1999 IEEE International Fuzzy Systems Conference Proceedings
August 22-25, 1999, Seoul, Korea

A Practical Fuzzy Interpolator for Prediction of Reservoir Permeability

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Abstract

In this paper, we propose a practical fuzzy interpolator (PFI) to represent imprecise relationships between inputs and outputs in high-dimensional data systems. The method employs expert knowledge and sample data to dynamically generate piece-wise linear inference rules, and then the values to be estimated are interpolated and extrapolated based on these rules. We demonstrate the use of this methodology in petroleum reservoir engineering where the permeability is estimated among oil wells. The results are compared to a neural-fuzzy technique for the same petroleum reservoir data set. This shows that the PFI is not only simple, and computationally fast, but also gives better performance than the neural-fuzzy technique.

1. Introduction

A petroleum reservoir is a volume of porous sedimentary rock which has been filled with hydrocarbon, such as oil or gas. Reservoir properties are a set of parameters which are usually used to recognise the geologic information in spatial variability. Permeability, porosity, and fluid saturation are the three most important parameters of reservoir properties in petroleum engineering, and are widely used to determine the oil well or field production rate of such hydrocarbon. Well logs are a series of multi-type digital measurements along the vertical depth of drilled wells.

In petroleum reservoirs, the magnitude of the permeability value directly affects hydrocarbon production. The determination of such a property is very complex and expensive because laboratory measured permeability values on rock samples are only available in limited and isolated drilled well locations. At these locations, measurements take the form of actual rock samples (cores) and well log readings (simply called well logs). In this paper, a cored well means the permeability, most geological parameters, geological facies (rock types) and well logs are available. An un-cored well means only geological facies and well logs are available. A drilled well can be a cored or un-cored well.

The prediction of permeability is becoming increasingly important as more wells are drilled for petroleum exploration and development. Normally more than five well logs plus geological facies are available in un-cored wells. We use X to denote well logs and geological facies, and Y to denote permeability. Currently two kinds of methods have been proposed for permeability estimation. One kind is the regression model, such as multiple linear regression [3, 9], nonlinear neural networks [20, 12], and neural-fuzzy techniques [4, 5]. This model can be written in the following form:

$$Y \approx f(\theta(X_0, Y_0), X) \tag{1}$$

where f is a mapping from inputs to outputs, (X_0, Y_0) are the past observation values, and θ is a parameter set depending on (X_0, Y_0). So we call X the input and Y the output. In fact, equation (1) coincides with the definition of function approximation.

The other kind of methods is the linear weighted averaging model, such as inverse distance [2], Kriging or Cokriging [10, 1], and DFFE [15]. This model can be written in the following form:

$$Y \approx W(X, X_0).Y_0 \tag{2}$$

where X_0 and Y_0 are input-output samples in cored wells, and W is a set of weights depending on (X, X_0). In fact, equation (2) coincides with numerical interpolation and extrapolation.

All of the above models are objective. That is, they only use numerical information, and do not use expert knowledge. Recent research shows that nonlinear neural networks give much higher accuracies than multiple linear regression [6],

* This paper prepared at the University of New South Wales.

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and Kriging or Cokriging generally gives better results than inverse distance [8]. Nevertheless, currently reservoir engineers prefer to use multi-linear regression and inverse distance methods to estimate permeability because of their simplicity and fast computation times. In the petroleum industry, commercial packages with numerical interpolators or extrapolators mostly use simple and fast algorithms.

In this paper, we will follow the form of equation (2) and propose a new practical method, called a practical fuzzy interpolator (PFI), which not only combines both numerical and expert knowledge but also is simple and computationally fast as well as highly accurate, for predicting reservoir permeability in un-cored wells.

As we know, the environment in a petroleum reservoir is so complex and uncertain that no mathematical model exists between well logs and permeability. Therefore, we have to design a model-free method. In our problem domain, there are two kinds of information always available. One is the knowledge of reservoir engineers and the other is numerical information in drilled wells. That is, we know sampled input-output pairs in cored wells as well as inputs in un-cored wells. Usually in drilled wells the input dimension is more than five.

The main ideas of our fuzzy interpolator are firstly to define the neighbouring cored wells for each un-cored well in the petroleum exploration and development area. Each neighbouring cored well is seen as a fuzzy system. The numerical ranges of the inputs in drilled wells are given by experts (reservoir engineers) because the drilled wells usually cannot cover all the ranges of inputs due to the sparseness of sampling. Then we develop a model which dynamically generates dynamic piece-wise-linear inference rules in cored well based on inputs in un-cored wells. Finally, the permeability values in un-cored wells are interpolated and extrapolated based on the above rules and Euclidean distances between the sample point and the estimated point of the input.

In Section 2, we give a detail description of the proposal fuzzy interpolator. In Section 3, we show that our new method is used in a real case study, and the results are compared with those obtained using a neural-fuzzy technique. Results, discussions, and conclusions are given in Sections 4 and 5.

2. Fuzzy Interpolator

2.1. Theoretical Fuzzy Interpolator

Theoretical investigations into fuzzy systems as universal approximators have been done by some researchers [21, 19]. In fact, they provide a theoretical proof for fuzzy interpolators with some conditional constraints. Of course, in the high-dimensional input spaces (say more than five-dimensional) of the real world, the conditional constraints are never satisfied because of the limitations on the number of real sample data.

In order to make fuzzy interpolators predict reservoir permeability, we assume, without loss of generality in real space, that the fuzzy system F^i (the i^{th} cored well, $i=1,2,...,r$) is an m-input system $X^i=X_1^i\times X_2^i\times...\times X_m^i$ (i.e. well logs), with a single output Y^i (permeability). That is: $F^i{:}X^i\rightarrow Y^i$. Where $X_1^i\subset R_1, X_2^i\subset R_2, ..., X_m^i\subset R_m$, and $Y^i\subset R$, the range of every $\{R_j\}$ is given by experts. Now for any input $x=(x_1,x_2,...,x_m)\in X^i$ which comes from an un-cored well, there exists a piecewise linear interpolator:

$$y^i=\sum_{i_1=0}^{N}\sum_{i_2=0}^{N}\cdots\sum_{i_m=0}^{N}A_{i_1}^{i}(x_1)A_{i_2}^{i}(x_2)\ldots A_{i_m}^{i}(x_m)f^i\left(\frac{i_1}{N},\frac{i_2}{N},...,\frac{i_n}{N}\right) \quad (3)$$

where N is a enough large natural number which depends on the desired accuracy. We assume p is the number of samples and $N=p$. $\{A_{i_j}^i(x_j)\}$, $j=1,2,...,m$ is one group of fuzzy membership function sets defined in the known universe of discourse, and $f^i\left(\frac{i_1}{N},\frac{i_2}{N},...,\frac{i_n}{N}\right)\in y^i$. Usually each of $\{A_{i_j}^i(x_j)\}$ may be orthogonal with triangular shapes. That is, for the i^{th} cored well, the triangular membership function of the j^{th} input variable after sorting $x_{j_1}\le x_{j_2}\le...\le x_{j_p}$ can be defined as:

$$A_{i_j}^i(x_j)=\begin{cases}\dfrac{x_{j_k}-x_j}{x_{j_k}-x_{j_{k-1}}} & x_{j_{k-1}}\le x_j\le x_{j_k}\\ \dfrac{x_j-x_{j_k}}{x_{j_{k+1}}-x_{j_k}} & x_{j_k}\le x_j\le x_{j_{k+1}}\\ 0 & others\end{cases} \quad (4)$$

Figure 1 illustrates the triangular membership function for equation (4), where $\sum_{k=1}^{p}A_{i_k}^i(x_j)=1$. As we see, the number of membership functions equals the number of sample points, and such a definition shows that the more sample points, the finer the membership functions.

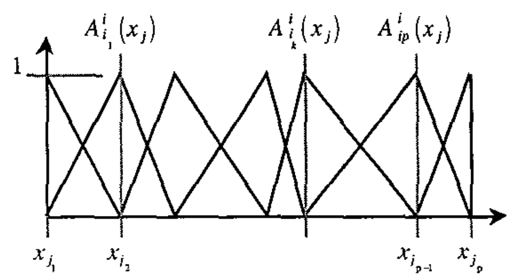

Figure 1. Orthogonal membership function definitions for the variable

When $x_{j_{k-1}} \le x_j \le x_{j_k}$,

$$A^i_{i_{k-1}}(x_j)+A^i_{i_k}(x_j)=\sum_{k=1}^{p} A^i_{i_k}(x_j)=1 \ \ and \ \ A^i_{i_k}\left(x_{j_k}\right)=1 \quad (5)$$

Clearly it is impossible to solve equation (3), because most of the $f\left(\frac{i_1}{N}, \frac{i_2}{N}, ..., \frac{i_n}{N}\right)$ are unknown. Therefore we have to look for another piecewise linear interpolator instead of equation (3). One of the possibilities is to select $m+1$ points whose convex hull contains the given input $x=(x_1, x_2, ..., x_m)$ and then construct local linear interpolators instead of equation (3) [14]. Unfortunately, for high dimensional input spaces (say more than five), it is hardly of practical use because the total of the possible selections is $m!$. Even if $m+1$ points have been selected, some of the output points of the input points are not among the sample points. That is, some of $f(x)$'s are still unknown.

2.2. Practical fuzzy interpolator

As we see in the above, the theoretical fuzzy interpolator is of limited practical use. Actually it is unnecessary to use the theoretical fuzzy interpolator. The reason is that reservoir engineering is a soft science, not only using quantitative data but also qualitative information. Sometimes in the extreme case, no quantitative data are needed, and all we need is a reservoir engineers' expert knowledge. Of course, if quantitative data are available, they can certainly strengthen predictions.

The basic idea of the practical fuzzy interpolator (PFI) is to simulate local fuzzy reasoning. When a new input vector is given, the PFI will select two past observation vectors which are the nearest to the new input vector to build a set of fuzzy rules with their observation true values. The details are as follows.

First suppose that in the input space the well logs are continuous curves, and geological facies are discrete points. The membership functions of the logs have the forms of equation (4). Geological facies are a crisp set, not fuzzy. We use the natural number $1, 2, ..., G$ to donates the types of geological facies, for example, 1 represents sand rock, 2 represents shell rock, and so on. We have found that geological facies greatly effect the scale of the reservoir permeability, that is, the difference of permeability values among different geological facies is significant. It is reasonable that each rock type uses its own data set to generate the orthogonal membership functions. Therefore, the input logs with a particular rock type only fire the sample data with the same rock type, and use the same fuzzy interpolator.

The next step is separately sorting sample points for every input variable. We get

$$x^n_{j_{min}} < x^n_{j_1} < x^n_{j_2} < ... < x^n_{j_t} < x^n_{j_{max}} \quad (6)$$

and the corresponding sample outputs are $\left(y^n_{j_1}, y^n_{j_2}, ..., y^n_{j_t}\right)$, where n is the rock index, t_n is the number of samples with rock type n, j is the j^{th} input variable index for the input vector, and $x^n_{j_{min}}$ and $x^n_{j_{max}}$ are given by reservoir engineers. Note that for $x^n_{j_{min}}$ and $x^n_{j_{max}}$ there are no corresponding outputs. In equation (6), we use '<' rather than '≤' because it is practically impossible for different sample points of the same log to have the same values. Equation (4) has to be modified based on figure 2.

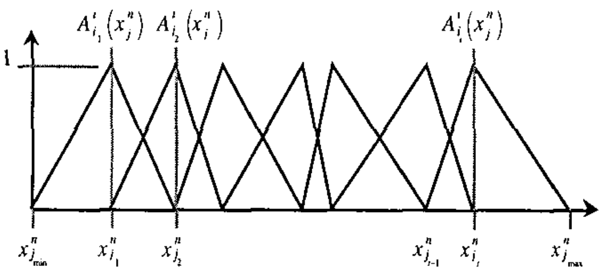

Figure 2. Universe of discourse and membership function over the input variable based on equation (5)

As we see from figure 2, it is only when x^n_j falls in $\left[x^n_{j_{min}}, x^n_{j_1}\right)$ or $\left(x^n_{j_t}, x^n_{j_{max}}\right]$, that the orthogonal property (equation (5)) is not satisfied.

Finally, for any input $x^n=(x^n_1, x^n_2, ..., x^n_m)\in X^i$, the output is:

$$y=\frac{\sum_{i=1}^{r}\sum_{j=1}^{m}\mu^i_j \cdot y^i_j}{\sum_{i=1}^{r}\sum_{j=1}^{m}\mu^i_j} \quad (7)$$

where

$$y^i_j=\begin{cases} y^n_{j_1} & x^n_{j_{min}} \le x^n_j \le x^n_{j_1} \\ A^i_{i_{k-1}}(x^n_j).y^n_{j_{k-1}}+A^i_{i_k}(x^n_j).y^n_{j_k} & x^n_{j_{k-1}} \le x^n_j \le x^n_{j_k} \\ y^n_{j_{t_n}} & x^n_{j_{t_n}} \le x^n_j \le x^n_{j_{max}} \end{cases} \quad (8)$$

where $k=1, 2, ..., t_n$ and

$$\mu^i_j=\begin{cases} \left[1-\frac{\left(x^n_{j_1}-x^n_j\right).\left(x^n_j-x^n_{j_{min}}\right)}{\left(x^n_{j_1}-x^n_{j_{min}}\right)^2}\right].\left[1-\frac{x^n_{j_1}-x^n_{j_{min}}}{x^n_{j_{max}}-x^n_{j_{min}}}\right] & x^n_{j_{min}} \le x^n_j \le x^n_{j_1} \\ \left[1-A^i_{i_{k-1}}(x^n_j).A^i_{i_k}(x^n_j)\right].\left[1-\frac{x^n_{j_k}-x^n_{j_{k-1}}}{x^n_{j_{max}}-x^n_{j_{min}}}\right] & x^n_{j_{k-1}} \le x^n_j \le x^n_{j_k} \\ \left[1-\frac{\left(x^n_{j_{max}}-x^n_j\right)\left(x^n_j-x^n_{j_{t_n}}\right)}{\left(x^n_{j_{max}}-x^n_{j_{t_n}}\right)^2}\right].\left[1-\frac{x^n_{j_{max}}-x^n_{j_{t_n}}}{x^n_{j_{max}}-x^n_{j_{min}}}\right] & x^n_{j_{t_n}} \le x^n_j \le x^n_{j_{max}} \end{cases} \quad (9)$$

We call equation (7) the practical fuzzy interpolator. Equations (8) and (9) show the properties of dynamic piece-wise-linear interpolation and extrapolation.

It should be pointed out that for PFI, dealing with multiple outputs is the same as for a single output. The only thing to do is to decompose a multi-output vector into a number of single outputs, and then separately mapping multiple-inputs to single-outputs.

3. Case Studies

In this example, we had data from two oil wells, namely well 1 and well 2. They are separated by approximately 2.5 kilometres. The data was obtained from the North West Shelf, Offshore Australia. From wells 1 and 2 there is available 173 and 225 recorded data points respectively at various depths, together with the corresponding permeability measurements.

There are eight different well logs ($m = 8$) from these wells. The rock types (geological facies) were identified by visual inspection of the samples by expert geologists. Nine rock classifications were obtained.

The objective of this example is to predict the permeability values of another well, namely well 3, using the PFI model. This well is located approximately half-way between wells 1 and 2. Based on experts' opinions, wells 1, 2, and 3 have similar geological environments. Thus, wells 1 and 2 can be seen as the neighbouring cored wells of well 3.

We will compare the prediction using the neural-fuzzy technique (NFT) [5]. Measured permeability values (157 samples) are actually available at this well for performance evaluation, but are not used in the prediction process. The ranges of each well log in the neighbouring wells are given by experts. The minimum and maximum values of these logs for these wells are shown in Table 1.

	Sample Min	Sample Max	Expert Min	Expert Max
Log 1	60.113	185.823	50	200
Log 2	3.921	18.127	3	20
Log 3	3.475	16.049	3	20
Log 4	1.102	15.454	1	16
Log 5	2.353	2.762	2.3	2.8
Log 6	0.149	0.344	0.14	0.35
Log 7	2.906	12.706	2.5	15
Log 8	68.887	87.837	65	90

Table 1. The characteristics of minimum and maximum values of three wells and given by experts.

In NFT, all the well log data (inputs) were normalised in the range of $[0,1]$. This is normally done in neural computation as the network will then give comparable magnitudes of weight values for equivalently important inputs.

All the permeability values (output) were normalised in the range of $[0.1, 0.9]$. On the other hand, PFI does not need normalisation. Table 2 shows the comparison of relative errors and some important properties for Well 3 using PFI and NFT. T donates true values, E donates estimate values, and $|.|$ donates the absolute value.

	PFI	NFT
100*|E-T| / T	24.30434	25.23625
100*|E-T| / E	19.1546	19.39391
Data preprocessing	None	Normalisation
Expert knowledge	Yes	No
CPU time	< 5 seconds	> 10 hours

Table 2. The comparison of performance when predicting Well 3 using PFI and NFT.

4. Results And Discussions

The results of prediction for permeability as well as the comparison between PFI and NFT are given in Table 2, and Figures 3 and 4.

Clearly, the PFI produces slightly better results than NFT, but PFI is much simpler than NFI. Besides normalisation for the data set, the NFT needs different neural networks for each different rock type, and selection of the training and the validation data set for each neural network.

We can see in Table 2 that the computation speed of the NFT is much slower, more than 5,000 times compared to that of PFI. In contrast, the computation of the PFI is quite straightforward and simple with no need of a number of training or iteration cycles.

It must be pointed out that the PFI does not use the look up table methods to define the fuzzy rules like some researchers [18, 13], but uses fuzzy interpolation and Euclidean distances between the sample points. The look up table methods are of limited practical use in the reservoir engineering area. Let us assume the dimension of inputs is m and there are l membership functions for each input variable, then there will be the total of l^m rules. For instance, if we assume that there are 7 membership functions for each well log in the above sample, then the rules total $n.l^m = 9.7^8 = 51{,}883{,}209$. Obviously, it is impossible to generate so many rules from the numerical samples and expert knowledge. That is the reason why in reservoir engineering Takagi-Sugeno's fuzzy approximation rules [17] of the form:

$$\text{Rule } i: \text{ if } x_1 \text{ is } A_{i_1} \text{ and } x_2 \text{ is } A_{i_2} \text{ and} \ldots \text{ and } x_m \text{ is } A_{i_m}, \text{ then } y=f(x_1,x_2,\ldots,x_m)$$

are more useful than Zadeh-Mamdani's fuzzy inference rules [11]:

$$\text{Rule } i: \text{ if } x_1 \text{ is } A_{i_1} \text{ and } x_2 \text{ is } A_{i_2} \text{ and} \ldots \text{ and } x_m \text{ is } A_{i_m}, \text{ then } y \text{ is } B_i$$

unfortunately Takagi-Sugeno's fuzzy approximation function $y=f(.)$ (whether linear or nonlinear) is not easy to find.

Currently training neural networks [16, 7, 5] is an efficient method to generalise $y=f(.)$. Unfortunately, training neural networks costs a lot in extra CPU time. In addition, the underground situation for any petroleum reservoir is very complex. It is very unlikely that there exists any consistent analytic mathematical equation between inputs (well logs) and output (permeability) in all drilled wells. Hence we require expert advise in deciding the context in which to use NFT.

However, PFI directly uses output values to interpolate or extrapolate, and also asks reservoir engineers to give ranges for input variables. This is particularly useful when it is desirable to incorporate interpretive knowledge based on a more complex understanding of the data. The results are shown in Figures 3 and 4.

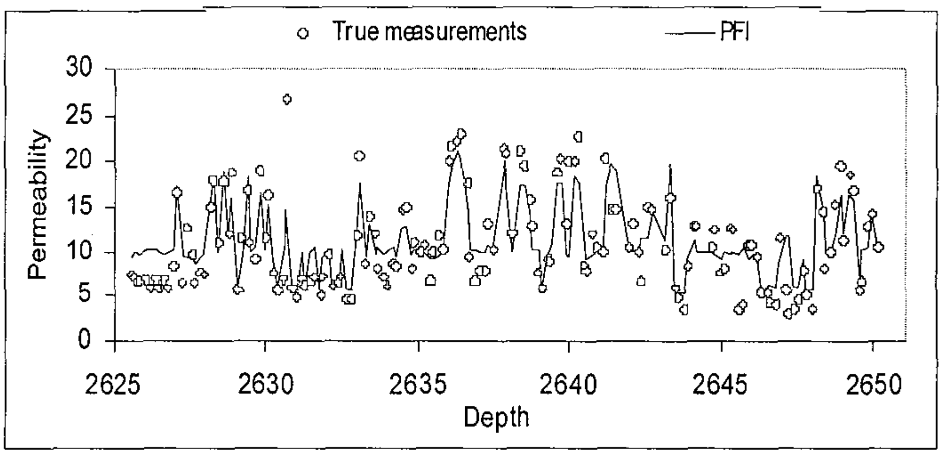

Figure 3. Permeability versus PFI predictions

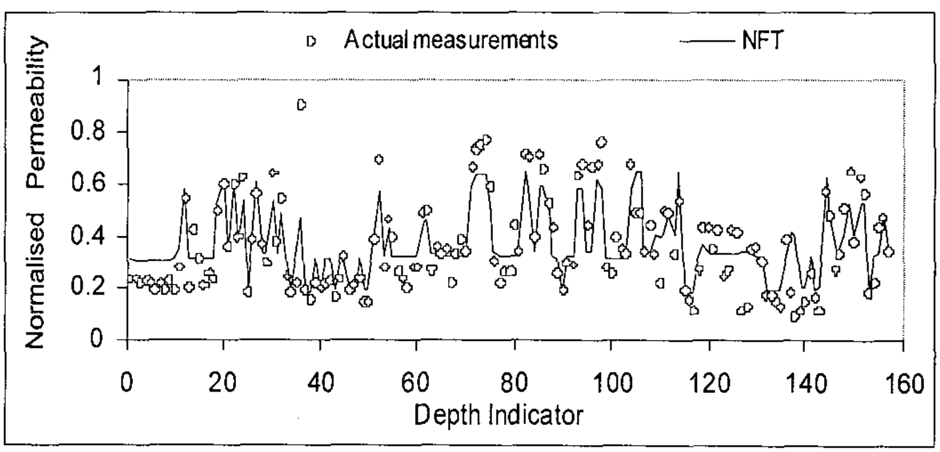

Figure 4. Normalised permeability versus NFT predictions

5. Conclusions

In this paper, we presented the practical fuzzy interpolator (PFI) in predicting reservoir permeability in petroleum engineering. PFI employs expert knowledge and numerical sample data to dynamically generate piece-wise-linear inference rules and then the estimated values are interpolated and extrapolated. The results of this study show that PFI is suitable for estimating reservoir permeability in petroleum engineering.

The PFI has many advantages. The great one is computational speed and the other is simplicity. Also it is an assumption-free, model-free, and adaptive estimator and is suitable for handling multi-dimensional inputs and outputs. The PFI can incorporate the use of expert knowledge such as determination of geologically neighbouring cored wells and the ranges of input variables based on geological information and expert judgment.

Generally speaking, the PFI does not use a traditional strictly fuzzy mathematical foundation. However, it is a practical estimation technique for petroleum engineering since it uses multiple, dynamic piece-wise-linear inference rules and can incorporate geological and expert knowledge. In addition, the PFI does not require a structured knowledge base. It has lots of freedom in choosing the neighbouring cored wells and fuzzy inference rule equations ((8) and (9)). Although in this paper PFI was applied to permeability estimates, the principles illustrated are generic. Our further work will concentrate on improving equations (8) and (9).

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