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AN IMPROVED MULTIDIMENSIONAL ALPHA-CUT BASED FUZZY INTERPOLATION TECHNIQUE

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ABSTRACT

Fuzzy rule based systems have been very popular in many engineering applications. However, when generating fuzzy rules from the available information, it may result in a sparse fuzzy rule base. Fuzzy rule interpolation techniques have been established to solve the problems encountered by sparse rule bases. In most engineering applications, the use of more than one input variable is common. This paper proposes an improved multidimensional fuzzy rule interpolation technique to handle large dimensional input spaces. Illustration examples are also generated and results shown that this improved multidimensional α-cut based fuzzy interpolation technique can be used in engineering applications.

1. INTRODUCTION

In most fuzzy engineering applications, the fuzzy rule base is set up using any available information. The information can be in the form of measure data or from some computational simulation. Quite often, the information provided is not enough to construct a complete and comprehensive fuzzy rule base. In the case when a fuzzy rule base contains gaps, which is a sparse rule base, classical fuzzy reasoning methods can no longer be used. This is due to the lack of inference mechanism in the case when observations find no fuzzy rule to fire. Fuzzy rule interpolation techniques provide a tool for specifying an output fuzzy set whenever at least one of the input spaces is sparse. Kóczy and Hirota [1] introduced the first interpolation approach known as (linear) KH interpolation. This is based on the Fundamental Equation of Rule Interpolation (refer to equation (1)). This method determines the conclusion by its α-cuts in such a way that the ratio of distances between the conclusion and the consequents should be identical with that among observation and the antecedents for all important α-cuts (breakpoint levels). This is shown in the equation as follow (refer to Figure 1 for notations):

$$d(A^*, A_1) : d(A^*, A_2) = d(B^*, B_1) : d(B^*, B_2) \qquad (1)$$

Two conditions apply for the usage of the linear interpolation. First, there should exist an ordering on the input and output universes. This allows us to introduce a notion of distance between the fuzzy sets. Second, the input sets (antecedents, consequents and the observation) should be convex and normal fuzzy sets.

The KH interpolation possesses several advantageous properties. Firstly, it behaves approximately linearly between the breakpoint levels. Secondly, its computational complexity is low, as it is sufficient to calculate the conclusion for the breakpoint level set. Moreover, its extension is found to be a universal approximator [2]. However, for some input situation it fails to result in a directly interpretable fuzzy set, because the slopes of the conclusion can collapse as shown in Figure 1**.**

Several approaches were proposed in the last decade to alleviate this inconvenience [3, 4, 5, 6]. These approaches either determine conditions with respect to the input sets [3, 4] or implement conceptually different method to avoid abnormal conclusions [5, 6]. The new concepts, however, do not preserve the low computational complexity of the original KH method. Recently, a modification of the original method has been proposed which solves the problem of abnormal conclusions while maintaining its advantageous properties [7, 8]. This is known as Modified Alpha-Cut fuzzy Interpolation (MACI). This method is selected for this study of interpolating multidimensional input. However, our study in this paper is limited to triangular and trapezoidal membership functions.

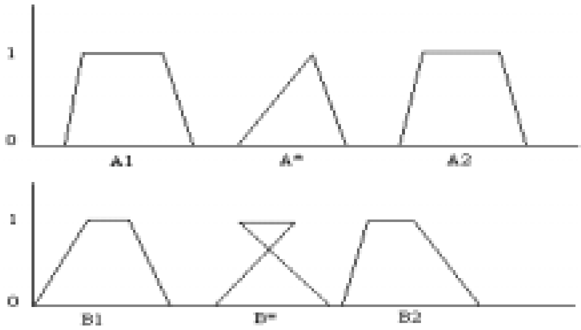

Figure 1: Problem of linear KH fuzzy interpolation.

In most fuzzy applications, the input vector normally involve more than one variable, therefore the characteristics of multidimensional fuzzy rule interpolation is of much interest [9,10]. In this paper, an improved multidimensional α-cut based fuzzy interpolation technique is proposed here to make use of the advantages of the MACI technique [8], as well as the conservation of fuzziness technique [6].

2. IMPROVED MULTIDIMENTIONAL FUZZY RULE INTERPOLATION

This method incorporates features of the MACI and the conservation of fuzziness technique. It makes use of the vector description of the fuzzy sets by representing them in characteristic points, and the coordinate transformation features of the MACI. At the same time, it can take the fuzziness of the fuzzy sets in the input spaces at the conclusion as those presented in the conservation of fuzziness technique. The advantage of this improved fuzzy interpolation technique not only takes the fuzziness of the sets at the input spaces, but also make use of the information of the core at the consequents.

Refer to Figure 2 for notations used in the following formulas. For k input dimensions, the reference characteristic point of the interpolated conclusion with the use of Euclidean distance is:

$$RB^* = (1-\lambda_{core})RB_1 + \lambda_{core}RB_2 \qquad (2)$$

where $$\lambda_{core} = \frac{\sqrt{\sum_{i=1}^{k}\left(RA_i^* - RA_{i1}\right)^2}}{\sqrt{\sum_{i=1}^{k}\left(RA_{i2} - RA_{i1}\right)^2}}$$

By using the above reference point, the left and right cores of the conclusion are calculated as follows:

For right core:

$$\begin{aligned} &RCB^* = \\ &(1-\lambda_{right})RCB_1 + \lambda_{right}RCB_2 + \\ &(\lambda_{core} - \lambda_{right})(RB_2 - RB_1) \end{aligned} \qquad (3)$$

where $$\lambda_{right} = \frac{\sqrt{\sum_{i=1}^{k}\left(RCA_i^* - RCA_{i1}\right)^2}}{\sqrt{\sum_{i=1}^{k}\left(RCA_{i2} - RCA_{i1}\right)^2}}$$

For left core:

$$\begin{aligned} &LCB^* = \\ &(1-\lambda_{left})LCB_1 + \lambda_{left}LCB_2 + \\ &(\lambda_{core} - \lambda_{left})(RB_2 - RB_1) \end{aligned} \qquad (4)$$

where $$\lambda_{left} = \frac{\sqrt{\sum_{i=1}^{k}\left(LCA_i^* - LCA_{i1}\right)^2}}{\sqrt{\sum_{i=1}^{k}\left(LCA_{i2} - LCA_{i1}\right)^2}}$$

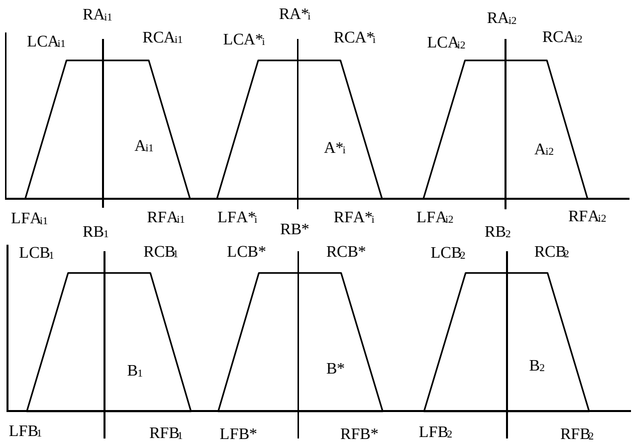

Figure 2: Notations for the formulae

After calculating the cores of the two sides, the two flanks can then be calculated. When calculating the left and right flanks of the conclusion, the relative fuzziness of the fuzzy sets in all the input spaces are taken into consideration as follows:

Based on A_{i1} and B_1

$$s_i = RFA_{i1} - RCA_{i1} \quad (5)$$

$$s' = RFB_1 - RCB_1 \quad (6)$$

$$r_i = LCA_i^* - LFA_i^* \quad (7)$$

$$r' = LCB^* - LFB^* \quad (8)$$

$$u_i = RA_i^* - RA_{i1} \quad (9)$$

$$u' = RB^* - RB_1 \quad (10)$$

In multidimensional input spaces,

$$s = \sqrt{\sum_{i=1}^{k} (s_i)^2} \quad (11)$$

$$r = \sqrt{\sum_{i=1}^{k} (r_i)^2} \quad (12)$$

$$u = \sqrt{\sum_{i=1}^{k} (u_i)^2} \quad (13)$$

For left flank:

$$LFB^* = LCB^* - r_k \left(1 + \left| \frac{s'}{u'} - \frac{s}{u} \right| \right) \quad (14)$$

3. CASE RESULTS AND DISCUSSION

3.1 One Dimensional Input Space

To illustrate the features of this improved multidimensional (IMUL) fuzzy rule interpolation technique, it is used for one-dimensional input space first before applying to multidimensional input spaces. This is necessary to ensure that the proposed IMUL technique can interpolate fuzzy rule just like the other fuzzy interpolation techniques.

A few examples in [6] are used for one-dimensional illustrations as follows. The notations used in the figures are as follow: KH for the KH technique in [9], MACI for the MACI technique in [8], and IMUL for this proposed IMUL fuzzy interpolation technique.

Test 1: (refer to Figure 3)

A_1: 0, 5, 25, 30
A_2: 70, 75, 95, 100
B_1: 0, 15, 20, 30
B_2: 70, 85, 90, 100
A*: 35, 55, 55, 60

B* (KH): 35, 65, 50, 60
B* (MACI): 40, 55, 60, 70
B* (IMUL): 32, 55, 60, 67

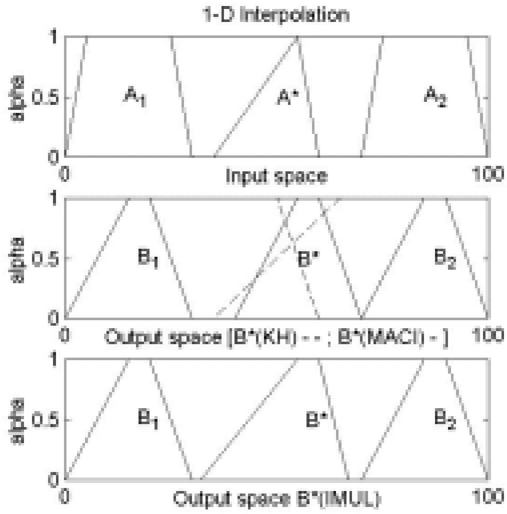

Figure 3: Fuzzy Interpolations for Test 1.

Figure 3 has shown the case where KH fuzzy interpolation technique generated abnormal conclusion. However, MACI and this IMUL produced results that avoided the abnormal conclusion. By using IMUL, the fuzziness of the left flank and right flank of the conclusion are taken from the observation as oppose to MACI, which takes its fuzziness from the neighbouring consequents. Beside this, the results generated from MACI and IMUL are both acceptable.

Test 2: (refer to Figure 4)

A_1: 10, 10, 10, 10
A_2: 90, 90, 90, 90
B_1: 30, 30, 30, 30
B_2: 95, 95, 95, 95
A*: 40, 45, 55, 70

B* (KH): 54, 58, 67, 79
B* (MACI): 62, 62, 62, 62
B* (IMUL): 58, 62, 62, 79

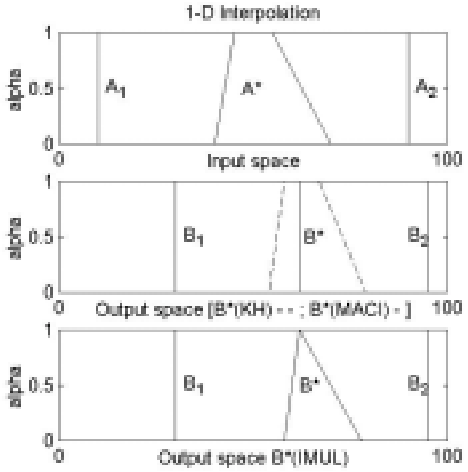

Figure 4: Fuzzy Interpolations for Test 2.

Figure 4 has shown the case where the three interpolation technique produce reasonable conclusions. However, MACI produces crisp conclusion due to the influence of the two neighbouring consequents. In this test, IMUL technique produces conclusion that is reasonable and comparable to those generated from the original KH technique.

Test 3: (refer to Figure 5)

A_1: 0, 20, 30, 40
A_2: 70, 80, 90, 100
B_1: 0, 30, 35, 40
B_2: 80, 85, 95, 100
A*: 45, 45, 60, 60

B* (KH): 51, 53, 69, 60
B* (MACI): 44, 58, 65, 70
B* (IMUL): 58, 58, 65, 65

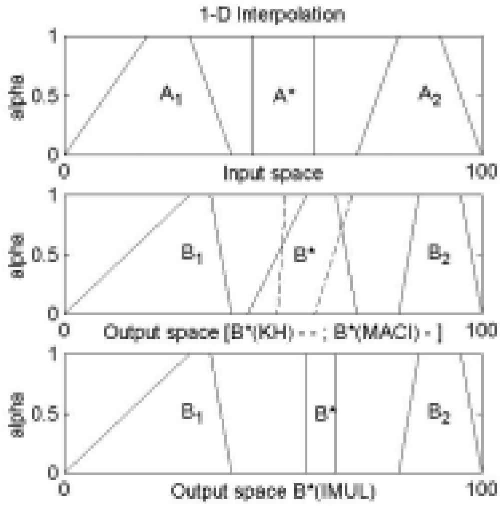

Figure 5: Fuzzy Interpolations for Test 3.

Figure 5 has again shown the case where KH fuzzy interpolation technique generated abnormal conclusion. From the above three illustration examples with one input space, it has been shown that the proposed interpolation technique can be used to generate results. From all the tests, it has also been shown that this IMUL not only uses the information of the core from the consequents but also takes the fuzziness of the sets at the input space in producing the interpolated conclusion.

3.2 Multidimensional Input Spaces

After showing the results in one-dimensional problems, it is now used in cases where multidimensional input spaces are used. The illustration example used in [9] with 5 input variables is used for this purpose as follows:

A_{11}: 6, 13, 13, 20
A_{12}: 61, 69, 69, 76
A_{21}: 10, 20, 20, 30
A_{22}: 77, 86, 86, 96
A_{31}: 6, 14, 14, 24
A_{32}: 86, 93, 93, 100
A_{41}: 21, 26, 26, 30
A_{42}: 47, 59, 59, 70
A_{51}: 16, 26, 26, 36
A_{52}: 81, 83, 83, 85
B_1: 24, 31, 31, 39
B_2: 81, 86, 86, 90

Test 4:
In this test, fuzzy observations are used in all the input spaces as those in [9]. Refer to Figure 6 and Figure 7.

A_1*: 31, 39, 39, 46
A_2*: 67, 69, 69, 70
A_3*: 51, 66, 66, 80
A_4*: 26, 33, 33, 40
A_5*: 61, 66, 66, 70

B* (KH): 59, 65, 65, 69
B* (MACI): 61, 66, 66, 72
B* (IMUL): 47, 66, 66, 84

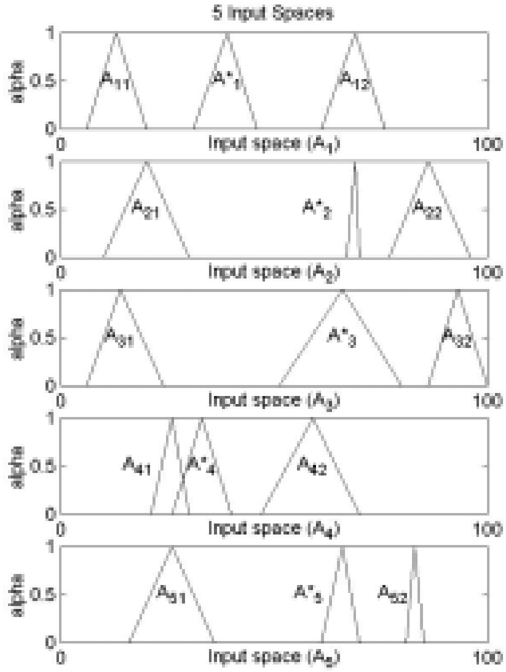

Figure 6: Input fuzzy sets for Test 4.

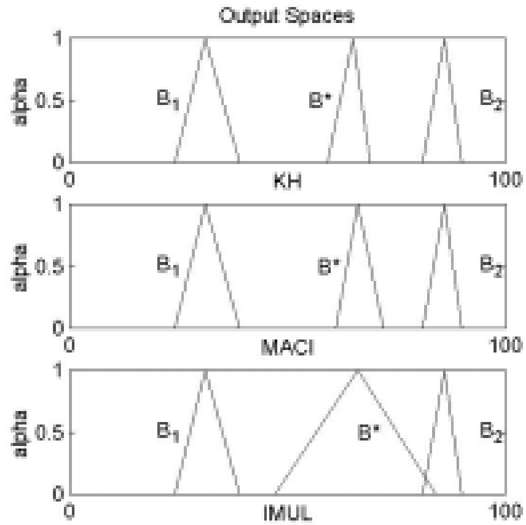

Figure 7: Output fuzzy sets for Test 4.

In this test, the characteristic of the IMUL by using the fuzziness from all the input observations and the cores of the neighbouring consequents can also be observed.

Test 5:
In this test, all the input fuzzy observations have been changed to singleton values as shown in Figure 8. The output fuzzy interpolation results are shown in Figure 9.

A_1*: 39, 39, 39, 39
A_2*: 69, 69, 69, 69
A_3*: 66, 66, 66, 66
A_4*: 33, 33, 33, 33
A_5*: 66, 66, 66, 66

B* (KH): 66, 65, 65, 64
B* (MACI): 61, 66, 66, 72
B* (IMUL): 66, 66, 66, 66

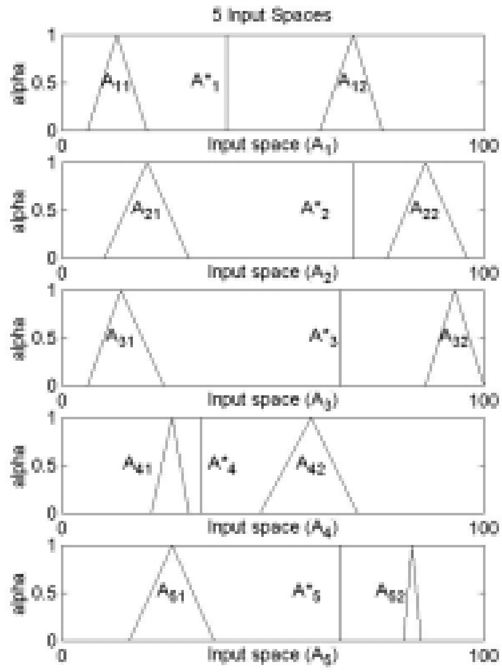

Figure 8: Input fuzzy sets for Test 5.

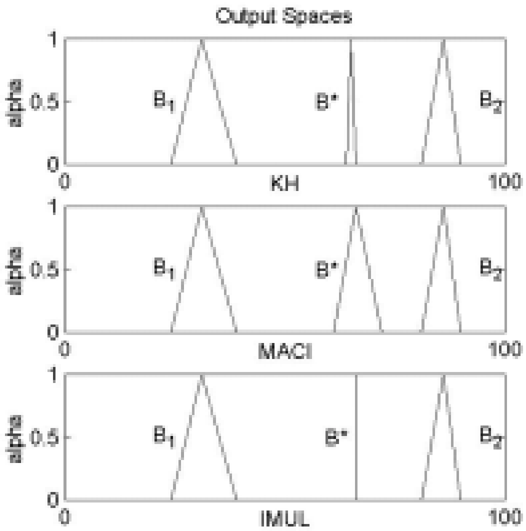

Figure 9: Output fuzzy sets for Test 5.

In this test, all the observations in Test 4 are changed to crisp observations, but the interpolated conclusion from MACI remains the same as compared to the result in Test 4. This seems to be inappropriate, as the observations have no effect to the interpolated conclusion. As for the KH technique, abnormal conclusion is generated. The conclusion generated from IMUL technique changes to crisp value with the changes in the observations. This seems to be more reasonable in this case.

From Test 4 and 5 where 5 input variables are used, the characteristics of the improved multidimensional α-cut fuzzy interpolation technique present consistent results. In this case, the relative fuzziness of the output fuzzy set is calculated from the effective fuzziness of all the input sets. The core information of the conclusion is again based on the information provided by the core of the consequents.

4. CONCLUSION

In this paper, a technique for interpolating multidimensional fuzzy rules using α-cut fuzzy interpolation is presented. An improved multidimensional α-cut based fuzzy interpolation technique is proposed here to make use of the advantages of the modified α-cut fuzzy interpolation technique [8] and the conservation of fuzziness technique [6]. This proposed technique takes into consideration the degree of fuzziness in the rule base, by measuring from the neighbouring rules in the multidimensional fuzzy rule base. Illustration test examples have also been presented and shown that this proposed technique can be used efficiently for interpolating fuzzy rules with multidimensional input variables. This technique has also shown successful application in the field of hydrocyclone control modeling [11].

5. ACKNOWLEDGEMENT

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