A general method for fuzzy rule interpolation: specialised for crisp triangular and trapezoidal rules

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Abstract

We have devised a general method for fuzzy rule interpolation in spare rule bases. In contrast to existing methods which require rules to be convex and normal, our method works for all cases.

We show in this paper a specialisation of our method to the crisp, normal triangular and trapezoidal rules used in practice. The result shown in this paper demonstrates that our method performs better than previously published methods.

1.Introduction

The main exercise of fuzzy controllers is to generate a conclusion fuzzy set for an input fuzzy set or observation. The Mamdani controller generates a B' conclusion for an A' observation knowing A_1 -A_n defined on powerset A, and B_1 -B_m defined on powerset B [1,2,3]. It is necessary for generating a conclusion, that every element of the input universe X belongs to at least two of the A_1 -A_n sets with a positive degree except if x_0 has degree 1 in some A_i. This means more restriction, that applications need high density of sets A_i. This needs knowledge and operation of A_1 -A_n and B_1 -B_m sets which consist of numerous rules. There are several methods in the literature to eliminate these problems [4,5,6]. These methods give conclusions even if the intersection of A' with A_1 -A_n is the nullset. Let A be the powerset of observations and B the powerset of conclusions. Let $A_1,A_2 \in A$ be such observations, that the conclusions $B_1,B_2 \in B$ are known. Let A' be the new observation, where the conclusion has to be generated by knowledge of set A_1,A_2,B_1,B_2.

Previous linear and other methods define such a transitional observation-sequence between A_1 and A_2, for which a conclusion-sequence can be given between B_1 and B_2. Therefore as A' can be found in this observation-sequence, B' can be found in the conclusion-sequence between B_1 and B_2 in the same way. There are observations that can be found only partially in this observation-sequence. In this case the methods do not give any conclusion. If these methods are used, than the given conclusion would be meaningless. Therefore the problem with these methods is, that they are not useful or can not give understandable conclusions in all cases. In this paper a new method will be introduced, which solves these problems and is useful in every case. Every interpolation method defines a different observation-sequence. Because of this some methods can, and others can not give a conclusion to the same observation. That is why a meaningless conclusion does not mean that a conclusion does not exist but that the method being used can not provide it. With people, when asking someone a question, a clear answer is expected even if it is incorrect in some respect. The accurancy of the answer depends on the knowledge of the respondent.

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The method introduced in this paper looks for conclusions in a manner similar to our way of thinking, and different from previous methods. When a new question is asked, then one would arrange such knowledge from their factual knowledge, which can come closest to the new question (A”,B”). Then, having this knowledge the conclusion B’ can be deduced. Avoiding the problems of previous methods, a conclusion can be given to any observation. A simplier version of the general method is introduced in this paper which is limited to the practically used crisp, triangular and trapezoidal fuzzy sets.

2.Definitions

1) $F(x_1,p_1,p_2,c_1,c_2)$. Let be $x_2=F(x_1,p_1,p_2,c_1,c_2)$. The function gives x_2 the value for which a/b=c/d is true in a) and b) cases of fig.1.

a) $x_1 \le p_1$ b) $p_1 \le x_1$

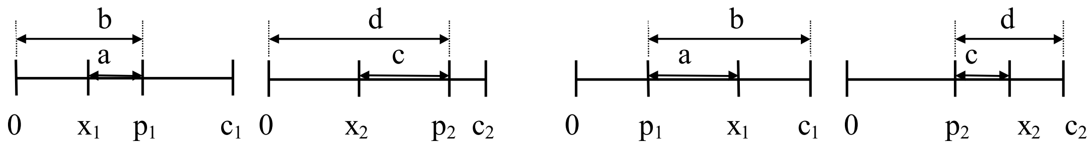


Fig.1

$$x_2=F(x_1,p_1,p_2,c_1,c_2)=\begin{cases} p_2 & \text{if } x_1=p_1 \\ p_2-a\dfrac{b\cdot c_2-p_2}{b\cdot c_1-p_1} & \end{cases} \quad \text{where: } \begin{aligned} a&=p_1-x_1 \\ b&=\frac{1}{2}\left(1-\operatorname{sgn}(a)\right) \end{aligned} \qquad (1)$$

2) Central points of a fuzzy set / CP(A) /. We have the A ($\langle \forall x \in X,\ \mu_A(x)\rangle$) fuzzy set, given an X universe. Let: $CP(A)=\dfrac{\sup(A_\alpha)+\inf(A_\alpha)}{2}$; where $\alpha=\text{height}(A)$; (2)

3) Support normalization for the values SL,SU: $\text{suppnorm}_{SU}^{SL}(A)$. A is defined in 2) above.

Let: $supp_L(A)$ = inf(supp(A)) and $supp_U$ = sup(supp(A)). The normalized support of fuzzy set A (SNA fuzzy set) for given values of SL=$supp_L$(SNA) and SU=$supp_U$(SNA): $SL \ne CP(A); SU \ne CP(A)$;

$$\mu_{SNA}(x)=\mu_A\left[(x-CP(A))\cdot\left(\frac{a-CP(A)}{b-CP(A)}\right)+CP(A)\right] \quad \begin{aligned} &\text{if } x<CP(A) \text{ then } a=supp_L; b=SL; \\ &\text{if } x>CP(A) \text{ then } a=supp_U; b=SU; \end{aligned} \qquad (3)$$

3.General Method

In the first step let us define A” in such a way, that: 1) A” should be as close as possible to A’. 2) The closer A” is to A_1 the more similar to it. 3) The closer A” is to A_2 the more similar to it. B” can be defined under the same conditions. Let the values of the A,B powersets be normalized to the range [0,100], for convenience of discussion in this paper let be ma = max(supp(A)) = mb = max(supp(B)) = 100.

I) Turning sets A_1,A_2 and A’ around their central point as shown in figure 2 a solid will be yielded. The surface of this solid is a line surface seating on the membership functions A_1 , A_2 as generatrix. In this way an univocal surface can be determinated. We then intersect this solid at the centerpoint of A’ and turn back the received intersection. This will be the fuzzy set A”, which suits the conditions given before. A solid can be created from B_1,B_2 in the same way. Intersecting it at the same relative point, set B” can be received as shown in figure 4.

II) In the second step, B’ can be given knowing A’,A” and B” (fig.3.). The center point of B’ in demand is: CP(B’)=CP(B”). The course of the second step: a) determination of the support of B’. b) Normalization of A’, A”, B” sets to the same support (SNA’,SNA”,SNB”). c) Determination of SNB' by the F function. d) Normalization of SNB' fuzzy set for the determined support B'. The details are as follows:

a) $$\text{supp}_L(B') = F\left(\text{supp}_L(A'), \text{supp}_L(A''), \text{supp}_L(B''), CP(A''), CP(B'')\right); \qquad (4)$$

$$\text{supp}_U(B') = CP(B') + F\left(\text{supp}_U(A') - CP(A'), \text{supp}_U(A'') - CP(A''), \text{supp}_U(B'') - CP(B''),\right.$$
$$\left. ma - CP(A''), mb - CP(B'')\right); \qquad (5)$$

b) Normalization of A',A",B" fuzzy sets to equivalent supports.

$$SL=CP(A')-0.5;\ SU=CP(A')+0.5;\ SNA'=\text{suppnorm}_{SU}^{SL}(A');$$

(6)

$$SL=CP(A'')-0.5;\ SU=CP(A'')+0.5;\ SNA''=\text{suppnorm}_{SU}^{SL}(A''); \qquad (7)$$

$$SL=CP(B'')-0.5;\ SU=CP(B'')+0.5;\ SNB''=\text{suppnorm}_{SU}^{SL}(B''); \qquad (8)$$

Of course any optional value instead of 0.5 can be used, depending on the normalization applied.

c) Determination of set SNB' by the function F: $y \in [0, 1]$;

$$x_1 = \mu_{SNA'}\left(\text{supp}_L(SNA') + y\right);\ p_1 = \mu_{SNA''}\left(\text{supp}_L(SNA'') + y\right);\ p_2 = \mu_{SNB''}\left(\text{supp}_L(SNB'') + y\right);$$

$$\mu_{SNB'}\left(\text{supp}_L(SNB') + y\right) = F(x_1, p_1, p_2, c_1, c_2); \qquad \text{where } c_1=1;\ c_2=1; \qquad (9)$$

d) Normalization of given fuzzy set SNB' to the determined support:

$$SL=\text{supp}_L(B');\ SU=\text{supp}_U(B'); \qquad (10)$$

The conclusion fuzzy set is: $B' = \text{suppnorm}_{SU}^{SL}(SNB')$; (11)

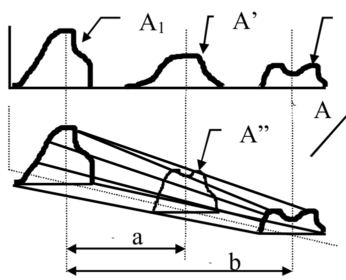


fig.2.

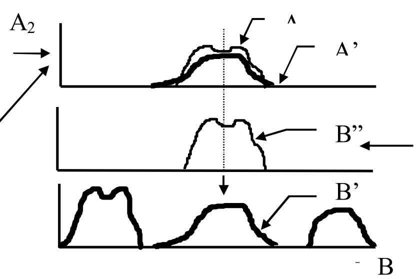


fig.3.

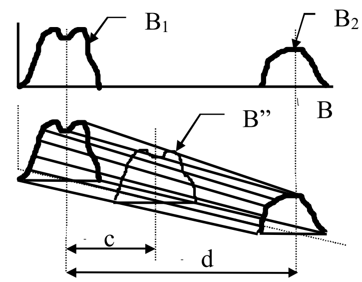
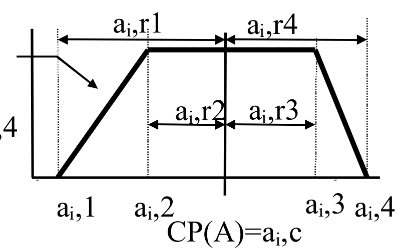


fig.4. (a/b=c/d)

4.Specialised method

This method is a simplified version of the general method limited to trapezoidal, triangular and crisp sets used in practice. These sets can be characterized by four key points. Naturally it is not suitable to apply the equations of the general method immediately, since the method can be simplified in this case.



1) Determination of A" and B" by cutting the solid.

$$a'',rj = a_1,rj + (a_2,rj - a_1,rj)C \qquad b'',rj = b_1,rj + (b_2,rj - b_1,rj)C;\quad C = \frac{a',c - a_1,c}{a_2,c - a_1,c};\ j=1,2,3,4$$

b',c = b",c = b_1,c + (b_2,c - b_1,c)C;

2) Determination of B' knowing A',A" and B".

a) Determination of core(B') / points b',2; b',3 /:

Calculation of b',2 and b',3 are similar. In the case of b',2 it is necessary to compare to the minimum element, in the case of b',3 to the maximum element of the powerset.

b',2=F(a',2; a",2; b",2; a',c; b',c); b',3=b',c+F(a',r3; a",r3; b",r3; ma-a',c; mb-b',c); (12)

b) Determination of membership function of set B' between b',1 and b',2 : $\alpha \in [0,1]$

$$\inf(B'_\alpha) = \inf(\mathrm{core}(B')) \cdot$$
$$F\Big(\big[\inf(A'_\alpha)/\inf(\mathrm{core}(A'))\big];\big[\inf(A''_\alpha)/\inf(\mathrm{core}(A''))\big];\big[\inf(B''_\alpha)/\inf(\mathrm{core}(B''))\big];1;1\Big) \quad (13)$$

Set B’ will be linear in this interval. In the same way linearity will be true between b’,3 and b’,4 also. We omit the proof of this part due to lack of space. Thus it is enough to count the ponts b’,1 and b’,4 for α=0.

Determination of supp(B') / points b’,1;b’,4 /:

b’,1=b’,2 · F([a’,1/a’,2]; [a”,1/a”,2]; [b”,1/b”,2]; 1; 1); (14)

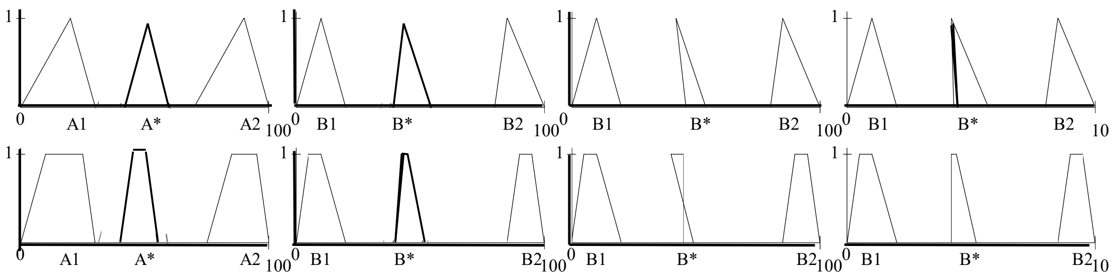
Equation 14 is meaningless in an extreme case, when inf(core(A’)) or inf(core(B’))=0. In this time the problem can be eliminated by shifting the universe.

Determination of b’,4 is done similary: where ap>ma; bp>mb;

b’,4=bp-(bp-b’,3) · F([(ap-a',4)/(ap-a',3)]; [(ap-a",4)/(ap-a",3)]; [(bp-b",4)/(bp-b",3)]; 1; 1); (15)

5.Results

observation conclusion



i) ii) iii)

In the above two diagrams, the rule antecedents and observations are on the left, and the rule consequents and the conclusions as found by i) our method described in this paper, by ii) linear interpolation [4,6], and iii) by Vass et al [5]. Note that in the first case, both of the latter methods produce meaningless conclusions, while in the second case, only one of these methods has this problem. Our method described in this paper guarantees convex normal fuzzy set conclusions.

6.Conclusion

The existing interpolation methods can be used only for normal and convex fuzzy sets. The general method shown in this paper is usable for any type of fuzzy set. Of course the value of the membership function has to be calculated for every element in this case. Based on the theory of the general method, a special method was introduced for application which needs low computational capacity. This method can be used for trapezoidal, triangular and crisp sets. There is a great problem with existing interpolation methods: they can give meaningless conclusions in many cases. Our special method gives reasonable conclusions in every case. The other problem with existing interpolation methods is that the conclusion is not convex in every case. So it is not enough to use only four points of each set. This problem does not occur with the special method shown in this paper, because the conclusion will be trapezoidal, triangular or crisp and thus the calculation using only four points of sets is enough. We are continuing work on the multi-dimensional version of our method.

7.References

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