A New Construction Method for a Lightweight Submerged Radial Gate

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Abstract- A new design method in hydraulic engineering is presented for the construction of a lightweight submerged radial gate (SRG) with two vertical arms. In the analysis, the structure is decomposed into several main components and each component is shaped using a topology optimization approach. In particular, a SRG can be divided into three major parts: the water-retaining face plate, the support frame, and the arms. The whole design process for the SRG generally includes five steps: (1) the optimal position of arms is determined in terms of avoiding bending moment of arm adjacent to the support frame; (2) using the topology optimization method, the shape of each arm is determined subject to the symmetry condition with respect to the whole structure; (3) a frame made of horizontal and vertical ribs is constructed to support the face plate; (4) the new radial gate is assembled based on these optimized components, e.g., two arms, the face plate and its support frame; and (5) the sizes of these components are verified using size optimization by considering the stiffness and strength of the new structure under a critical loading condition. To show the validity of the method, a traditional SRG in a real project is adopted as an example and is redesigned using the present method. The evaluation result shows that the new structure is about 27% lighter than the original. At the same time, the mechanical properties of the SRG are significantly improved.

Keywords- Radial Gate; Optimal Design; Topology Optimization; Size Optimization; Lightweight Structure

I. INTRODUCTION

In a hydropower structure, a water gate is an important component for controlling water storage for the purposes of energy generation, flood protection and water level control in a channel. In recent years, radial gates have been more popular than plane gates because of their ease and safety in operation and because they have no gate slot. In hydropower engineering, steel radial gates are in wide use. With the traditional design method, designing a radial gate is very complex and the design is usually based on an earlier specification. Moreover, many simplifications are required to resolve problems with the traditional design method. For instance, a three-dimensional (3D) radial gate is treated as a two-dimensional (2D) structure for design convenience, which can induce significant errors in the resulting radial gate, such that it may not be reliable. Much attention has been paid in the past few years, therefore, to developing reliable design methods for SRGs. Lian et al. used the Sequential quadratic programming method to give an optimization for radial gate performance. Cai and Zhang presented an optimization algorithm for designing an integrated emersed radial gate. Peng optimized a radial gate using a genetic algorithm (GA) method. However, the gates involved in these articles must initially be designed by traditional methods. This initial design of the radial gate still relies on an old specification. Furthermore, size optimization or parameter optimization, rather than topology optimization, is employed in these studies.

Structural topology optimization, in contrast, is a method for optimizing the layout of materials in a structure. Therefore it is also called layout optimization. It is a powerful computational tool in structural design in addition to size/shape optimization. In topology optimization both the amount and layout of the material in the design domain can be changed freely. The topology optimization method for continuum structures became popular after the homogenization design method was proposed by Bendsoe and Kikuchi in the late 1980s. Since then, many effective approaches have been proposed, e.g., the method of solid isotropic microstructures with penalization (SIMP), the evolutional structural optimization (ESO) method, the level sets (LST) method and the reference interval method. The SIMP method has recently been widely used in water gate design in conjunction with some commercial finite element software such as ABAQUS, ANSYS, and Hyperworks. It should be mentioned that Zhu and Wang first discussed the layout of the arms in a radial gate by using topology optimization module in ANSYS. However, only a cross-section of the radial gate was discussed in their work, and the results were unfeasible for use in practical design. To address this situation, a novel design scheme of a deep-hole steel radial gate, for which loading conditions are different from and more critical than those on an emersed radial gate, is presented in this work. With the method presented here, the whole structure is first decomposed into several major components according to their unique functions. Then, the shape of each component is optimized using a topology optimization method. The optimal radial gate is assembled from these optimized components.

II. METHOD OF TOPOLOGY OPTIMIZATION FOR CONTINUUMS

A. SIMP Method

In this work, structural analysis is conducted using commercial software Hyperworks, which adopts the SIMP method as its core solution approach for topology
optimization. Before describing the whole design process, SIMP is briefly summarized. In the SIMP method, the pseudo-density of a finite element is used as the design variable (DV). Each DV is in the interval (0 1]. For an element, if its pseudo-density is close to zero, the element is considered as void (i.e., without material). Otherwise, the element with its pseudo-density equal to unity is considered as a solid (i.e., full of material). An element with a pseudodensity between the lower and upper bounds of the interval is called a mid-density element. To obtain a clear structural topology for further (size) optimization or manufacture, the number of mid-density elements is reduced by using a penalization approach. Generally, the stiffness maximization problem for the structure can be expressed as

\[
\text{Find: } X = [x_1 \ x_2 \ \cdots \ x_N] \\
\text{min: } c(X) = U^T KU = \sum_{j=1}^{N} (x_j)^p u_j^T k_j u_j \\
\text{subject to: } \phi_j(X) \leq 0 \quad j = 1, 2, \ldots, J \\
: \quad KU = F \\
: \quad 0 < x_{min} \leq x_j \leq 1, \quad e = 1, 2, \ldots, N
\]

where \(X\) is a set of DVs, \(U\) and \(F\) are the global displacement and force vectors in FEM, respectively, \(K\) is the global stiffness matrix, \(F\) and \(K\) can be obtained using the well-established FEM. \(u_j\) and \(k_j\) are the element displacement vector and element stiffness matrix, respectively. The penalization factor \(p\) is set at 3 in the present work. \(DV\) \(x_j\) is the pseudo-density of the \(e\)th element. A small positive scalar \(x_{min}\), rather than zero, is used as the lower bound of the pseudo-density interval, to avoid singularity of \(K\) for a given finite element mesh. \(N\) is the total number of elements in design domain. \(N\) is the total number of constraints. \(\phi_j\) is a constraint function such as volume constraint, displacement constraint, stress constraint, buckling constraint, frequency constraints, etc. \(J\) is the total number of constraints. Meanwhile, to avoid checkerboard patterns, a filtering technique is applied to the sensitivity of the objective function in the optimization process.

B. Flowchart of SIMP Method

Eq. (1) gives the mathematical formula of topology optimization of a continuum with \(N\) design variables. The flowchart of the algorithm is as follows

1) Create the FE model of structure;
2) Define the design variables, e.g., the relative densities of elements in structure;
3) Calculate the deformation of structure by FEM;
4) Calculate the sensitivities of objective function and constraint functions on design variables;
5) Update the design variables according to the sensitivities by such optimization methods as SLP, SQP, MMA etc.;
6) Check convergence of algorithm, if yes, go to 7), no return to 3);
7) Stop to operate post-processing.

In fact, in the following simulation, only a volume constraint is considered in determining the shape of components. The other types of constraints are only considered in size optimization when the new structure is reconstructed.

\[
\text{Find: } X = [x_1 \ x_2 \ \cdots \ x_N] \\
\text{min: } c(X) = U^T KU = \sum_{j=1}^{N} (x_j)^p u_j^T k_j u_j \\
\text{subject to: } V(X)/V_0 - \text{Rat} = 0 \quad (2)
\]

where \(V(X)\) is the total volume of the current design domain, \(V_0\) is the total volume of initial design domain, the parameter \(\text{Rat}\) is specified before optimization and is usually within the interval of \([5\%, 30\%]\). The flowchart corresponding to Eq. (2) is the same as that of Eq. (1) if the constraint function is specified as volume constraint.

C. Steps of Design for a Complex Structure Using Topology Optimization

Generally, it is not possible to obtain the optimal design of a complex structure for manufacture by using topology optimization directly. To meet manufacturing needs, it is necessary to decompose the structure into several main components and to determine the optimal design of the components individually. The design procedure using topology optimization therefore generally involves the following steps:

Step 1. Decompose a structure into several components;
Step 2. Find the optimal shape/topology of each component using topology optimization;
Step 3. Integrate the structure with the newly obtained components;
Step 4. Carry out size optimization for the structure under necessary constraints;
Step 5. Obtain the detailed sizes of the components of the structure for manufacturing.

The decomposition of structure into major components depends on the unique functions of those components. Optimization of each component should be easily performed and the result will also be convenient for further use. In Step 2, manual verification is usually undertaken to obtain a result which is easily manufactured.

III. EVALUATION OF NEW DESIGN

A. Statement of the Problem

The skeleton of a steel radial gate used for spillway (as shown in Fig. 1) is from a traditional design based on China’s national specification [1]. There are two major box-sectional crossbeams, three major vertical ribs/beams and 10 horizontal ribs/beams in the frame made of Q345B steel.
material properties are: Young’s modulus 206 GPa, Poisson’s ratio 0.3, and density 7800 kg/m³. The maximum radius of the vertical ribs (Fig. 1) is 12.00 m. The design head/water level is 90.42 m. The span is 6.00 m. The thickness of face plate is 36 mm. The width of each arm is 1.0 m. The total weight of the gate is about 140 tons.

5) Construct a new radial gate with the new components obtained
6) Perform size optimization for the new structure to provide structural dimensions for manufacture

Consistent with industry standard, all the ribs are made of I-beam. The initial thicknesses of the webs and the flange of a rib are the same as those in the initial structure. Achievement of the new design can be simplified by considering the symmetry of the gate.

C. Design Process

In the present design, the global coordinate system is set to be a right-hand Cartesian coordinate (OXYZ), in which the direction of axis OX is parallel to the flow direction, axis OY is in the vertical direction (see Fig. 1a).

1) Location of the Arms:

Before optimizing the shape of an arm (with box-section), the position of the arm should first be determined. Clearly, the arms are under compression. To make full use of the material in the arms, their deformation should be of uniform distribution, i.e., each arm should be under pure compression, and bending deformation is not permitted as considering stability of arms. Accordingly, the gate with two arms can be simplified as an extended beam under uniform pressure (see Fig. 2) and the positions of the arms are the same as those of two fixed supports of the beam. To avoid bending of the two supports, Eq. (3) below must be satisfied.

\[ l_1 = 0.207(2l_1 + l_2) \]  

As the span of the gate, i.e., \( 2l_1 + l_2 \), is 6.0 m, the distance between the centres of each arm and the vertical symmetry plane of the gate, i.e., \( 0.5l_2 \), is 1.758 m.

2) Topology and Shape of the Two Arms:

Due to the symmetry condition of the gate, only one arm needs to be determined. For simplicity, the following assumptions are used: (1) the arm is considered first, as a component experiencing plane stress in the OXY plane only; (2) the initial design domain for topology optimization is assumed to be a circular sector with radius of 12.0 m; and (3) the radial line of the sector is considered as the boundary of face plate being subjected to hydrostatic pressure.

In the process of topology optimization, the face plate together with its support frame can be considered as a non-design domain whose thickness has an obvious effect on the final result. The thickness of the non-design domain can be
obtained using the condition of equivalent bend stiffness per length of structure.

$$T_N = \alpha \cdot \sqrt{L/D} \cdot T_0$$  (4)

where the parameter $\alpha \in [2, 20]$, $L$ is the span of the gate, $D$ is the total thickness of the webs of the two arms, $T_0$ is the thickness of the face plate, and $T_N$ is assumed to be around 1800 mm in the present design. Higher value of $T_N$ will lead to “T” type of structure which is unstable, and lower value of $T_N$ will lead to too many branches to support face plate.

In this optimization process, the objective is to minimize the structural compliance subject to any volume constraints (2). In particular, the parameter $R_T$ in Eq. (2) is assumed to be 15% (in other words, the material remaining in the final structure must be at least 15% of the total amount) in our analysis. Fig. 3a shows the initial design. Fig. 3b gives the topology optimization result using Eqs. (2) and (3).

It should be mentioned that the result shown in Fig. 3b is not an optimal design for manufacture, for two reasons. Firstly, the shape of the arm is not smooth. Secondly, the long slim pole on the left side of the trunk is under compression and may become unstable due to buckling. The situation can be improved by adding a bar to support the long slim pole, as shown in Fig. 3c.

3) Vertical Ribs in the Support Frame:

The frame supporting the face plate, is traditionally composed of vertical ribs and horizontal/cross ribs. The function of vertical ribs is to reduce the bending deformation of the face plate around the OZ axis. To provide a reasonable layout of the vertical ribs in the frame, topology optimization methods Eqs. (1) – (3) are used to determine the total number of (vertical) ribs, their positions and depths.

In the analysis, the left half part of the beam shown in Fig. 2 is selected as the initial design in topology optimization. But the depth of beam and the width of the fixed support can be enlarged. The depth of the beam is set to be 2.0 m and the width of the support to be 1.0 m. The position of the centre of the new support is the same as that of the support in Fig. 2. The depth of the beam is slightly greater than the maximum of the depths of the ribs, i.e., 1.8 m. The load, i.e., the uniform pressure applied on the beam in Fig. 4a, is unchanged.

The revised initial design domain used in this analysis is shown in Fig. 4a. Fig. 4b presents the result of topology optimization using Eq. (2), and indicates that five vertical ribs are required in the left half. Rib No. 5 is in the centre of symmetry plane of the gate. Thus there are in total nine vertical ribs in the frame.

The positions of the ribs are shown in Fig. 4c, showing Ribs No. 2 and No. 3 are connected with arm.

4) Horizontal Ribs in Support Frame:

In the analysis, the depth of the support frame is assumed to be 2.0 m in the design domain shown in Fig. 5a, which is different from that shown in Fig. 3. The areas with fixed degrees of freedom are the interfaces between the arms and the frame. Their initial positions are shown in Fig. 5a. The face plate is the non-design domain, which is
subjected to hydrostatic pressure. The bottom of the structure is simply supported.

![Initial design domain](image1)

![Optimal topology](image2)

![Layout of horizontal ribs in support frame](image3)

Fig. 5 Initial design and optimal layout of the horizontal ribs

Fig. 5b shows the optimal topology of the structure using Eq. (2). Fig. 5c shows the layout of the horizontal ribs. Clearly, there are 11 level ribs, including 9 mid-ribs and 2 boundary ribs, i.e., No. 0 and No. 10. On rib No.10 the hangers are assembled. Ribs No. 2, No. 3, No. 5 and No. 6 are connected to the arms. They can be considered as extensions of the arms. For convenience of manufacture, it is suggested that ribs No. 0, No. 1, No. 4, No. 7, and No. 8 have the same shape. The angles between the rib No. 0 and some mid-ribs are as follows: 2.8° for \( \theta_1 \), 13.5° for \( \theta_2 \), 24.3° for \( \theta_3 \), and 27.5° for \( \theta_4 \), where \( \theta_i \) is the angle between rib No. 0 and rib No. \( i \).

5) Integration of the Structure with New Components:

Using the components, namely the face plate, the arms (in Fig. 3c) and the support frame with the ribs shown in Figs. 4c and 5c, the new radial gate is obtained as shown in Fig. 6. In the new gate, the arms are made of box-sectional beams. The ribs in the support frame are made of I-beams with the thickness of 40mm, which is close to the initial design value.

Before conducting size optimization, the strength and stiffness of the new design are examined. Under the critical loading condition, namely 90.42 m of water pressure (the normal water level) and the weight of the structure itself, the maximum displacement of the gate is only 36.7 mm at the lifting point of the gate. Simultaneously, most parts of the structure are under low stress except for the areas near the ends of the arms where the maximum von Mises stress reaches 324 MPa. This stress concentration is caused primarily by the shape of the edge which has no fillet. In manufacture, a fillet is often adopted to release a local stress concentration. The size optimization described below is used to improve this situation.

![New structure](image4)

![Half part](image5)

Fig. 6 New design of the radial gate in Fig. 2

6) Size Optimization of the New Gate:

In size optimization, the objective is to minimize weight, subject to the displacement constraint and stress constraint (5). In our analysis, the maximum displacement \( u_{\text{max}} \leq [u] = 50 \text{mm} \) and the maximum von Mises stress \( \sigma_{\text{max}} \leq [\sigma] = 190 \text{MPa} \), which is 55% of the yield stress of Q345B steel. The size optimization algorithm can then be written as

\[
\text{Find: } Y = [h1/01 \ hl/02 \cdots hl/10 \ zl/01 \ zl/02 \cdots zl/05] \\
\text{min: } V(Y) \\
\text{subject to: } \sigma_{\text{max}} - [\sigma] \leq 0 \\
: u_{\text{max}} - [u] \leq 0 \\
: Y(i) \leq Y_{\text{up}}, \quad i = 1, 2, \ldots, 15
\]

where \( V \) is the total weight of the current design. And the flowchart of Eq. (5) is as following:

(1) Select the design variables, e.g., the thicknesses of the ribs in support frame;

(2) Build the geometry model of structure according to the design variables and discretize the structure with FEs(shell elements);

(3) Calculate the deformation of structure;

(4) Calculate the sensitivities of objective function (volume of structure) and constraint functions (maximum displacement and maximum von Mises stress in structure);

(5) Update the design variables according to the sensitivities;
(6) Check convergence of algorithm: if yes, go to (7), no return to (3);
(7) Stop to carry out post-processing.

Fig. 6b shows one of the components to be optimized in our analysis. The vertical ribs zl03 and zl04 connected with the arm and the horizontal ribs hl00, hl09 and hl10 near the edges will not be optimized. Considering the specifications for manufacturing and steel size, the thickness of components of the structure has lower and upper limits of \( y_{\text{inf}} = 20 \text{ mm} \) and \( y_{\text{sup}} = 60 \text{ mm} \), respectively. All the initial values of the DVs are 40 mm. The increment of a DV is 1 mm. In optimization, the stress near the support pivot (within the dark area) and lifting points is not used as a reference (see Fig. 6b).

In our size optimization, the structure (see Fig. 6b) is discretized with 60008 shell elements and only the critical loading condition is considered. That loading condition is 90.42 \( \text{m} \) induced by water pressure on the face plate, the weight of structure itself, and the single point supporting it at the lifting point on the frame.

![Contour plot of displacement (mm)](image)

(a) Contour plot of displacement (mm)

![Contour plot of von Mises stress](image)

(b) Contour plot of von Mises stress

Fig. 7 Displacement and Stress Distributions in the Gate after Size Optimization (Without Considering the End of Arm)

After size optimization, DVs with the labels of hl03, hl04, hl06, hl10, zl02 and zl05 reach their infimum, i.e., 20 mm. The DV with the label of hl09 reaches its supremum, 60 mm. Simultaneously, the DV with the label of hl05 reaches 21 mm, hl07 reaches 26 mm, hl08 reaches 32 mm, and zl01 reaches 26 mm.

Fig. 7 illustrates that both the displacement and the stress of the gate are distributed uniformly and their maxima (u_{max}=44.5 \text{ mm}, see Fig. 7a; \( \sigma_{\text{max}}=256 \text{ Mpa}, \) see Fig. 7b) occur at the lifting point of the gate. The greater part of the structure is under a low stress condition, e.g., less than 190 MPa. In fact, the local high deformation and high stress can also be reduced by arranging more lifting points near the current one.

The significant result is that the weight of the new gate is about 102 tons, which is 38 tons or 27% lighter than the original gate with weight of 140 tons.

IV. CONCLUSIONS AND PROSPECTS

Using the topology optimization approach, a new lightweight design method is developed for a submerged radial gate. In the design process, the whole structure is first decomposed into several major components according to their unique functions. Then the optimal shapes of these components are obtained using volume-constrained topology optimization. The optimal radial gate is subsequently obtained by integrating these optimized components. In each step, the design procedure is discussed so that the method can be easily understood by designers. The most important result is that, compared with the traditional design, the weight of the new gate with stricter mechanical requirements is much lighter, implying that the new gate can be installed and operated more easily.

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