

An Improved Learning Algorithm for Laterally Interconnected Synergetically Self-organizing Map

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Abstract *LISSOM (Laterally Interconnected Synergetically Self-organizing Map) is a biologically motivated self-organizing neural network for the simultaneous development of topographic maps and lateral interactions in the visual cortex. However, the simple Hebbian mechanism for afferent connections requires a redundant dimension to be added to the input and normalization is necessary. Another shortcoming of LISSOM is that several parameters must be chosen before the LISSOM can be used as a model of topographic map formation. To solve these problems, in this paper, we propose to apply the Least Mean Square Error Reconstruction (LMSER) learning rule as an alternative to the simple Hebbian rule for the afferent connections. Experiments demonstrate the essential topographic map property from the improved LISSOM model.*

1. Introduction

The topographic map is a ubiquitous property in many cortical areas by which nearby neurons respond to nearby regions of the retina. Such retinotopic maps usually result from the self-organization of the afferent connections to the cortex, which is driven by external input. In the past, several self-organization models have been proposed to demonstrate the topographic map property, which are often based on predetermined lateral interaction, and focus on explaining how the afferent connections become ordered. Kohonen's self-organizing map (SOM) is a representative model [1-2]. The critical factor in generating topology-preserving mappings with the SOM is the use of a neighborhood function for updating neuron weights. As a result, neighboring neurons cooperate and specialize for similar input signals and the neuron lattice organizes into an orderly topology-preserving state. In SOM, how the lateral interactions are self-organized has not been addressed. Recently, many neurobiological experi-

ments have stressed the importance of the development of lateral connections. Therefore, a plausible cortical map model must demonstrate that both afferent and lateral connections can organize simultaneously.

In [3-5], a cortical self-organization model was proposed toward the aforementioned goal, which is called LISSOM (Laterally Interconnected Synergetically Self-organizing Map). In LISSOM, lateral and afferent connections self-organize cooperatively, and with suitably abstracted visual inputs, LISSOM can model self-organization of visual cortical structures, for example, the development of ocular dominance. Afferent connections in LISSOM are adapted via Hebbian learning, in exactly the same way as the lateral connections. As such, the input to LISSOM must be normalized. Another shortcoming of LISSOM is that several parameters must be carefully selected beforehand, especially the lower and upper thresholds in the piecewise linear approximation of the sigmoidal activation.

Though Hebbian synaptic plasticity is a generally accepted concept, its mechanism is not limited to the most simple form, i.e., the increasing synaptic efficacy is proportional to the correlation between pre- and post synaptic activity. A well-known result is that a linear neuron with an appropriately defined Hebbian learning objective can be regarded as a principal component analyzer. From this consideration, we study an alternative general form of Hebbian learning, based on a learning principle called Least Mean Square Error Reconstruction (LMSER) [6]. LMSER usually results in a principal component subspace, similar to an autoencoder trained by the error-propagation algorithm. For a nonlinear neuron model with sigmoidal activation, LMSER can be considered as a general realization of the Hebbian mechanism. When applied to LISSOM, the problems of input normalization and parameters se-

lection can be avoided. In our improved LISSOM, lateral interactions are still adjusted by the basic Hebbian learning.

2. Least Mean Square Error Reconstruction (LMSER) Learning in a Non-linear Neuron

Given random pre-synaptic inputs ξ_l , $l = 1, \dots, L$, weight vector $w = [w_1, \dots, w_L]^T$ and output $\eta = \sum_{l=1}^L \xi_l w_l$, Hebbian learning can be studied by a criterion which optimizes the reconstruction mean-square-error

$$\begin{aligned} \text{minimize } J &= E\left\{\frac{1}{2} \sum_{l=1}^L (\xi_l - \hat{\xi}_l)^2\right\} \\ &= E\left\{\frac{1}{2} \sum_{l=1}^L (\xi_l - \eta w_l)^2\right\} \end{aligned} \quad (1)$$

with

$$\hat{\xi}_l = \eta w_l \quad (2)$$

as a reconstruction of input ξ_l . In (1), E stands for statistical expectation. The corresponding stochastic gradient descent learning rule is

$$w(t + \Delta t) = w(t) + \mu_t (\varepsilon_l \eta + \xi_l \sum_{l=1}^L \varepsilon_l w_l) \quad (3)$$

where $\varepsilon_l = \xi_l - \hat{\xi}_l$ is the component of the reconstruction error.

The best reconstruction objective (1) can be extended to a nonlinear neuron $\eta = \sigma(\sum_{l=1}^L \xi_l w_l)$. In general, different nonlinear activation functions yield different learning results. Similar to (3), a nonlinear neuron's weights can be adaptively adjusted to solve the optimization problem in the same form of (1), and a stochastic approximation approach will lead to the following learning rule

$$w(t + \Delta t) = w(t) + \mu_t (\varepsilon_l \eta + \vartheta \xi_l \sum_{l=1}^L \varepsilon_l w_l) \quad (4)$$

where ϑ is the derivative of σ , $\vartheta = \sigma'(\sum_{l=1}^L \xi_l w_l)$. The proof of eqn (4) can be similarly performed as those in [6-7].

Learning rule (4) is a special case of the Least Mean Square Error Reconstruction (LMSER) learning proposed in [6] (see also [7]), which claimed best reconstruction as a general self-organization principle for any network structure. For a multilayer network, LSMER learning results in principal subspace

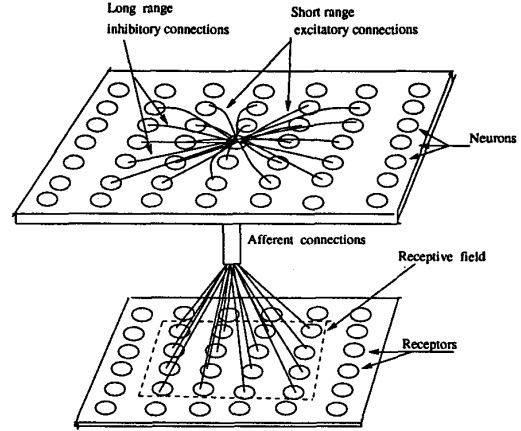


Figure 1: Architecture of the LISSOM self-organizing network. The lateral excitatory and lateral inhibitory connections of a single neuron in the network are shown, as well as the afferent connections.

solution.

The nonlinear neuron model can be considered as a special case of a symmetrical autoencoder with one hidden unit. In previous studies of autoencoders, the role played by individual units is addressed less. On the contrary, our focus here is to study the nonlinear neuron model and the corresponding learning algorithm (4) as a building block in a neural network.

3. An Improved LISSOM Model

The architecture of LISSOM is shown in figure 1, which is a two dimensional $N \times N$ grid of neurons. In LISSOM, a neuron has a set of afferent input connections (i.e. from the external input to the map) and a set of lateral input connections (i.e. from other neurons in the map). Each neuron has an initial response from a weighted sum of the activations through its afferent connections. The lateral interactions between the neurons contribute to the response to reach an equilibrium. Then all the connection weights are modified.

The afferent and lateral connections are organized through a self-organization process. At each training step, neurons start out with zero activity. The initial response η_{ij} of neuron (i, j) is based on the inner product of the input and weight vectors:

$$\eta_{ij} = \sigma\left(\sum_h w_{ij,h} \xi_h\right) \quad (5)$$

In the original form of LISSOM, σ is taken as a piecewise linear function, with low and high thresholds being carefully chosen beforehand. In the following, we take σ as a continuous sigmoidal function bounded between 0 and 1, for example, $\sigma(t) = 1/(1 + e^{-\beta t})$, with a nonlinearity parameter β being dynamically adapted in the self-organization process. Specifically, at the start of learning, the dynamic range of $\sigma(t)$ is relatively large. As learning progresses, β becomes large and the dynamic range narrows.

The response in (5) evolves over time through lateral interactions. At each time step, the neuron combines external activation with lateral excitation and inhibition according to

$$\eta_{ij}(t) = \sigma\left(\sum_h w_{ij,h}\xi_h + \gamma_e \sum_{k,l} E_{kl,ij}\eta_{kl}(t - \Delta t) + \gamma_i \sum_{k,l} I_{kl,ij}\eta_{kl}(t - \Delta t)\right) \quad (6)$$

where $E_{kl,ij}$ is the excitation lateral connection weight on the connection from unit (k, l) to unit (i, j) , $I_{kl,ij}$ is the inhibitory connection weight, and $\eta_{kl}(t - \Delta t)$ is the activity of unit (k, l) during the previous time step. The constants γ_e and γ_i control the relative strength of the lateral excitation and inhibition. The activity pattern then quickly converges into a stable focused patch of activity, usually within a few iterations of eqn (6).

Before learning starts, all the connection weights are random and the initial activity on the unordered network lattice shows a random distribution, as illustrated in Fig.2(a). The repeated exchange of short-range excitation and long-range inhibition enhances activity at the center and suppress activity away from it. The focusing of the response is demonstrated in Fig.2(b). When the network becomes organized, the initial activity bubbles become smoother and lateral interactions then focus the bubble.

In each learning step, after the activity has settled, the connection weights of each neuron are then modified. The lateral weights are modified by a Hebbian rule, keeping the sum of weights constant:

$$\gamma_{ij,kl}(t + \Delta t) = \frac{\gamma_{ij,kl}(t) + \alpha_L \eta_{ij}\eta_{kl}}{\sum_{kl} \gamma_{ij,kl}(t) + \alpha_L \eta_{ij}\eta_{kl}} \quad (7)$$

where η_{ij} stands for the activity of the unit (i, j) in the settled activity bubble, the γ s are the lateral

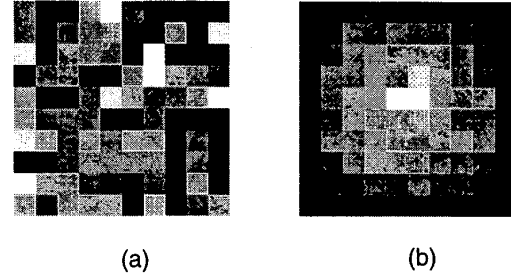


Figure 2: The focusing effect from lateral interaction on an unordered network. (a). Before the exchange of activations, all the connection weights are random and the initial response is also random. (b). After the exchange of activations, the response at the center of the net becomes enhanced.

interaction weights ($E_{ij,kl}$ or $I_{ij,kl}$) and α_L is the learning rate for lateral interaction (α_E for excitatory weights and α_I for inhibitory).

The afferent connections are modified according the LMSER learning rule:

$$w_{ij,h}(t + \Delta t) = w_{ij,h}(t) + \mu_t(\varepsilon_{ij,h}w_{ij,h}\xi_h g_{ij} + \eta_{ij}\varepsilon_{ij,h}) \quad (8)$$

Obviously, the above learning rule is local to an individual neuron and its connections.

4. Simulations

In the simulations, each neuron initially connects to its neighbors within distance d with excitatory lateral connections and to all neighbors within a larger distance d' with inhibitory connections. In our experiment, we choose $d = 5$ for self-organizing a 10×10 LISSOM network and $d = 10$ for a 20×20 network. We also choose $\gamma_e = 0.9$ and $\gamma_i = 0.9$ as in [3]. As the self-organization progresses, the neurons grow more nonlinear. In our simulations, the sigmoidal nonlinearity parameter is dynamically adjusted from 0.01 to 1. In Figure 3, we demonstrate the self-organization of the afferent input weights. The weight vector of each neuron in the 10×10 network is plotted as a point in the original input space. Each weight vector is connected to its four immediate neighbors by a line. The resulting grid depicts the topological organization of the map.

The self-organization process of afferent connections in the improved LISSOM is quite similar to

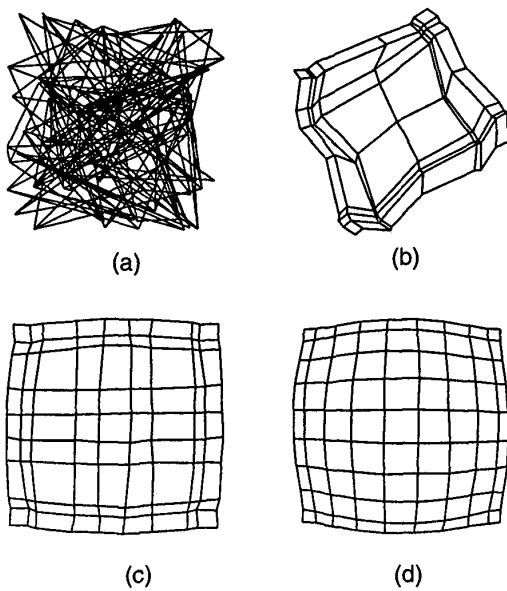


Figure 3: Demonstration of the self-organization of afferent connections. The network is trained with a 2-dimensional input vector x , with elements uniformly distributed in $\{-1, +1\}$. The random initial values is shown in (a). Three stages after 1000, 20000 and 30000 iterations are illustrated in (b) (c) (d) respectively.

that in Kohonen's Self-organizing Map algorithm. However, in the SOM process, the maximally responding unit is chosen through global supervision, and adaptation neighborhoods are reduced according to a predetermined schedule. In contrast, in the improved LISSOM process, self-organization is set up on purely local rules without global supervision. The shape of the lateral interaction is automatically extracted from the statistical properties of the external input.

5. Discussion

The LISSOM model demonstrates how lateral interaction and topological organization of cortical maps can be learned simultaneously from correlation in the input information. The model is biologically motivated, and its predictions agree well with experimental observations on cortical development. LISSOM is potentially capable of explaining various aspects of lateral and afferent connection development on the cortex, as well as the nature of lateral interactions in the cortex. LISSOM can be utilized

as an abstract self-organizing algorithm, with most promising application in modeling the development of the neocortex. LISSOM has shown how ocular dominance and lateral connections develop simultaneously in the visual cortex. It will be interesting to simulate our improved LISSOM model on the same tasks.

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