

# A General Interpolation Technique in Fuzzy Rule Bases with Arbitrary Membership Functions

P. Baranyi\*, T.D. Gedeon\*\* and L.T. Kóczy\*\*\*

\*Department of Electrical Engineering

\*\*\*Department of Telecommunications and Telematics

Technical University of Budapest

Budapest \*Budafoki u. 8, \*\*\* Sztoczek u.2, H-1111 HUNGARY

\*\*School of Computer Science and Engineering

University of New South Wales Sydney 2052 Australia

baranyi@elektro.get.bme.hu, tom@cse.unsw.edu.au, koczy@ttt-202.ttt.bme.hu

## ABSTRACT

In this article we will present a new general multidimensional fuzzy rule interpolation method. This method, compared to the existing interpolation methods, can be applied for arbitrary type of fuzzy sets, does not require convex and normal sets in the rules. Other important difference: the new method gives an interpretable conclusion in every case, unlike the previously published methods. As a matter of course, to apply arbitrary type of sets, the general method makes calculation necessary for "every point" of the sets. A special method, based on the theory of the general method, will be introduced for application in practice, which needs low computational capacity. The specialised method uses three of the most wide spread set types in practice: the crisp, the triangular, and the trapezoidal fuzzy sets. The difference between the new and the former methods will be pointed out by examples and the results of different formal methods.

## 1. INTRODUCTION

The advantages of fuzzy controllers in practice, and the growing number of applications are well known. The basic job of fuzzy controllers is to transform the input signal into membership degrees (observation fuzzy set), then to generate conclusion fuzzy set and, depending on needs, to defuzzify the conclusion [1]. Naturally a conclusion to an observation can be only given on the base of the knowledge got from earlier information, so a conclusion set can only be generated starting from the fuzzy sets obtained from a knowledge of the system to be controlled, and the complete rules for mapping of these sets. A conclusion fuzzy set can be generated using different theories to apply known knowledge base.

The traditional method like Mamdani-algorithm generates a  $B'$  conclusion to an  $A'_j$  observation knowing  $A_{i,1}-A_{i,m}$  defined on universe  $X_i$ , and  $B_1 -B_k$  defined on universe  $Y$  (fig.1)[1,2,3]. It is necessary for generating a conclusion, that every element of the input universe  $X_i$  belongs to at least two of the sets  $A_{i,1} -A_{i,m}$  with a positive degree except if  $x_{i,0}$  has degree 1 in some  $A_{i,j}$ . As a matter of course, in this way we need the complete knowledge that contains the possible observations. So the support of the union of  $A_{i,1} -A_{i,m}$  and the

input universe itself must be equal. In the practice, even stricter conditions apply:  $A_{i,j}$  should be located densely. The result of this "rule of thumb" is that even in case of a small system, the number of rules is increasing considerably. Having a large number of rules brings a lot of problems both in respect to calculation time and to storage space [4].

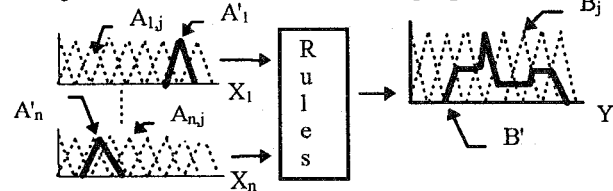


Figure 1: Traditional method

Original aim in the interpolation methods eliminate this problem, namely, reduce the number of rules [4,5,6]. To caught this purpose the interpolation methods use sparse rule base contrast with traditional method like Mamdani-algorithm (fig.2). The importance and the difference from traditional method of the fuzzy rule interpolation is founded upon the ability of the method to deduce form, not only to interconnect the existing knowledge. That is, it is not necessary to have in the knowledge (up to this point) a fuzzy set whose intersection with the observation is not empty. Therefore these methods can be fruitfully used in applications where the available amount of knowledge is limited. Other important result: conclusion by interpolation (fig.1,2) can be observation easily for next controller while the conclusion of traditional algorithm is more difficult to apply as observation.

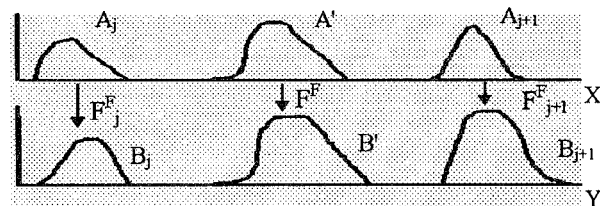


Figure 2: Interpolation method

So, knowing antecedents  $A_{1,m}$  on  $X$ , and consequent  $B_{1,k}$  on  $Y$ , relations  $B_j = F_j^F(A_j)$  are already known as fuzzy sets (here  $F^F$  denotes a relation. It is not a mathematical function,

but a mapping from antecedent fuzzy sets to consequent fuzzy sets). In case of observation  $A'$  conclusion  $B'$  can be deduced by known relations between sets  $A_j$  and  $A_{j+1}$  surrounding  $A'$  and their known consequents  $B_j$  and  $B_{j+1}$ . Therefore relation  $B' = F^F(A')$  can be given by some weighted combination of relations  $F_j^F$  and  $F_{j+1}^F$ . So these methods give conclusion  $B' = B_j$  for different fuzzy sets  $A' = A_j$  and generate conclusions for arbitrary observations between  $A_j$  and  $A_{j+1}$ .

Hence the basic difference between the main idea of the traditional and interpolation methods:

-Traditional methods generate the conclusion basically by a weighted considering of the consequent sets chosen by the system of rules. The weighting can be given as the intersection of the  $A_{ij}$  and the observation.

-Interpolation methods generate the conclusion by means of weighted considering of  $F^F$  relations between antecedent and consequent sets, not by some kind of weighting of consequent fuzzy sets. Making possible in this manner the deduction of the conclusion on the base of limited information.

Multidimensional extension of some interpolation methods exist. In this article we show a multidimensional version of Baranyi Gedeon Kóczy's interpolation method [6]. Since we use only the terms flanking the observation let us use the following denotations:  $X_i$  is the input universe ( $i = 1..n$ ,  $n$  = the number of input universe),  $A_{i,1}$  and  $A_{i,2}$  are the antecedents defined on  $X_i$ ,  $Y$  is the output universe  $B_1$  and  $B_2$  are the consequents defined on  $Y$ ,  $A'_i$  is the observed input fuzzy set on  $X_i$  and  $B'$  is the conclusion which is generated knowing  $A'_{1..n}, A_{1..n,1}, A_{1..n,2}, B_1$  and  $B_2$ .

The former interpolation methods induce several problems [7]:

- They can be applied only for convex and normal sets.
- They are not even interpretable for arbitrary convex and normal observation fuzzy sets, namely, ordering must hold ( $A_{ij} \prec A'_i \prec A_{i,j+1}, B_j \prec B_{j+1}$  where the observation set is  $A'_i$ )
- The method does not give a directly interpretable conclusion fuzzy set in every case ("loops" in the membership functions) (fig.3).

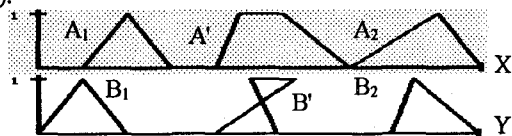


Figure 3: Conclusion  $B'$  is not fuzzy set.

- Using trapezoidal, triangular or crisp sets, the shape is not preserved for the conclusion. It means that in case of fuzzy triangular or trapezoidal terms calculation by the three or four characteristic points, e.g. by linear interpolation, is not always sufficient as it gives only a rough approximation (except if some rather strict conditions apply) (fig.4). This is an important problem because of the computational complexity aspect.

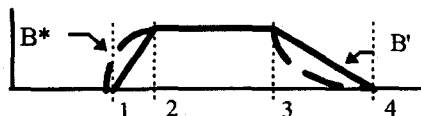


Figure 4. Conclusion  $B'$  was calculated by characteristic points.  $B^*$  was calculated point by point of sets.

In this article we will present a new method that can be applied on arbitrary shaped fuzzy terms and that always results in directly "acceptable" sets, further on eliminate all the mentioned problems. To show the essential new points in this method, we will classify the former interpolation methods by their key idea:

First class contains single term deduction methods. The conclusion ( $B'$ ) is generated from observation  $A'$  and  $A_1$  a single rule,  $B_1$  using some kind of General Modus Ponens (GMP see e.g. the Revision Principle [8]). These methods can not give a conclusion if the intersection of  $A'$  and  $A_1$  is empty. The problem is that in this case the distance between  $A'$  and  $A_1$  is not meaningful.

Methods in the second class, use at least two rules. The Mamdani-method and other reasoning methods alike use the degree of matching between observation and at least two antecedents by calculating a weighted average (see e.g.) [2]. This is a natural way of interpolation.

The third class applies approximation for the  $\alpha$ -cuts and this can be used even if there is no formal matching. The linear interpolation method is the prototype of the methods in this class. These methods generate conclusion  $B'$  to observation  $A'$  using at least two rules ( $A_1 \rightarrow B_1, A_2 \rightarrow B_2$ ) [4,5]. The basic idea of these methods is the following: If given are sequence of observations  $A_1, A', A_2$  and a corresponding of sequence of conclusions  $B_1, B', B_2$  where  $B'$  is unknown,  $B'$  is found by considering  $A'$ 's relative location in  $X$  and determining  $B'$  from  $B_1$  and  $B_2$  accordingly. If  $A'$  is not comparable with  $A_1$  ( $A_1 \prec A' \prec A_2$ ) then this method can not be applied. However generalized interpolation and approximation methods (where polynomial or rational functions are used on the characteristic points of the  $\alpha$ -cuts, eliminate the difficulty of orderedness condition [9]).

The method introduced in this article, goes back to the basic interpolation of fuzzy rules and searches for a conclusion similarly to the way of human thinking. When we hear a new question, at first we summarize our knowledge that is closest to the topic of the question, and we try to find questions that approach the new question as close as possible and whose answers are known. Then based on the comparison of our factual knowledge and the new question we deduce an approximate answer. Following this method, at first, we look for a fitting  $A'_i$  "between" known  $A_{i,1}$  and  $A_{i,2}$ , which is the closest to  $A'_i$  and we determinate the corresponding conclusion  $B''$  "between"  $B_1$  and  $B_2$ . Actually the first step gives the multidimensional fuzzy set  $A''$  nearest to a multidimensional fuzzy set  $A'$ . Because the output base set is one-dimensional we have to transform the multidimensional fuzzy sets  $A'$  and  $A''$  into the one-dimensional sets  $S'$  and  $S''$  with the same properties. Then, the conclusion  $B'$  to  $A'_i$  will be found by evaluating  $S', S''$  and  $B''$  according to the Baranyi Gedeon Kóczy's one-dimensional interpolation method [6].

Of course, if we use general type fuzzy terms, the operations with these sets will need much computational effort, because enough points of the sets should be taken into consideration to have a good enough approximation. Therefore, for practical applications we prepare a special method that can be applied

for crisp, triangular and trapezoidal fuzzy sets (that can be described by 2,3 and 4 characteristic points respectively). In this article we will present the way in which the new method can be used as an extrapolation.

## 2. DEFINITIONS

**Revision function:**  $x_2 = F(x_1, p_1, p_2, c_1, c_2)$

Let  $x_2 = F(x_1, p_1, p_2, c_1, c_2)$  be that function, which results  $x_2$  in such a way that  $a/b = c/d$  is true both if  $x_1 \leq p_1$  and  $p_1 < x_1$  (fig. 5).

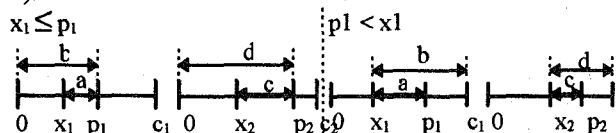


Figure 5

$$x_2 = F(x_1, p_1, p_2, c_1, c_2) = \begin{cases} p_2; & \text{if } x_1 = p_1 \\ p_2 - a \frac{b \cdot c_2 - p_2}{b \cdot c_1 - p_1}; & \text{otherwise} \end{cases} \quad (1)$$

where:

$$a = p_1 - x_1;$$

$$b = \frac{1}{2} (1 - \text{sgn}(a));$$

**Central point of fuzzy set A: cp(A)**

A ( $\langle \forall x \in X, \mu_A(x) \rangle$ ) fuzzy set is given on base set X. The center of the fuzzy set is:

$$\text{cp}(A) = \frac{\sup(A_\alpha) + \inf(A_\alpha)}{2}; \text{ where } \alpha = \text{height}(A); \quad (2)$$

(This is a generalization of the concept of the centre of the core.)

**Normalisation of the support of fuzzy set A for given SU and SL:  $\text{suppnorm}_{SU}^{SL}(A)$**

Let A ( $\langle \forall x \in X, \mu_A(x) \rangle$ ) fuzzy set be given on base set X, then let  $\text{suppnorm}_{SU}^{SL}(A)$  be such a fuzzy set SNA, whose support's minimum value is SL and maximum value is SU. Let the membership function of  $SNA = \text{suppnorm}_{SU}^{SL}(A)$  be:

$$\mu_{SNA}(x) = \mu_A \left( \left( x - \text{cp}(A) \right) \cdot \left( \frac{a - \text{cp}(A)}{b - \text{cp}(A)} \right) + \text{cp}(A) \right) \quad (3)$$

Where:  $SU < \text{cp}(A) < SL$ ;

If  $x < \text{cp}(A)$  then:  $a = \text{supp}_L(A) = \inf(\text{supp}(A))$ ;  $b = SL$ ;

If  $x > \text{cp}(A)$  then:  $a = \text{supp}_U(A) = \sup(\text{supp}(A))$ ;  $b = SU$ ;

**Symmetrical normalisation of the support of fuzzy set A for given d:  $\text{suppnorm}^d(A)$**

Let A ( $\langle \forall x \in X, \mu_A(x) \rangle$ ) fuzzy set be given on base set X, then let be: where:  $SL = \text{cp}(A) - d/2$ ;  $SU = \text{cp}(A) + d/2$ ;

$$\text{suppnorm}^d(A) = \text{suppnorm}_{SU}^{SL}(A), \quad (4)$$

## 3. GENERAL METHOD

According to the introduction, the determination of the conclusion fuzzy set can be divided into three main steps. First, the fuzzy set  $A''_i$  is to be determined on every input universe  $X_i$ . Second, the sets  $A'_i$  and  $A''_i$  are to be transformed into the sets  $S'$  and  $S''$  on the universe S. Third, conclusion B' can be determined using  $S'$ ,  $S''$  and  $B''$ .

For determining  $A''_i$  and  $B''$  there are more conditions to be satisfied:  $A''_i$  should be as close to  $A'_i$  as possible. The closer is  $A''_i$  to  $A_{ij}$  the more similar they are. The same "similarity relation" is required between  $B''$  and  $B_j$ . In an extreme case, when observation  $A'_i$  is identical with  $A_{ij}$  (an already known antecedent) then  $A''_i$  the closest information to  $A'_i$  should be also identical with  $A_{ij}$ . Similarly,  $B''$  should be identical with  $B_j$ , which implies that conclusions B' and  $B''$  are also the same. Therefore in the extreme case, when the observation is  $A_{ij}$ , the conclusion should be  $B_j$ .

Let us define the crisp distance between two fuzzy sets with the distance of their centres [4]:  $d(A_1, A_2) = d(\text{cp}(A_1), \text{cp}(A_2))$ . To avoid the problem of abnormal membership function shapes for B' no other distance will be calculated, and all points of the membership function will be generated by this distance as a reference. For simplicity, let us consider  $X_i$  and Y be normalized for the interval [0,100] i.e.

$$Mx_i = \max(\text{supp}(X_i)) = My = \max(\text{supp}(Y)) = Ms = \max(\text{supp}(S)) = 100,$$

$$mx_i = \min(\text{supp}(X_i)) = my = \min(\text{supp}(Y)) = ms = \min(\text{supp}(S)) = 0.$$

**First step**

Let us turn fuzzy sets  $A_{i,1}$  and  $A_{i,2}$  around their centers as it is shown in figure 6. The rotated curves ( $A_{i,1}$  and  $A_{i,2}$ ) are considered as the cross-sections of a geometric solid. Fuzzy set  $A''_i$  can be found between  $A_{i,1}$  and  $A_{i,2}$  as the cross section of this imaginary geometric solid. To get  $A''_i$ , the closest fuzzy set to  $A'_i$ , we have to cut the solid at the position of  $A'_i$ , using the above introduced distance measure.

Turning back the cross-section into its original position we will obtain  $A''_i$ , a set that satisfies our conditions, namely that is equal to  $A_{i,1}$  or  $A_{i,2}$  in an extreme case.

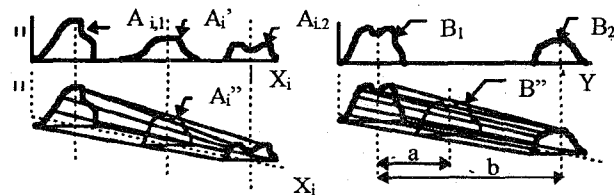


Figure 6: Determination of  $A''_i$  and  $B''$  by geometric solid cutting.

There are more possibilities to define the solid based on the rotated fuzzy sets. Therefore, it is possible to handle more than two sets  $A_j$  as one solid. In a simple case, when we consider only two fuzzy sets, the geometric solid based on  $A_{i,1}$  and  $A_{i,2}$  can be easily constructed as a line surface seating on the membership functions  $A_{i,1}$ ,  $A_{i,2}$  as generatrix.

## Second step

Determination of sets  $S'$  and  $S''$  using  $A'_{1..n}$  and  $A''_{1..n}$  (fig.7). This transformation can be divided into four parts:

1. Determination of centers and supports of fuzzy sets  $S'$  and  $S''$ .

$$cp(S') = \frac{Ms}{n} \sum_{i=1}^n \frac{cp(A'_i)}{Mx_i}; \quad cp(S'') = \frac{Ms}{n} \sum_{i=1}^n \frac{cp(A''_i)}{Mx_i}; \quad (5)$$

$$supp_L(S') = \frac{Ms}{n} \sum_{i=1}^n \frac{supp_L(A'_i)}{Mx_i}; \quad (6)$$

$$supp_U(S') = \frac{Ms}{n} \sum_{i=1}^n \frac{supp_U(A'_i)}{Mx_i}; \quad (7)$$

$$supp_L(S'') = \frac{Ms}{n} \sum_{i=1}^n \frac{supp_L(A''_i)}{Mx_i}; \quad (8)$$

$$supp_U(S'') = \frac{Ms}{n} \sum_{i=1}^n \frac{supp_U(A''_i)}{Mx_i}; \quad (9)$$

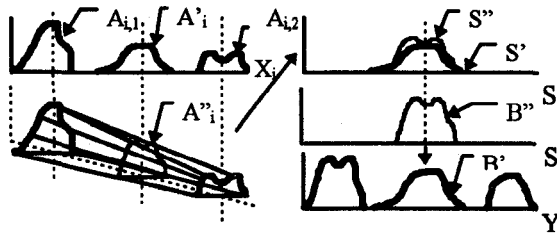


Figure 7: Main steps of new method.

2. Normalization of fuzzy sets  $A'_i$  and  $A''_i$  to the same supports ( $SNA'_i$  and  $SNA''_i$ ), while their centers does not change. The size of the common support is arbitrary, in this case it is  $d$ .

$$SNA'_i = \text{suppnorm}^d(A'_i); \quad SNA''_i = \text{suppnorm}^d(A''_i); \quad (10)$$

where:  $i = 1..n$ ;

3. Determination of fuzzy sets  $SNS'$  and  $SNS''$  point by point, using sets  $SNA'$  and  $SNA''$ .  $y \in [0, d]$ ; Sets  $A'_i$  and  $A''_i$  could be weighted, depending on needs, by  $w'_i$  and  $w''_i$  ( $w'_i, w''_i \in [0,1]$ ).

$$\mu_{SNS'}(cp(S') - d/2 + y) = \frac{1}{n} \sum_{i=1}^n w'_i \mu_{SNA'}(cp(A'_i) - d/2 + y); \quad (11)$$

$$\mu_{SNS''}(cp(S'') - d/2 + y) = \frac{1}{n} \sum_{i=1}^n w''_i \mu_{SNA''}(cp(A''_i) - d/2 + y); \quad (12)$$

4. Determination of fuzzy sets  $S'$  and  $S''$ .

$$SL = \text{supp}_L(S'); \quad SU = \text{supp}_U(S'); \quad S' = \text{suppnorm}_{SU}^{SL}(SNS'); \quad (13)$$

$$SL = \text{supp}_L(S''); \quad SU = \text{supp}_U(S''); \quad S'' = \text{suppnorm}_{SU}^{SL}(SNS''); \quad (14)$$

The determination of  $S'$  and  $S''$  by suppnorm using  $SNS'$  and  $SNS''$  is not necessary step in the method. It could be over stepped.

Determination of  $B''$  is similar to determination of  $A''_i$  fuzzy sets. Satisfying the conditions, the geometrical solid, which is created by turning  $B_1$  and  $B_2$  (fig. 6), should be cut in such way that  $a/b$  is equal to:

$$a/b = \frac{\sum_{i=1}^n \frac{cp(A'_{i,1}) - cp(A_{i,1})}{Mx_i}}{\sum_{i=1}^n \frac{cp(A_{i,2}) - cp(A_{i,1})}{Mx_i}}; \quad (15)$$

## Third step

The determination of  $B'$  based on  $S'$ ,  $S''$  and  $B''$  and the determination of  $B'$  based on  $A'$ ,  $A''$  and  $B''$  in one-dimensional case [6] are same. Difference between them that  $S'$  and  $S''$  will be used instead of  $A'$  and  $A''$ .

Determination of  $SNB''$ .

$$SNB'' = \text{suppnorm}^d(B''); \quad (16)$$

Determination of  $SNB'$  point by point using sets  $SNS'$ ,  $SNS''$ ,  $SNB''$  and the revision function  $F$ .

$$x_1 = \mu_{SNS'}(\text{supp}_L(SNS') + y); \quad (17)$$

$$p_1 = \mu_{SNS''}(\text{supp}_L(SNS'') + y); \quad (18)$$

$$p_2 = \mu_{SNB''}(\text{supp}_L(SNB'') + y); \quad (19)$$

$$c_1 = 1; \quad c_2 = 1; \quad y \in [0, d];$$

$$\mu_{SNB'}(\text{supp}_L(SNB'') + y) = F(x_1, p_1, p_2, c_1, c_2); \quad (20)$$

Determination of conclusion  $B'$  by suppnorm using set  $SNB''$ .

$$\text{supp}_L(B') = F(\text{supp}_L(S'), \text{supp}_L(S''), \text{supp}_L(SNB''), cp(S'), cp(SNB'')); \quad (21)$$

$$\text{supp}_U(B') = cp(SNB'') + F(\text{supp}_U(S') - cp(S'), \text{supp}_U(S'') - cp(S''), \text{supp}_U(SNB'') - cp(SNB''), Ms - cp(S'), My - cp(SNB'')); \quad (22)$$

$$SL = \text{supp}_L(B'); \quad SU = \text{supp}_U(B'); \quad (23)$$

$$\text{The conclusion is: } B' = \text{suppnorm}_{SU}^{SL}(SNB''); \quad (24)$$

The conclusion is fuzzy set in every case, since the normalization of the support must always result into a fuzzy set. Using this method we can get a conclusion for any type of fuzzy sets.

## 4.SPECIALISED METHOD

This method is a simplified version of the general method, which does not calculate all the points of the membership function, but only the four characteristic points. So this method is applicable for crisp, triangular and trapezoidal fuzzy sets, while it requires only a small amount of computational time. According to the expressed in the introduction, we can consider the observation and the conclusion formed by the multidimensional  $A'$  set and the one-dimensional  $B'$  fuzzy set respectively, while the knowledge is composed of fuzzy sets  $A_1$ ,  $A_2$  (multidimensional) and  $B_1$  and  $B_2$  (one-dimensional). So, as mentioned earlier, first we have to determine the fuzzy sets  $A''$  and  $B''$  (fig. 8). On the base of the sets  $A_{i,1}$  and  $A_{i,2}$  determined on every input base set the cross sections  $A'_i$

must be found. The sets  $S'$  and  $S''$  the orthogonal shadows of  $A'$  and  $A''$  solids on the surface  $s$  (fig.8), carry all particularities of the  $A'$  and  $A''$  fuzzy sets. For simplicity let us denote:  $A_{1,3}=A'_{i,3}$ ,  $A_{1,4}=A''_{i,4}$ ,  $B_3=B'$ ,  $B_4=B''$ ,  $S_3=S'$ ,  $S_4=S''$  and  $a_{i,j,k}$  the  $k$ -th characteristic point of  $A_{i,j}$  in  $\alpha$ , and  $d_{A_{i,j,k}}$  is the distance from  $cp(A_{i,j})$ . The method can be divided into three parts.

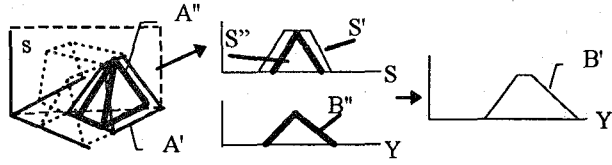


Figure 8: Key-steps in specialised method.

1. Determination of fuzzy sets  $A''_i$  and  $S'$ ,  $S''$ :  
where:  $i = 1..n$ ;  $k = 1..4$ ;  $j = 3,4$ ;

$$d_{A_{i,4,k}} = d_{A_{i,1,k}} + (d_{A_{i,2,k}} - d_{A_{i,1,k}}) \cdot C_i; \quad (25)$$

$$\text{where: } C_i = \frac{cp(A_{i,3}) - cp(A_{i,1})}{cp(A_{i,2}) - cp(A_{i,1})}; \quad S_{j,k} = \frac{Ms}{n} \sum_{i=1}^n \frac{a_{i,j,k}}{Mx_i}; \quad (26)$$

2. Determination of the fuzzy set  $B''$ :  
where:  $j = 1..3$ ;

$$d_{B_{4,k}} = d_{B_{1,k}} + (d_{B_{2,k}} - d_{B_{1,k}}) \cdot C; \quad (27)$$

$$cp(B_3) = cp(B_4) = cp(B_1) + (cp(B_2) - cp(B_1)) \cdot C; \quad (28)$$

$$\text{where: } C = \frac{cp(S_3) - cp(S_1)}{cp(S_2) - cp(S_1)}; \quad cp(S_j) = \frac{Ms}{n} \sum_{i=1}^n \frac{cp(A_{i,j})}{Mx_i}; \quad (29)$$

3. Determination of the conclusion set  $B'$  from  $S'$ ,  $S''$  and  $B''$ .  
where  $Ms' \geq Ms$ ;  $My' \geq My$ ;

$$b_{3,2} = F(s_{3,2}; s_{4,2}; b_{4,2}; cp(S_3); cp(B_3)) \quad (30)$$

$$b_{3,3} = cp(B_3) + F(d_{s_{3,3}}; d_{s_{4,3}}; d_{B_{4,3}}; Mx - cp(S_3); My - cp(B_3)); \quad (31)$$

$$b_{3,1} = b_{3,2} \cdot F([s_{3,1}/s_{3,2}]; [s_{4,1}/s_{4,2}]; [b_{4,1}/b_{4,2}]; 1; 1); \quad (32)$$

$$b_{3,4} = My' - (My' - b_{3,3}) \cdot F([(Ms' - s_{3,4}) / (Ms' - s_{3,3})]; [(Ms' - s_{4,4}) / (Ms' - s_{4,3})]; [(My' - b_{4,4}) / (My' - b_{4,3})]; 1; 1); \quad (33)$$

## 5. EXTRAPOLATION

The basic idea in the new method can be applied as an extrapolation.  $A''_i$  can be interpretable where the shape of the cut of solid, which is built from  $A_{i,1}$  and  $A_{i,2}$ , can be as a fuzzy set. The conclusion depends on  $A'$ ,  $A''$  and  $B''$  directly not on  $A_{i,1}$  and  $A_{i,2}$ . Therefore if  $A'_i$  is not „between“  $A_{i,1}$  and  $A_{i,2}$  and the cut of solid at the position of  $A'_i$  is interpretable as fuzzy set  $A''$  then the conclusion can be generated.  $B''$  can be generated in same way. So the method which is introduced in this article can be applied as a fuzzy rule extrapolation too. To get  $A''_i$  around  $A_{i,1}$  and  $A_{i,2}$  can be defined an interval  $[I_L, I_U]$  where the extrapolation can be used. In the case of general method the definition of interval depends on the type of solid. In the case of specialised method where the solid is built by line surface the interval can be defined easily. To get interpretable set  $A''_i$  some conditions have to be satisfied.

Based on these conditions can be defined the interval of extrapolation on every each input universe.

$$1) a_{i,4,1} \leq a_{i,4,2} \leq a_{i,4,3} \leq a_{i,4,4}$$

$$I_{i,1} = \frac{d_{A_{i,1,2}} - d_{A_{i,1,1}}}{d_{A_{i,2,1}} + d_{A_{i,1,2}} - d_{A_{i,1,1}} - d_{A_{i,2,2}}} (cp(A_{i,2}) - cp(A_{i,1})); \quad (34)$$

$$I_{i,2} = \frac{d_{A_{i,1,3}} - d_{A_{i,1,4}}}{d_{A_{i,2,4}} + d_{A_{i,1,3}} - d_{A_{i,1,4}} - d_{A_{i,2,3}}} (cp(A_{i,2}) - cp(A_{i,1})); \quad (35)$$

$$2) a_{i,4,1} \leq a_{i,4,2} \leq cp(A''_i) \leq a_{i,4,3} \leq a_{i,4,4}$$

$$I_{i,2+j} = -d_{A_{i,1,j}} \cdot \frac{cp(A_2) - cp(A_1)}{d_{A_{i,2,j}} - d_{A_{i,1,j}}}; \quad \text{where: } j = 1..4 \quad (36)$$

$$I_{i,7} = Mx_i; \quad I_{i,8} = mx_i; \quad (37)$$

3) To get  $B''$  have to be satisfied same conditions on universe  $Y$ :  $I_{B,1..8}$   
4) Transformation the interval on  $Y$  to interval  $X$ .

$$I_{i,8+j} = I_{B,j} \cdot \frac{cp(A_{i,2}) - cp(A_{i,3})}{cp(B_2) - cp(B_1)}; \quad \text{where: } j = 1..8; \quad (38)$$

5) Determination of the interval of extrapolation.

$$I_{L,i} = \frac{1}{2} \max_j ((1 + a_{i,j,1}) \cdot mx_i + (1 - a_{i,j,1}) \cdot I_{i,j}); \quad (43)$$

$$I_{U,i} = \frac{1}{2} \min_j ((1 + a_{i,j,2}) \cdot I_{i,j} + (1 - a_{i,j,2}) \cdot Mx_i); \quad (44)$$

$$\text{where: } j = 1..16; \quad a_{i,j,n} = \text{sgn}(I_{i,j} - cp(A_n));$$

## 6. EXAMPLES

**Examples of the general method.** In the figure 10. the results obtained with the general method in the two-dimensional case can be seen. There are three diagrams in each one of the figures (a,b,c,d). The first and second diagrams represent the input universe  $X_1$  and  $X_2$ , while the third one shows the output universe  $Y$ . The computer simulation made possible the utilization of fuzzy sets drawn by hand, permitting observation of all the particularities in the process of conclusion generation in any type of fuzzy sets (fig. 10. a,b,c). The fig. c) shows a case, when the conclusion set can be difficultly given using only human comprehension. The fig. d) shows an example of the theory in a extreme case, with crisp sets. The response shows convincingly that  $B_1$  is the relation AND between sets  $A_{11}$  and  $A_{21}$  and  $B_2$  is between  $A_{12}$  and  $A_{22}$  as obtained by classical logic operation. And the same is true for the relations between  $A'_1$ ,  $A'_2$  and  $B'$ . The limits of the conclusion crisp set  $B'$  corresponds with the result of the min-max operation with minimal divergence.

**Examples of the specialised method** In the fig.11 the results obtained with the specialised method in the two-dimensional case can be seen. The first and second diagrams show in a,b and c figures the input universe sets and the observation fuzzy sets  $A'_1$ ,  $A'_2$ , and the antecedent sets  $A_{11}$ ,  $A_{12}$ ,  $A_{21}$ ,  $A_{22}$ . On the third diagram can be found the conclusion fuzzy sets  $B'$  generated by Kóczy and Hirota's interpolation. The fourth diagram contains the results of the Vas-Kalmár and Kóczy-

method, and the fifth diagram the results of specialised method.

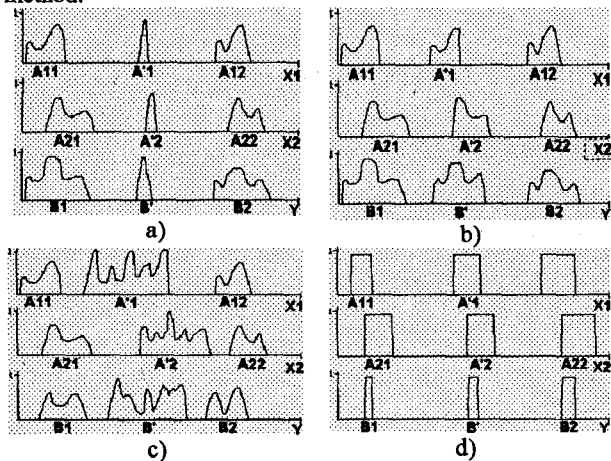


Figure 10: Results of general method

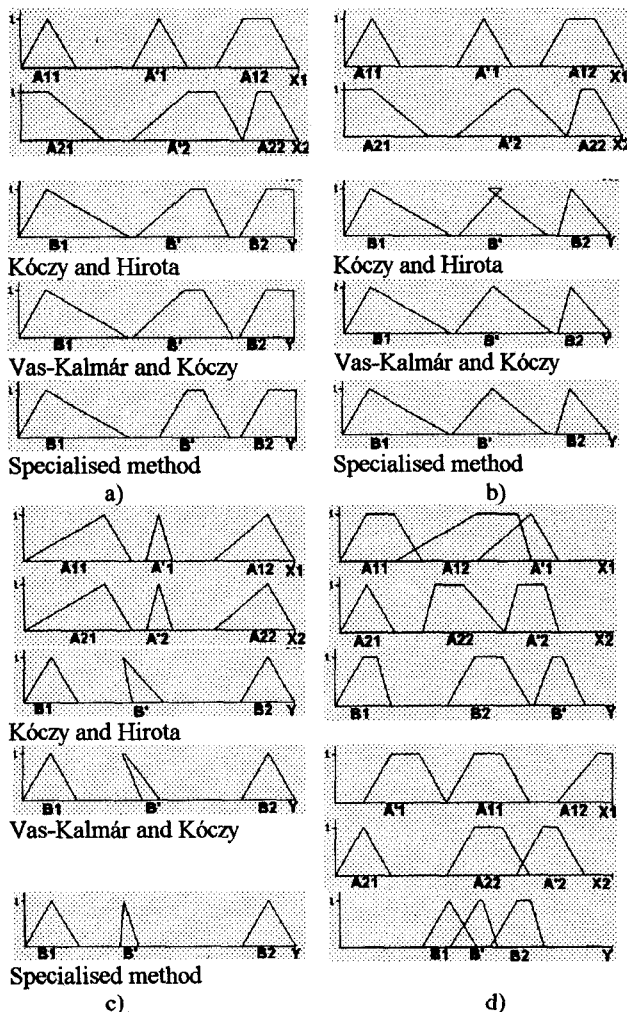


Figure 11: Comparison of three different methods

In the figure a) every method resulted into a directly interpretable conclusion. Comparing these three different methods it can be said that the Kóczy and Hirota's method usually gives an almost identical conclusion (if it is directly

interpretable) with the newly introduced specialized method. The figure b) and c) present examples where the specialized method still gives directly interpretable conclusions while the others do not. Figure d) shows two examples, where the specialised method is used as extrapolation.

## 7. CONCLUSION

The interpolation methods in the previous literature can be applied only on convex sets, while the generalized method introduced in this paper is applicable for arbitrary type fuzzy sets. As a matter of course, the general method makes calculation necessary for "every point" of the set. The specialized method can be applied on the most commonly used crisp, triangular and trapezoidal fuzzy sets, and requires only small computational time. The original linear and nonlinear interpolation methods present another problem, namely the conclusion set is not always convex, therefore it is not sufficient to calculate only the four characteristic points of the set. Our specialized method eliminates this problem, because it results always into crisp, triangular or trapezoidal conclusions, therefore it is enough to calculate only the four characteristic points. Another difference is that the new method offers a directly interpretable conclusion in every case. The basic idea in the new method can be applied as fuzzy extrapolation. We are working now to use different type of solid to generate conclusion.

Péter Baranyi was supported by Foundation for Hungarian Science.

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