Bearing only visual servo control of a non-holonomic mobile robot.

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ANU (RSISE) Nomad 4000 mobile robot

Long term project towards the full automation of a mobile robot. Goal is to have robot running independently for long periods.

http://robot.anu.edu.au/

This presentation considers the development of an automated docking process based on visual sensor system.
Kinematic model of robot

\[
\begin{align*}
\dot{x} &= u \cos \theta \\
\dot{y} &= u \sin \theta \\
\dot{\theta} &= \omega
\end{align*}
\]

\(u\) linear velocity of the vehicle.
\(\theta\) rotation angle from global frame \(\langle g \rangle\) to body fixed frame \(\langle b \rangle\).
\(\omega\) angular velocity.

A docking manoeuvre \((x, y, \theta) \to 0\) is a state stabilisation problem for the kinematic a mobile robot.
Key Issues for docking manoeuvres

➤ In practice, several sensor systems are available to provide feedback during a docking manoeuvre.

➤ Considering just the vision system (and undertaking similar developments based on other sensor configurations) provides redundancy in the control/sensing configuration of the robot.

➤ The docking manoeuvre is highly sensitivity to lateral offset from docking station. In practice lateral accuracy of around 1 cm is required during the docking process.

➤ Maintaining good calibration of a visual system for extended periods of operation is a significant problem. It is preferable to have a highly robust visual/control loop that does not rely on precise identification of image features.
Omni-directional (Panoramic) Camera

➤ Allows the robot to observe targets in entire room.
➤ Low pixel resolution leads to bearing only measurements.
➤ Widely spaced targets provides precise positioning of robot despite low precision vision system.

Excellent sensor modality for low cost, highly reliable visual positioning tasks.
Multi-coloured visual targets

Target identification uses a combination of color segmentation, image moments along with a simple matching criterion to obtain 98% reliable image recognition in a wide range of lighting conditions.
Image space with identified landmarks
Each identified landmark ‘bearing’ is represented as a unit vector expressed in the body-fixed frame

\[ p_i = R(\theta) \frac{\xi_i - \xi}{|\xi_i - \xi|} \]

where \( R(\theta) \) is the rotation matrix from the global frame to the body-fixed frame.

Image feature used is the Average Landmark Vector (AVL), defined as

\[ q^g(\xi) = \sum_{i=1}^{n} p^g_i \]
1) Approach inspired by insect navigation.
2) Image based feature - advantages for closed-loop robustness - does not require detailed world model.
3) Image features are discontinuous at target points due to lack of range information in bearing only measurements.
Bio-mimetic vision systems


- Theory developed in two dimensions.
- Holonomic kinematic control law $v = q - q_0 \in \mathbb{R}^2$.
- Goal pose is distinct from landmark points.
- Requires an inertial direction in addition to visual data.
Image based visual servo-control of dynamic systems

➤ Theory developed in three dimensions.
➤ Control of dynamic system - exploit passivity of spherical image plane.
➤ Kinematic part of control
\[ \nu_{\text{des.}} = q - q_0 \in \mathbb{R}^3. \]
➤ Goal pose is distinct from landmark points.
➤ Requires an inertial direction in addition to visual data.
The kinematics of the ALV in $\langle g \rangle$ are given by

$$\dot{q}_g = -Q(\xi)v, \quad D_\xi q_g = -Q(\xi)$$

where

$$Q = \sum_{i=1}^{n} \frac{(I_2 - p_ip_i^T)}{|\xi_i - \xi|} > 0, \quad \text{Interaction Matrix}$$

and

$$v = \dot{\xi} = \begin{pmatrix} u \cos(\theta) \\ u \sin(\theta) \end{pmatrix},$$

is the velocity vector of the vehicle in the global frame.
Choose a holonomic control design

\[ v_{\text{des.}} = q(\xi) \]

Leads to closed-loop kinematics

\[ \dot{\xi} = q(\xi), \]

Consider the two Lyapunov functions

\[ U(\xi) = \sum_{i=1}^{n} |\xi_i - \xi| \]

\[ \phi(\xi) = q(\xi)^T q(\xi) = |q(\xi)|^2 \]

Note that \( \phi(\xi) \) can be measured on-line while \( U(\xi) \) requires knowledge of \( \xi \) and \( \xi_i \).
Holonomic Stability Analysis

Stability analysis (Moeller, 2000)

\[ \frac{d}{dt} U(\xi) = \sum_{i=1}^{n} \frac{-(\xi_i - \xi)^T \dot{\xi}}{|\xi_i - \xi|} = -|q|^2 \]

Stability analysis (Hamel et al., 2001)

\[ \frac{d}{dt} \phi(\xi) = -2q^T Q(\xi) \dot{\xi} = -2q^T Q(\xi)q \]

Note that \( U(\xi) \) is convex, \( U(\xi) \geq 0 \) and \( DU(\xi) = 0 \) at a local minimum of \( U \),

\[ \phi(\xi) = |DU(\xi)|^2, \quad D^2U(\xi) = Q(\xi) > 0 \]
Placing the targets.

The stabilizing control design depends on exploiting the singularity of the image feature at a landmark point. Thus, a target must be placed at the docking station.

For three or more landmark features there exists a unique point $\xi^*$ in the closed convex hull of the landmarks that minimizes both $U(\xi^*)$ and $\phi(x^*)$.

By placing the targets appropriately it is possible to arrange that $x^*$ is co-located on the docking station.

If the exact placing of targets is not possible then a target weighting scheme has been developed, (Wei, et al., 2005) that can be used to move the minimum value $\xi^*$ anywhere within the convex hull of landmarks.
Structure of $\phi$ at $\xi^\ast$.

- The cost surface $\phi$ is discontinuous at the minimum $\xi^\ast$ since it is located at a landmark.
- The holonomic control design causes smooth decrease of $\phi$.
- It is expected that the closed-loop trajectory of the holonomic system will fall into the trough of the cost surface $\phi$ and converge to the target point along the asymptotic direction.

The graph of $\phi$ at an optimal landmark.
Level sets of cost functions

It is interesting to note that the cost function $U(\xi)$ does not show the discontinuity to the same extent as the function $\phi$. 
Simulation of closed-loop trajectories for holonomic control strategy

Note that the closed loop system converges to the target point along a suitable trajectories to complete the docking manoeuvre.
Non-holonomic control strategy

For actual system the holonomic velocity control

\[ v_{\text{des.}} = q \]

will not normally satisfy the non-holonomic constraints.

The approach to the non-holonomic control design is to design a control that will approximately track the desired behaviour

\[ \dot{\xi} = q_g \]

This is similar to designing a controller to track a family of trajectories, any one of which will satisfy the control requirements.
Tracking error

Tracking error

\[ \epsilon_{\text{track.}} = \dot{\xi} - v_{\text{des}}. \]

Define normalized vector directions

\[ \bar{q}_g = \frac{q_g}{|q_g|}, \quad \bar{v}_g = \frac{v_g}{|v_g|}. \]

Robot scalar velocity is \( u = |v_g|. \)

Normalized tracking error and tracking angle

\[ \Delta = \bar{q}_g - \bar{v}_g, \quad \gamma = \angle(\bar{v}_g, \bar{q}_g). \]
Stability Analysis

Consider the Lyapunov function

\[ L = U + |q_g| + |q_g| (1 - \cos(\gamma)) \]

The first term is the stability function introduced by Moeller 2001.

The second term \(|q| = \sqrt{\phi}\) is highly sensitive to the discontinuous structure of the cost function.

The third term provides an error criterion on the tracking error, scaled by convergence to the goal point.

In the detailed analysis it appears that the \(C_0\) nature of the cost terms \(|q|\) in \(L\) are crucial to obtaining the bounds.
Derivative of the cost function

Differentiating $L$ along trajectories

$$\dot{L} = -u \bar{q}_g^T Q \bar{q}_g + u \left( \Delta^T Q \Delta - \langle q_g, \bar{v}_g \rangle \right) - \omega \langle q_g, A \bar{v}_g \rangle,$$

$A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

Using $T = \tan(\gamma/2)$ and making some approximations one obtains

$$\dot{L} \leq -u \bar{q}_g^T Q \bar{q}_g + \left[ u \left( \frac{4nT^2}{r_0(1 + T^2) |q_g|} - \frac{1 - T^2}{1 + T^2} \right) - \omega \frac{2T}{1 + T^2} \right] |q_g|,$$

$$= -u \bar{q}_g^T Q \bar{q}_g - (u\alpha + \omega\beta) |q_g|$$

$$\leq - (u\alpha + \omega\beta) |q_g|$$

where $\alpha$ and $\beta$ are functions of the observed quantities $T$ and $|q_g|$.
Control design

The simplified form of the Lyapunov derivative is

\[ \dot{L} \leq -(u\alpha + \omega \beta) |q_g| \]

Minimize the right-hand side subject to a control constraints

\[ \frac{1}{2} |u|^2 + \frac{1}{2} k|\omega|^2 \leq \frac{1}{2} B |q_g|^2, \quad u \geq 0. \]

Control inputs

\[
\begin{align*}
 u &= \frac{c_0 \alpha |q|}{\sqrt{k\alpha^2 + \beta^2}} \quad \text{for } \alpha \geq 0 \\
 u &= 0 \quad \text{for } \alpha < 0
\end{align*}
\]

\[
\begin{align*}
 \omega &= \frac{c_0 \beta |q_g|}{\sqrt{k\alpha^2 + \beta^2}} \quad \text{for } \alpha \geq 0 \\
 \omega &= \text{sgn}(\beta) c_0 |q| \quad \text{for } \alpha < 0
\end{align*}
\]
Convergence analysis

Several calculations yield the following bound on the decrease in the Lyapunov function

\[ \dot{L} \leq -c_0 |q_g|^{2.5} \sqrt{r_0(t)} \sqrt{\frac{2}{(k + 1)n r_0(t) + 4kn}} \]

The constant \( r_0(t) \) is a bound \( r_0(t) \leq \min\{ |\xi - \xi_i| \} \).

Applying Lyapunov’s 2nd method shows that \( \xi \to \xi^* \).

With some care it is also possible to use LaSalles principal to show that \( \gamma \to 0 \) for generic initial conditions.
Asymptotic analysis

The nature of the Lyapunov function allows one to obtain the following bound close to the limit point

\[ L \leq A|q|, \quad |\xi - \xi_*| \leq \epsilon \]

Consequently asymptotically

\[ \dot{L} \leq -BL^3 \]

One obtains

\[ L(t) \leq \frac{1}{\sqrt{(c_1t + c_2)}} \]

The convergence implied in \( |\xi - \xi_*| \to 0 \) is slower than exponential. This is to be expected for stabilization of a non-holonomic system.
Results of 21 experimental runs
**Final convergence results**

The key statistics are:

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>St. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_\infty$</td>
<td>0.01 cm</td>
<td>0.38 cm</td>
</tr>
<tr>
<td>$y_\infty$</td>
<td>0.389 cm</td>
<td>0.5 cm</td>
</tr>
<tr>
<td>$\gamma_\infty$</td>
<td>-1.75 deg.</td>
<td>6.37 deg.</td>
</tr>
</tbody>
</table>

Final position and orientation for a set of 21 experiments.
Video Demonstration

Unwrapped image sequence from a typical trajectory.

Raw image sequence from a typical trajectory
Conclusions

➤ Precise position information obtained from low quality vision system by using a panoramic camera and widely spaced landmarks.

➤ Inherent discontinuity of bearing only measurements used to construct a discontinuous error feature that has nice properties for stabilization of non-holonomic systems.

➤ Non-holonomic control design based on minimizing decrease in a control Lyapunov function subject to steering and speed constraints.

➤ Excellent experimental results.
Work presented was undertaken in collaboration with

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