RHOB is [0.17 0.42 0.51 0.88] and
NPHI is [0.10 0.40 0.52 0.98] and
PEF is [0.01 0.15 0.19 0.76] and
DT is [0.35 0.58 0.66 0.87] Then y is y4

5) if GR is [0.09 0.44 0.57 0.99] and
RDEV is [0.06 0.39 0.51 1.00] and
RMEV is [0.08 0.40 0.51 0.99] and
RXO is [0.00 0.25 0.34 0.83] and
RHOB is [0.00 0.34 0.46 0.85] and
NPHI is [0.14 0.47 0.58 1.00] and
PEF is [0.00 0.16 0.22 0.58] and
DT is [0.35 0.59 0.67 0.89] Then y is y5

6) if GR is [0.00 0.30 0.41 0.88] and
RDEV is [0.06 0.41 0.51 0.80] and
RMEV is [0.08 0.45 0.58 1.00] and
RXO is [0.00 0.23 0.32 0.77] and
RHOB is [0.07 0.32 0.41 0.77] and
NPHI is [0.27 0.42 0.48 0.91] and
PEF is [0.06 0.18 0.23 0.38] and
DT is [0.00 0.55 0.67 1.00] Then y is y6

Using the concept discussed in section 3.2.2, the fuzzy rules are merged to form the following hierarchical model:

Meta rules
If GR is [0.06 0.52 0.64 1.00] and
RMEV is [0.01 0.41 0.52 0.87] and
NPHI is [0.16 0.44 0.54 0.95] and
PEF is [0.02 0.13 0.17 0.41] Then use R1

If GR is [0.08 0.43 0.55 0.99] and
RDEV is [0.01 0.37 0.50 1.00] and
NPHI is [0.10 0.44 0.55 1.00] and
DT is [0.35 0.585 0.665 0.89] Then use R2

If RXO is [0.00 0.20 0.27 0.67] and
RHOB is [0.11 0.43 0.53 0.96] and
DT is [0.29 0.58 0.66 0.87] Then use R3

If RXO is [0.00 0.24 0.33 0.83] and
RHOB is [0.00 0.33 0.44 0.85] and
PEF is [0.00 0.17 0.23 0.58] Then use R4

R1:
if RDEV is [0.02 0.36 0.48 0.87] and
RXO is [0.03 0.31 0.42 1.00] and
RHOB is [0.31 0.54 0.62 1.00] and
DT is [0.17 0.46 0.55 0.80] Then y is y1

if RDEV is [0.07 0.41 0.53 0.96] and
RXO is [0.02 0.26 0.35 0.91] and
RHOB is [0.17 0.51 0.62 0.94] and
DT is [0.04 0.48 0.59 0.87] Then y is y2

R2:
if RMEV is [0.01 0.34 0.46 0.96] and
RXO is [0.01 0.19 0.26 0.67] and
RHOB is [0.17 0.42 0.51 0.88] and
PEF is [0.01 0.15 0.19 0.76] Then y is y4

if RMEV is [0.08 0.40 0.51 0.99] and
RXO is [0.00 0.25 0.34 0.83] and
RHOB is [0.00 0.34 0.46 0.85] and
PEF is [0.00 0.16 0.22 0.58] Then y is y5

R3:
if GR is [0.12 0.43 0.53 0.84] and
RDEV is [0.09 0.36 0.45 0.73] and
RMEV is [0.10 0.36 0.44 0.70] and
NPHI is [0.11 0.36 0.46 1.00] and
PEF is [0.01 0.16 0.20 1.00] Then y is y3

if GR is [0.08 0.41 0.53 0.96] and
RDEV is [0.01 0.35 0.48 0.94] and
RMEV is [0.01 0.34 0.46 0.96] and
NPHI is [0.10 0.40 0.52 0.98] and
PEF is [0.01 0.15 0.19 0.76] Then y is y4

R4:
if GR is [0.09 0.44 0.57 0.99] and
RDEV is [0.06 0.39 0.51 1.00] and
RMEV is [0.08 0.40 0.51 0.99] and
NPHI is [0.14 0.47 0.58 1.00] and
DT is [0.35 0.59 0.67 0.89] Then y is y5

if GR is [0.00 0.30 0.41 0.88] and
RDEV is [0.06 0.41 0.51 0.80] and
RMEV is [0.08 0.45 0.58 1.00] and
NPHI is [0.27 0.42 0.48 0.91] and
DT is [0.00 0.55 0.67 1.00] Then y is y6

To ensure that the model has reasonable accuracy, the mean square error has been used as a performance index:

$$PI = \sum_{i=1}^{m} (y^i - \hat{y}^i)^2 / m \qquad \text{Eqn 1}$$

Where m is the number of data, y^i is the i^{th} actual output and $\hat{y}^i$ is the i^{th} model output.

The performance index of the conventional fuzzy rule base is 0.07. In the merging process (section 3.2.2) , some accuracy has been traded off for inference speed and interpretability. The produced hierarchical fuzzy rule base has a slightly higher error, 0.08. After two iteration of the parameter identification process (section 4), the error becomes 0.025, which in our opinion is reasonably low.

It is also observed that the fuzzy rules in the hierarchical fuzzy rule base have fewer terms in the antecedents compared to the conventional rule base. This leads to higher interpretability of the model.

6. CONCLUSION

The hierarchical fuzzy rule base has been studied. A recursive algorithm for implementing the rule base has been proposed. Using a set of real world data, the practicality of the rule base has been verified.

The hierarchical fuzzy rule base studied in this paper has advantages over other conventional as well as hierarchical rule bases in the literature. The model is designed to reduce the complexity of the system and at the same time, improve the interpretability of the fuzzy rules.

In our next paper, the automatic construction of such hierarchical fuzzy rule bases from training data will be investigated. Recommended future researches include the comparison of the model with other existing hierarchical fuzzy model in terms of their performances.

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