Abstract
We present a tractable analytical framework for the exact calculation of probability of node isolation when \( N \) sensor nodes are independently and uniformly distributed inside a finite square region. The proposed framework can accurately account for the boundary effects by partitioning the square into subregions, based on the transmission range and the node location. We show that for each subregion, the probability that a random node falls inside a disk centered at an arbitrary node located in that subregion can be expressed analytically in closed-form. Using the results for the different subregions, we obtain the exact probability of node isolation. The proposed framework is validated by comparison with simulation results.

Problem Formulation
The basic building blocks in our approach to characterise the boundary effects are (i) the circular segment areas formed outside each side (border effects) and (ii) the corner overlap areas between two circular segments formed at each vertex (corner effects).

\( B_i(u; r_o) \) is the area of the circular segment formed outside the side \( S_i \), given by

\[
B_i(u; r_o) = r_o^2 \arccos \left( \frac{u}{r_o} \right) - u \sqrt{r_o^2 - u^2}. \tag{5}
\]

\( C_{ij}(u; r_o) \) denotes the area of the corner overlap region between two circular segments at vertex \( V_{ij} \), given by

\[
C_{ij}(u; r_o) = r_o^2 \arccos \left( \frac{\sqrt{r_o^2 - u^2}}{2r_o} \right) - \frac{1}{2} \left( \sqrt{r_o^2 - u^2} - y \right) - \frac{1}{2} \left( \sqrt{r_o^2 - x^2} - y \right), \tag{6}
\]

where \( \theta_i = 2\pi^2 - 2\pi \sqrt{r_o^2 - y^2} - 2\pi \sqrt{r_o^2 - x^2} \).

Similarly, the areas of the circular segments formed outside the sides \( S_2, S_3, S_4 \) and the areas of the corner overlap region formed at vertex \( V_2, V_3, V_4 \) can be calculated.

Proposed Framework
Let \( R_1, R_2, \ldots, R_M \) denotes the type of subregions and \( n_i, i \in \{1, 2, \ldots, M\} \) denotes the number of subregion of type \( R_i \). (3) can be written as

\[
P_{\text{iso}}(r_o) = \prod_{i=1}^{M} n_i \int_{R_i} \left( 1 - F_i(u; r_o) \right)^{N-1} du. \tag{7}
\]

where \( F_i(u; r_o) \) denotes the probability that a random node falls inside the disk \( D(u; r_o) \) of radius \( r_o \) centered at \( u \in R_i, R_i \) and \( F_i(u; r_o) \) are defined in Table 1 and Figure 2 for different \( r_o \).

Table 1: Subregions and conditional probabilities for calculation of \( P_{\text{iso}}(r_o) \) \( (0 < r_o \leq \frac{1}{2}) \).

<table>
<thead>
<tr>
<th>Subregion ( R_i )</th>
<th>( n_i )</th>
<th>( F_i(u; r_o) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_1 = { x \in (r_o, 1 - r_o), } )</td>
<td>1</td>
<td>( \pi r_o^2 )</td>
</tr>
<tr>
<td>( R_2 = { x \in (0, r_o), } )</td>
<td>4</td>
<td>( \pi r_o^2 - (B_1) )</td>
</tr>
<tr>
<td>( R_3 = { x \in (0, r_o), } )</td>
<td>4</td>
<td>( \pi r_o^2 - (B_1 + B_2) )</td>
</tr>
<tr>
<td>( R_4 = { x \in (0, r_o), } )</td>
<td>4</td>
<td>( \pi r_o^2 - (B_1 + B_2 - C_1) )</td>
</tr>
</tbody>
</table>

Results

Figure 3: \( P_{\text{iso}}(u; r_o) \) versus position of arbitrary node with \( r_o = 0.4, N = 10 \).