IEEE Instrumentation and Measurement
Technology Conference
Budapest, Hungary, May 21-23, 2001.

Determination Of Parameter d50c of Hydrocyclones Using Improved Multidimensional Alpha-cut Based Fuzzy Interpolation Technique

Kok Wai Wong[1], Chun Che Fung[2], Halit Eren[2], and Tamás Gedeon[1]

[1]Murdoch University
School of Information Technology
South St, Murdoch,
Western Australia 6150
Tel: (+61 8) 9360 6599 Fax: (+61 8) 9360 2941
Email: {k.wong / t.gedeon}@murdoch.edu.au

[2]Curtin University
School of Electrical and Computer Engineering
Kent Street, Bentley,
Western Australia 6102
Tel: (+61 8) 9266 2575 Fax:: (+61 8) 9266 2584
Email: {tfungcc / terenh}@cc.curtin.edu.au

Abstract ***– In most control and engineering applications, the use of fuzzy system as a way to improve the human-computer interaction has becoming popular. This paper reports on the use of fuzzy system in mineral processing specifically in determining the parameter d50c of hydrocyclone. However, with the input-output data provided to build the fuzzy rule base, it normally results in a sparse fuzzy rule base. This paper examines the use of the improved multidimensional alpha-cut based (IMUL) fuzzy interpolation technique to improve the prediction capability of the sparse fuzzy rule base.***

Keywords: ***Hydrocyclone, Fuzzy Interpolation, d50c, Sparse Fuzzy Rules, Mineral Processing.***

I. INTRODUCTION

Hydrocyclones [1] find extensive application in the mineral process industry where they are used for the classification and separations of solids suspended in fluids. They are manufactured in different shapes and sizes to suit specific purposes. Hydrocyclones normally have no moving parts. The feed slurry containing all sizes of particles enters the hydrocyclone. Inside, due to centrifugal force experienced by the slurry, the heavier particles are separated from the lighter ones thus leading to size separation.

After the particles suspended in the fluid are classified, they are discharged either from the vortex finder as overflow or from the spigot opening as underflow. Due to the complexity of the separation mechanism in the hydrocyclone, the interpretation of the physical behaviour and forces acting on the particles is not clear. Much work has been done on describing hydrocyclone performance using mathematical modelling [2, 3, 4, 5].

The performance of a hydrocyclone is normally described by a parameter known as the d50. This parameter determines the classification efficiency. It represents a particular particle size reporting 50% to the overflow and 50% to the underflow streams. The separation efficiency of hydrocyclones depends on the dimensions of the hydrocyclone and the operational parameters. Examples of the operational parameters are flowrates and densities of slurries. The d50 is not a monitored parameter, but can be determined from the separation curves, *eg* the tromp curve. They are used to provide the relationship between the weight fraction of each particle size in the overflow and underflow streams.

In practical applications, the d50 curve is corrected by assuming that a fraction of the heavier particles is entering the overflow stream. This correction of d50 is designated as d50c. The correct estimation of d50c is important since it is directly related to the efficiency of operations. Under normal industrial applications of hydrocyclones, any deviation from a desired d50c value cannot be restored without changing the operation conditions or/and the geometry of the hydrocyclone. Also, sensing the alterations in the d50c is a difficult task, requiring external interference by taking samples from the overflow and underflow streams and conducting lengthy size distribution analyses of these samples.

Gupta and Eren have discussed the automatic control of hydrocyclones [5]. The output signal d50c cannot be sensed or conditioned directly, thus d50c needs to be calculated from the operation parameters. The automatic control of hydrocyclones can be achieved by manipulation of the operational parameters such as diameters of the spigot opening (Du), the vortex finder height (H), the inlet flowrate (Qi), the density (Pi) and the temperature of slurries (T) for a set value of d50c. The correct prediction of d50c is essential to generate control signals.

Traditionally, mathematical models based on empirical methods and statistical techniques in describing the performance of the hydrocyclones are used. Although these approaches have long been established in the industry, they do have their shortcomings. The experimental conditions may vary, resulting in these empirical models to be unreliable. Hence this approach may not be applicable universally.

In recent years, Artificial Neural Network (ANN) [6, 7] and Neural-Fuzzy [8] techniques have been applied. Although ANN techniques have proven to be useful for the prediction of the d50c, the main disadvantage is their inability to convey the acquired knowledge to the user. As a trained network is represented by a collection of weights, the user will have difficulty to understand and modify the model. In most cases, the system may not even gain the confidence of the user. The Neural-Fuzzy approach shown to be better than the ANN approach as it can generate fuzzy rules for user to manipulate. However, the fuzzy rules generated to cover the whole sample space are too tedious for the user to examine.

In this paper, it will examine the possible use of fuzzy system in determining d50c. Next section will discuss on the fuzzy rules extraction technique used to extract sparse fuzzy rule base from the test examples. Section three will look at the improved multidimensional alpha cut based (IMUL) fuzzy rule interpolation technique, which is used to solve the problem in sparse fuzzy rule base. Case study and results will also be presented in this paper.

II. PRELIMINARIES

Fuzzy control systems are becoming popular in dealing with control problems that are normally handled by statistical approaches or Artificial Neural Networks. The most important application of the classical fuzzy set theory is still fuzzy control systems. A fuzzy set allows for the degree of membership of an item in a set to be any real number between 0 and 1. This allows human observations, expressions and expertise to be modelled more closely. Once the fuzzy sets have been defined, it is possible to use them in constructing rules for fuzzy control systems and in performing fuzzy inference. Fuzzy reasoning is expressed as linguistic rules in the form "If x is A, then y is B", where x and y are fuzzy variables, and A and B are fuzzy values. These rules correspond well to the rules expressed by humans.

The use of fuzzy systems simplifies the development of an intelligent control system. Sophisticated knowledge and rich human experience can be incorporated into the fuzzy knowledge base in a form that is close to natural language. The fuzzy control systems allow incorporation of knowledge that is not necessarily precise and complete. The input to be accessed in fuzzy inference need not necessarily be clear nor do they have to match the given knowledge exactly. The fuzzy systems also allow partially matched conclusions to be inferred from the fuzzy facts and the established fuzzy knowledge base. The advantages of using FL are the ability to interpret the control model built and to handle fuzzy data. The control model can also be changed easily by modifying the fuzzy rule base.

However, conventional fuzzy systems do not have any learning algorithm to build the analysis model. Rather, they make use of human knowledge, past experience or detailed analysis of the available data by other means in order to build the fuzzy rules for the control system. Therefore, the major limitation is the difficulty in building the fuzzy rules due to lack of learning capability. However, if a rule extraction technique can be used, the fuzzy control system still has its advantage. Recently, an automatic self-generating fuzzy rules inference system [9] had shown successful results in establishing the well log interpretation model. This method is used in this paper to extract fuzzy rules from the test examples generated by the hydrocyclone model.

The steps involved in the self-generating fuzzy rules inference system are summarise as follows:

(1) Normalise the data between 0 and 1 by using a linear or logarithmic transformation depending on the nature of the data. This is to ensure that the resolutions of all data are similar.
(2) Define the shape of the membership function, number of fuzzy regions and fuzzy terms for all data. In this approach, only triangular membership functions are used. The number of fuzzy regions used is the same for all inputs and output
(3) The space associated with each fuzzy term over the universal discourse for each variable is then calculated and divided evenly.
(4) For each available training data, a fuzzy rule is established by directly mapping the physical value of the variable to the corresponding fuzzy membership function. Most of the time for a given value, it will normally fall into more than one fuzzy region. In this case, a degree is given to that value in the fuzzy region. The value is then assigned to the fuzzy region with maximum degree.
(5) Go through Steps (1-4) with all available data and generate one rule for each input-output data pair.
(6) Reduce the fuzzy rule base. In this step, all rules are examined for similarity. Similar rules are then eliminated and taken out of the rule base.

(7) The set of reduced fuzzy rules together with the centroid defuzzification algorithm now forms the fuzzy rule base.

The fuzzy rule base established by the extraction technique is normally a sparse fuzzy rule base as it normally contains gaps. This is mainly due to the reason that the given examples used to construct the fuzzy rule base are normally not enough to construct a complete and comprehensive fuzzy rule base. In the case when a fuzzy rule base contains gaps, classical fuzzy reasoning methods can no longer be used. This is due to the lack of inference mechanism in the case when observations find no rule to fire.

III. CALCULATIONS

Kóczy and Hirota [10] introduce the first fuzzy interpolation technique in providing a tool for generating an output fuzzy set whenever at least one of the input universes is sparse. However as shown in [11], KH fuzzy interpolation has some undesirable conclusions. In this paper, the IMUL fuzzy interpolation technique [11] is used to generate d50c of hydrocyclone using the sparse fuzzy rule base.

For k input dimensions, the reference characteristic point of the interpolated conclusion can be calculated:

$$RB^* = (1-\lambda_{core})RB_1 + \lambda_{core}RB_2 \tag{1}$$

$$\text{where } \lambda_{core} = \frac{\sqrt{\sum_{i=1}^{k}\left(RA_i^* - RA_{i1}\right)^2}}{\sqrt{\sum_{i=1}^{k}(RA_{i2} - RA_{i1})^2}}$$

By using the above reference point, the left and right cores of the conclusion can then be calculated:

For right core:

$$\begin{aligned} &RCB^* = \\ &(1-\lambda_{right})RCB_1 + \lambda_{right}RCB_2 + \\ &(\lambda_{core} - \lambda_{right})(RB_2 - RB_1) \end{aligned} \tag{2}$$

$$\text{where } \lambda_{right} = \frac{\sqrt{\sum_{i=1}^{k}\left(RCA_i^* - RCA_{i1}\right)^2}}{\sqrt{\sum_{i=1}^{k}(RCA_{i2} - RCA_{i1})^2}}$$

After calculating the cores of the two sides, the two flanks can then be found. When calculating the left and right flanks of the conclusion, the relative fuzziness of the fuzzy sets in all the input spaces are taken into consideration as follows: Based on A_{i1} and B_1

$$s_i = RFA_{i1} - RCA_{i1} \tag{3}$$

$$s' = RFB_1 - RCB_1 \tag{4}$$

$$r_i = LCA_i^* - LFA_i^* \tag{5}$$

$$r' = LCB^* - LFB^* \tag{6}$$

$$u_i = RA_i^* - RA_{i1} \tag{7}$$

$$u' = RB^* - RB_1 \tag{8}$$

In multidimensional input spaces,

$$s = \sqrt{\sum_{i=1}^{k}(s_i)^2} \tag{9}$$

$$r = \sqrt{\sum_{i=1}^{k}(r_i)^2} \tag{10}$$

$$u = \sqrt{\sum_{i=1}^{k}(u_i)^2} \tag{11}$$

To calculate the left flank:

$$LFB^* = LCB^* - r_k\left(1 + \left|\frac{s'}{u'} - \frac{s}{u}\right|\right) \tag{12}$$

IV. RESULTS AND NOVELTIES

Data collected from a Krebs hydrocyclone model D6B-12°-839 have been used. There are a total of 70 training data and 69 testing data used in this study. The input parameters are diameters of the spigot opening (Du), the vortex finder height (H), the inlet flowrate (Qi), the density (Pi) and the temperature of slurries (T), and the output is d50c. The self-generating fuzzy rules technique is used to extract fuzzy rules from the 70 training data. 7-membership function has been selected as it gives the best result. There are a total of 64 fuzzy sparse rules generated from the rule extraction process.

In order to show the applicability of this proposed method, the results are also used to compare with results generated from the on-line control model shown in [5].

$$
d50c = 23.36\left[\frac{EXP(-0.0125Du + 0.1031)Pi}{EXP(0.2721Du)}\right]
$$
$$
*\left[(-0.0229Du - 0.0211)\left(\frac{Qi}{Q\min}\right) + 0.0739Du + 0.9138\right]
$$
$$
\left[-0.426\left(\frac{H}{H_D}\right) + 1.42\right]\left[0.2\left(\frac{T_n}{T}\right) + 0.8\right] \tag{13}
$$

where $T_n = 25$, $Q\min = 120$, and $H_D = 85.2$.

When the sparse rule base are used to perform control on the testing data, 4 sets of input instances cannot find any fuzzy rules to fire, and the fuzzy inference system default the output to zero. In this case study, the number of input sets that cannot find any rule to fire is considered minimal. However, in some cases, this may not always be true. If more than half the input instances cannot find any rule to fire, this control system may be considered useless.

From observation and Euclidean distance measure on each input variable, the nearest fuzzy rules of the four input instances are determined to be used for the IMUL fuzzy interpolation. After performing the IMUL fuzzy interpolation, d50c for the four input instances can be interpolated.

Percentage Similarity Coefficient (PSC) is used to perform the measurement of difference between the predicted d50c *(T)* and the observed d50c *(O)*. The calculation is perform using the following:

$$
PSC = 200\frac{\sum_{i=1}^{P}\min(T_i, O_i)}{\sum_{i=1}^{P}(T_i + O_i)} \tag{14}
$$

The PSC value between the observed d50c and the d50c calculated using (13) is 90.096%. As for d50c generated from the fuzzy inference system without the IMUL fuzzy rule interpolation as compared to the observed d50c is 88.211%. However, when the IMUL fuzzy rule interpolation is used the PSC value increased to 91.899%.

Fig. 1 shows the plot of all the predicted d50c as compared to the observed d50c. The sharp fall of value in the fuzzy curve is mainly due the default value of zero when no fuzzy rule is fired.

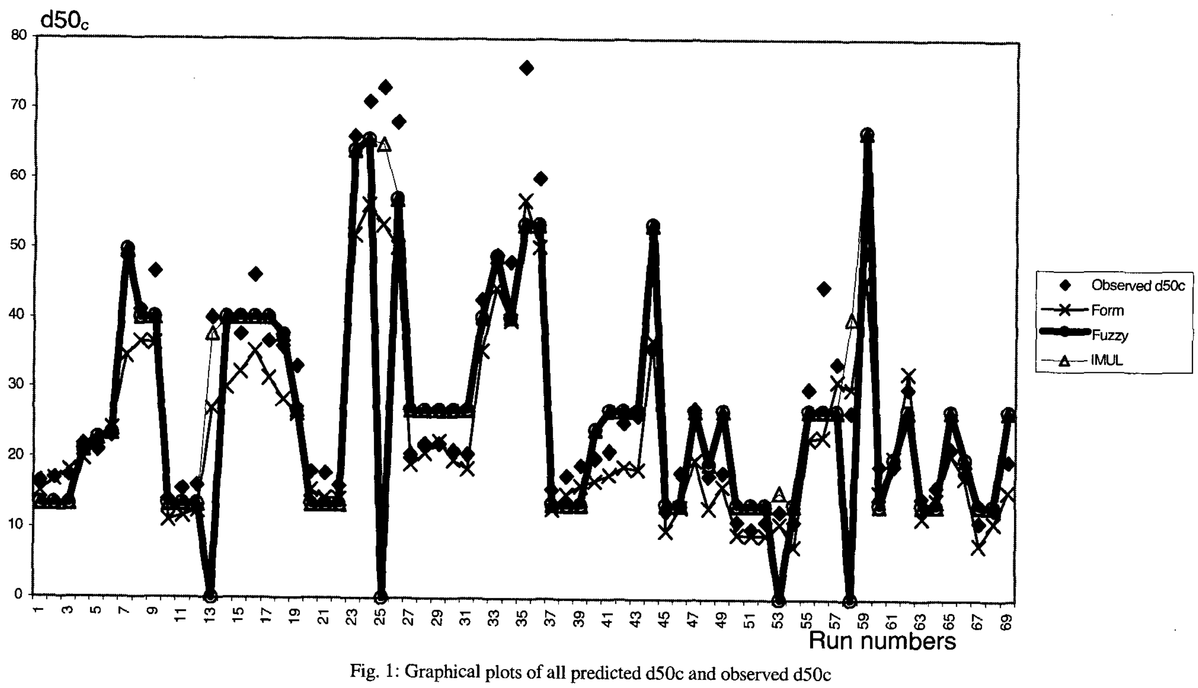

Fig. 1: Graphical plots of all predicted d50c and observed d50c

From these results, it has shown that the fuzzy system with the IMUL fuzzy rule interpolation technique has better prediction results to those of the formula. In this case study, it has also shown the importance of the IMUL fuzzy rule interpolation technique to be used in practical situation where the observed d50c can only generate sparse fuzzy rule base. However, in this case study the number of input instances that cannot find any fuzzy rule to fire is considered minimum. If more than 20% of the input instances have the similar problem, and then the fuzzy inference system will have no use in determining the d50c. Besides, with the use of the IMUL fuzzy interpolation technique, the number of fuzzy rules in the rule base is not increased. This is a desirable characteristic for on-line hydrocyclone control, as minimum fuzzy rules used will imply better human-computer interaction.

V. CONCLUSIONS

In the prediction of hydrocyclone parameter, $d50_C$, the results of the best-known conventional model has been compared with those results obtained from the fuzzy inference system with the IMUL fuzzy interpolation technique. Case study has shown that with assistance of the IMUL fuzzy interpolation technique, the sparse fuzzy rule base extracted from the observed d50c, can provide a good alternative in on-line hydrocyclone control. This is useful in the field of on-line hydrocyclone modelling as it allows the incorporation of fuzzy sets to enable better human-computer interaction.

REFERENCES

[1] D. Bradley, *The Hydrocyclone*, Pergamon Press Ltd, 1965.

[2] D. F. Kelsall, "A Study of the Motion of Solid Particles in a Hydrocyclone", *Transactions of Institution Chemical Engineering*, 30, 1952, pp. 87-104.

[3] A. J. Lynch, and T. C. Rao, "Modelling and scaling up of Hydrocyclone classifiers", *Proceedings Eleventh International Mineral Processing Congress*, 1975, pp. 245-270.

[4] L. R. Plitt, "A Mathematical Model for Hydrocyclone Classifier", *C.I.M. Bulletin*, 69(776), 1976, pp. 114-122.

[5] A. Gupta, and H. Eren, "Mathematical modelling and on-line control of Hydrocyclones", *Proceedings Aus. IMM*, 295 (2), 1990, pp. 31-41.

[6] H. Eren, C. C. Fung, K. W., Wong, and A. Gupta., "Artificial Neural Networks in Estimation of Hydrocyclone Parameter d50c with Unusual Input Vaiables," *IEEE Transactions on Instrumentation & Measurement*, 46(4), 1997, pp. 908-912.

[7] H. Eren, C. C. Fung, and K. W. Wong, "An Application of Artificial Neural Network for Prediction of Densities and Particle Size Distributions in Mineral Processing Industry," *Proceedings of IEEE Instrumentation and Measurement Technology Conference*, 1997, pp. 1118-1121.

[8] C. C. Fung, K. W. Wong, and H. Eren, "Developing a Generalised Neural-Fuzzy Hydrocyclone Model for Particle Separation," *Proceedings of IEEE Instrumentation and Measurement Technology Conference*, 1998, pp. 334-337.

[9] C. C. Fung, K. W. Wong, and P. M. Wong, "A Self-generating Fuzzy Inference Systems for Petrophysical Properties Prediction," *Proceedings of IEEE International Conference on Intelligent Processing Systems*, 1997, pp.205-208.

[10] L. T. Kóczy, and K. Hirota, "Approximate Reasoning by Linear Rule Interpolation and General Approximation," *Int. J. Approx. Reason*, 9, 1993, pp.197-225.

[11] K. W. Wong, T. D. Gedeon, and D. Tikk "An Improved Multidimensional Alpha-cut Based Fuzzy Interpolation Technique," *Proceedings of International Conference on Artificial Intelligence in Science and Technology AISAT'2000*, 2000, pp. 33-38.