Postbuckling analysis of plates on an elastic foundation by the boundary element method

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Received 21 May 1990
Revised manuscript received 28 October 1991

The postbuckling behavior of plates on an elastic foundation is studied by using the boundary element method (BEM). A new fundamental solution of lateral deflection is derived through the resolution theory of a differential operator, and a set of boundary element formulae in incremental form is presented. By using these formulae, the BEM solution procedure becomes relatively simple. The results of a number of numerical examples are compared with existing solutions and good agreement is observed. It shows that the proposed method is effective for solving the postbuckling problems of plates with arbitrary shape and various boundary conditions.

1. Introduction

The large deflection behavior of an elastic plate has been investigated recently using the boundary element method (BEM). Kamiya et al. [1] presented an integral equation approach for analyzing the large deflection of plates with the aid of Berger's equation, but its applications were restricted to a narrow range by discarding the second invariant of membrane strains included in the strain energy expression. Tanaka [2] later derived a coupled set of boundary and inner domain integral equations in terms of the stresses and deflection functions on the basis of Karman's equations. However, his formulae could not handle problems in which the in-plane displacements rather than the in-plane stresses on the boundary were prescribed because there were no explicit expressions describing the relationship between the in-plane displacements and the stress function.

In this paper, emphasis is placed on a BEM solution of the postbuckling behaviour of plates on elastic foundations in which a set of three nonlinear integral equations in three displacement functions \((U_1, U_2, W)\) is obtained. Accordingly, the limitation of the solution to the case where the in-plane stresses on the boundary of a plate are prescribed is removed. A new fundamental solution is also derived. With the aid of the boundary element formulae in incremental form obtained here, the BEM solution procedure can be greatly simplified. Finally, a number of numerical examples are presented to illustrate the proposed method.

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results are in good agreement with existing solutions. Thus the proposed method is effective for solving the problems mentioned above.

2. Basic equations and their fundamental solution

Consider a thin isotropic plate of uniform thickness $h$ and of arbitrary shape resting on an elastic foundation subjected to an external radius uniform compressive load, $p_0$ per unit length at the boundary (Fig. 1). The material constants of the plate are represented by $E$ (Young’s modulus) and $\nu$ (Poisson’s ratio). We use a Cartesian coordinate system in which the axes $x$ and $y$ lie in the plate middle plane as shown in Fig. 2. The field equations governing the postbuckling behavior can be written as follows:

\[
\begin{align*}
N_{x,x} + N_{x,y,y} &= 0 , \\
N_{y,y} + N_{y,x,x} &= 0 , \\
D\nabla^4 W + p_0 \nabla^2 W + k_0 W &= N_{x,x} W_{x,x} + 2N_{x,y} W_{x,y} + N_{y,y} W_{y,y} ,
\end{align*}
\]

where $N_x$, $N_y$ and $N_{xy}$ represent the membrane force components, $W$ is the deflection of the plate, $D = Eh^3/[12(1 - \nu^2)]$ the flexural rigidity, $k_0$ the foundation stiffness coefficient, the comma followed by a subscript indicates partial differentiation with respect to that subscript, and $\nabla^2 = (\partial_{xx} + (\partial_{yy})$.

Substituting the well known physical and geometric relations into (1) yields the following differential equations in three displacement functions ($U_1, U_2, W$):

\[
\begin{align*}
L_1 U_1 + L_2 U_2 &= P_1 , \\
L_2 U_1 + L_3 U_2 &= P_2 , \\
L_4 W &= P_3 ,
\end{align*}
\]

Fig. 1. Geometry and loading.

Fig. 2. Notation and coordinate system.
with
\[ L_1(\cdot) = (\cdot)_{xx} + d_1(\cdot)_{yy}, \quad L_2(\cdot) = d_2(\cdot)_{xy}, \]
\[ L_3(\cdot) = (\cdot)_{yy} + d_1(\cdot)_{xx}, \quad L_4(\cdot) = D(\nabla^4 + \lambda^2\nabla^2 + S)(\cdot), \]
\[ \lambda^2 = p_0/D, \quad d_1 = \frac{1}{2}(1 - \nu), \quad d_2 = \frac{1}{2}(1 + \nu), \quad S = k_0/D, \]

in which \( U_1 \) and \( U_2 \) represent the in-plane displacement components, and \( P_1, P_2, P_3 \) are the pseudo-distributed load components defined by

\[ P_1 = -W_x(W_{xx} + d_1W_{yy}) - d_2W_yW_{xy}, \]
\[ P_2 = -W_y(W_{yy} + d_1W_{xx}) - d_2W_xW_{xy}, \]
\[ P_3 = J\{((U_{1,x} + 0.5W_{xx})(W_{xx} + \nu W_{yy}) + (U_{2,y} + 0.5W_{yy})(W_{yy} + \nu W_{xx})) \]
\[ + J(1 - \nu)(U_{1,y} + U_{2,x} + W_xW_y)W_{xy}, \]

where
\[ J = \frac{Eh}{1 - \nu^2}. \]

Since an incremental formulation may have a wider applicability to higher nonlinear problems, it is necessary to express (2) in incremental form. Denoting the incremental variable by the superimposed dot and omitting the infinitesimal result from the product of incremental variables, one obtains

\[ L_1\dot{U}_1 + L_2\dot{U}_2 = \dot{P}_1, \]
\[ L_2\dot{U}_1 + L_3\dot{U}_2 = \dot{P}_2, \]
\[ L_4\dot{W} = \dot{P}_3, \]

where
\[ \dot{P}_1 = -\dot{W}_x(W_{xx} + d_1\dot{W}_{yy}) - W_x(\dot{W}_{xx} + d_1\dot{W}_{yy}) - d_2\dot{W}_yW_{xy} - d_2W_y\dot{W}_{xy}, \]
\[ \dot{P}_2 = -\dot{W}_y(W_{yy} + d_1\dot{W}_{xx}) - W_y(\dot{W}_{yy} + d_1\dot{W}_{xx}) - d_2\dot{W}_xW_{xy} - d_2W_x\dot{W}_{xy}, \]
\[ \dot{P}_3 = J\{(\dot{U}_{1,x} + W_x\dot{W}_x)(W_{xx} + \nu W_{yy}) + (U_{1,x} + 0.5W_{xx}^2)(W_{xx} + \nu \dot{W}_{yy}) \]
\[ + (\dot{U}_{2,y} + W_y\dot{W}_y)(W_{yy} + \nu W_{xx}) + (U_{2,y} + 0.5W_{yy}^2)(W_{yy} + \nu \dot{W}_{xx}) \]
\[ + J(1 - \nu)((\dot{U}_{1,y} + \dot{U}_{2,x} + \dot{W}_xW_y + \dot{W}_x\dot{W}_y)W_{xy} + (U_{1,y} + U_{2,x} + W_xW_y)\dot{W}_{xy}). \]

It follows that equations (3) are linear with respect to the incremental variables. Note that the total values at the current deformation state under consideration are assumed to be known in the incremental approach.

According to the BEM in 2-D elasticity, the fundamental solution corresponding to the first two equations of (3) is obviously Kelvin's solution (for the plane stress case) as
\[ U_{ij}^*(r) = \frac{1 + \nu}{4\pi E} \left[ (3 - \nu) \ln \left( \frac{1}{r} \right) \delta_{ij} + (1 + \nu) r_{,i} r_{,j} \right] , \]
\[ N_{ij}^*(r) = -\frac{1}{4\pi r} \left[ (1 - \nu) \delta_{ij} + 2(1 + \nu) r_{,i} r_{,j} - (1 - \nu) (r_{,i} n_j - r_{,j} n_i) \right] , \]

where all the notation is given in [3].

In the following, attention is focused on seeking the fundamental solution \( W^* \), corresponding to the third equation of (3), which is defined by

\[ D(\nabla^4 + \lambda^2 \nabla^2 + S)W^* = D(\nabla^2 + C_1)(\nabla^2 + C_2)W^* = \delta(P, Q) , \]

where \( \delta(P, Q) \) is the Dirac \( \delta \)-function, and

\[ C_1 = \frac{1}{2}(\lambda^2 - \sqrt{\lambda^4 - 4S}) , \quad C_2 = \frac{1}{2}(\lambda^2 + \sqrt{\lambda^4 - 4S}) . \]

For the sake of brevity, set

\[ (\nabla^2 + C_2)W^* = A . \]

It follows from (6) that

\[ D(\nabla^2 + C_1)A = \delta(P, Q) . \]

The solution of (9) can be easily obtained as

\[ A = N_0(\sqrt{C_1} r)/4D , \]

in which \( r \) is the distance from the source point \( P \) to the field point \( Q \) under consideration and \( N_0(\cdot) \) is the Bessel function of zeroth order of the second kind. In the same manner, we have

\[ D(\nabla^2 + C_2)B = \delta(P, Q) , \]

where

\[ (\nabla^2 + C_1)W^* = B . \]

By analogy with (9), the solution of (11) is

\[ B = N_0(\sqrt{C_2} r)/4D . \]

Therefore, subtracting (12) from (8), and according to (10) and (13), yields the fundamental solution \( W^* \):

\[ W^*(r) = \frac{A - B}{C_2 - C_1} = \frac{1}{4D(C_2 - C_1)} \left[ N_0(\sqrt{C_1} r) - N_0(\sqrt{C_2} r) \right] . \]

In the absence of elastic foundation \( (k_0 = C_1 = 0) \), the solution of (9) reduces to
Thus the fundamental solution of the postbuckling plates without elastic foundation becomes
\[ W^* = \frac{2 \ln r}{\pi} - \frac{N_0(\lambda r)}{4DC_2}. \]

3. Integral equation formulation

By virtue of Betti's reciprocal theorem, the integral equations corresponding to the postbuckling problems of elastic plates can be obtained in the following form:

\[
\begin{align*}
C(\mathbf{P})\dot{U}_i(\mathbf{P}) - & \int_{\Gamma} \left[ \dot{N}_k(Q)U^*_{ik}(P, Q) - \dot{U}_k(Q)N^*_{ik}(P, Q) \right] d\Gamma(Q) \\
= & \int_{\Omega} P_k(Q)U^*_{ik}(P, Q) d\Omega(Q), \\
\dot{C}(P)\dot{W}(P) + & \int_{\Gamma} \left[ V^*_n(P, Q)\dot{W}(Q) - M^*_n(P, Q)\dot{\theta}_n(Q) - \dot{V}_n(Q)W^*(P, Q) \\
& + \dot{M}_n(Q)\theta^*_n(P, Q) \right] d\Gamma(Q) \\
+ & \sum_{j=1}^{K} [\Delta M^*_n(P, Q_j)\dot{W}(Q_j) - \Delta \dot{M}_n(Q_j)W^*(P, Q_j)] = \int_{\Omega} P_3(Q)W^*(P, Q) d\Omega(Q), \\
\dot{C}(P)\dot{W}_{n_0}(P) + & \int_{\Gamma} \left[ V^*_{n, n_0}(P, Q)\dot{W}(Q) - M^*_{n, n_0}(P, Q)\dot{\theta}_{n_0}(Q) \\
& - \dot{V}_n(Q)W^*_{n_0}(P, Q) + \dot{M}_n(Q)\theta^*_{n_0}(P, Q) \right] d\Gamma(Q) \\
+ & \sum_{j=1}^{K} [\Delta M^*_n(P, Q_j)\dot{W}_n(Q_j) - \Delta \dot{M}_n(Q_j)W^*_{n_0}(P, Q_j)] \\
= & \int_{\Omega} P_3(Q)W^*_{n_0}(P, Q) d\Omega(Q),
\end{align*}
\]

where the repeated index \( k \) implies the Einstein summation convention with \( k \in \{1, 2\} \), \( n \) is the outward normal at the boundary \( \Gamma \), \( n_0 \) the outward normal at the source point on \( \Gamma \), \( \Delta(\cdot)_{Q_j} = (\cdot)_{Q_j^+} - (\cdot)_{Q_j^-} \) is the discontinuity jump at the corner point \( Q_j \), \( (\cdot)_{Q_j^-} \) and \( (\cdot)_{Q_j^+} \) stand for the quantities before and after the corner \( Q_j \), respectively, and \( K \) is the number of all the corners. According to the thin plate theory, the bending moment \( M^*_n \) and the equivalent shear force \( V^*_n \) corresponding to the fundamental solution \( W^* \) can be evaluated through the high order derivatives of \( W^* \). The details can be found in [4].

In order to obtain a weak solution of (16), as in the usual boundary element method, the boundary \( \Gamma \) and the domain \( \Omega \) of the plate are divided into a series of boundary elements and internal cells, respectively. After performing the discretization by use of various kinds of
boundary elements (e.g. constant element, linear element or high-order element), the boundary integral equations (16) become a set of linear algebraic equations including the incremental boundary variables $\dot{U}_1, \dot{U}_2, \dot{N}_1, \dot{N}_2, \dot{W}, \dot{\theta}, \dot{M}, \dot{V}$ for each loading step:

$$
\begin{bmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{bmatrix} \{\dot{U}\} - \begin{bmatrix}
Q_{11} & Q_{12} \\
Q_{21} & Q_{22}
\end{bmatrix} \{\dot{N}\} = \{\dot{R}_1\},
$$
\tag{17a}

$$
\begin{bmatrix}
H_{11} & H_{12} & H_{13} \\
H_{21} & H_{22} & H_{23}
\end{bmatrix} \{\dot{\Delta}\} - \begin{bmatrix}
G_{11} & G_{12} & G_{13} \\
G_{21} & G_{22} & G_{23}
\end{bmatrix} \{\dot{T}\} = \{\dot{R}_2\}.
$$
\tag{17b}

Taking the constant element as an example, we have

$$
\{\dot{U}\} = \{\dot{U}_1^{(1)} \dot{U}_1^{(2)} \ldots \dot{U}_1^{(g)} \dot{U}_2^{(1)} \ldots \dot{U}_2^{(g)}\}^t,
$$

$$
\{\dot{N}\} = \{\dot{N}_1^{(1)} \dot{N}_1^{(2)} \ldots \dot{N}_1^{(g)} \dot{N}_2^{(1)} \ldots \dot{N}_2^{(g)}\}^t,
$$

$$
\{\dot{R}_1\} = \{\dot{R}_{11}^{(1)} \dot{R}_{12}^{(1)} \ldots \dot{R}_{11}^{(g)} \dot{R}_{12}^{(g)}\}^t,
$$

$$
\{\dot{R}_2\} = \{\dot{R}_{21} \dot{R}_{22}\}^t,
$$

$$
\{\dot{\Delta}\} = \{\dot{W} \dot{\theta} \dot{W}_c\}^t,
$$

$$
\{\dot{T}\} = \{\dot{V} \Delta \dot{M}_{nt}\}^t,
$$

$$
\{\dot{W}\} = \{\dot{W}_1^{(1)} \dot{W}_1^{(2)} \ldots \dot{W}_1^{(g)}\}^t,
$$

$$
\{\dot{\theta}\} = \{\dot{\theta}_1^{(1)} \dot{\theta}_1^{(2)} \ldots \dot{\theta}_1^{(g)}\}^t,
$$

in which $\{\dot{W}_c\}$ denotes a corner deflection vector, while $\{\Delta \dot{M}_{nt}\}$ is the corresponding corner force vector, and

$$
\{\dot{V}\} = \{\dot{V}_n^{(1)} \dot{V}_n^{(2)} \ldots \dot{V}_n^{(g)}\}^t,
$$

$$
\{\dot{M}\} = \{\dot{M}_n^{(1)} \dot{M}_n^{(2)} \ldots \dot{M}_n^{(g)}\}^t,
$$

where the superscript $(i)$ means the element number, and $g$ is the total of the constant elements. In addition, the elements of $S_{km}, Q_{km}, R_{km} (k, m = 1, 2)$ and $H_{kt}, G_{kt} (t = 1, 2, 3)$ are

$$
(S_{km})_{ij} = \int_{\Gamma} N_{km}^*(P_i, Q) d\Gamma(Q) + C(P_i) \delta_{ij} \delta_{km} \quad (i, j = 1, 2, \ldots, g),
$$

$$
(Q_{km})_{ij} = \int_{\Gamma} U_{km}^*(P_i, Q) d\Gamma(Q),
$$

$$
(\dot{R}_{ik})_i = \int_{\Omega} \dot{P}_m U_{km}^*(P_i, Q) d\Omega(Q),
$$

$$
(\dot{R}_{21})_i = \int_{\Omega} \dot{P}_m W_{km}^*(P_i, Q) d\Omega(Q),
$$

$$
(\dot{R}_{22})_i = \int_{\Omega} \dot{P}_m W_{n0}^*(P_i, Q) d\Omega(Q),
$$

$$
(H_{11})_{ij} = \int_{\Gamma} V_{ik}^*(P_i, Q) d\Gamma(Q) + \bar{C}(P_i) \delta_{ij},
$$
\( (H_{12})_{ij} = - \int_{r_j} M^*(P_i, Q) \, d\Gamma(Q), \)

\( (H_{13})_{ij} = \Delta M^*_n(P_i, Q) \quad (f = 1, 2, \ldots, K), \)

\( (H_{21})_{ij} = \int_{r_j} V^*_{n,0}(P_i, Q) \, d\Gamma(Q), \)

\( (H_{22})_{ij} = - \int_{r_j} M^*_{n,0}(P_i, Q) \, d\Gamma(Q) + \tilde{c}(P_i) \delta_{ij}, \)

\( (H_{23})_{ij} = \Delta M^*_{n,0}(P_i, Q), \)

\( (G_{11})_{ij} = - \int_{r_j} W^*(P_i, Q) \, d\Gamma(Q), \)

\( (G_{12})_{ij} = \int_{r_j} \theta^*_n(P_i, Q) \, d\Gamma(Q), \)

\( (G_{13})_{ij} = - W^*(P_i, Q), \)

\( (G_{21})_{ij} = - \int_{r_j} W^*_{n,0}(P_i, Q) \, d\Gamma(Q), \)

\( (G_{22})_{ij} = \int_{r_j} \theta^*_{n,0}(P_i, Q) \, d\Gamma(Q), \)

\( (G_{23})_{ij} = - W^*_{n,0}(P_i, Q). \)

It is noted that the corner variables of \( \{\Delta M_n\} \) and \( \{\tilde{W}_c\} \) can be solved by using the corner conditions, a priori, and then expressed in terms of other unknowns.

It can be seen from (4) that \( \bar{P}_1 \) and \( \bar{P}_2 \) depend only upon \( \bar{W} \). Therefore as long as the value of \( \bar{W} \) in the domain \( \Omega \) is known a priori, we can compute the pseudo-loading vector \( \{\bar{R}_1\} \). After applying the boundary conditions and reordering (17a,b) we obtain

\[
\begin{bmatrix}
E_{11} & E_{12} \\
E_{21} & E_{22}
\end{bmatrix}
\begin{bmatrix}
\dot{U} \\
\dot{N}
\end{bmatrix} = \{\dot{R}_3\},
\]

\( (17c) \)

\[
\begin{bmatrix}
F_{11} & F_{12} \\
F_{21} & F_{22}
\end{bmatrix}
\begin{bmatrix}
\dot{W} \\
\dot{M}
\end{bmatrix} = \{\dot{R}_4\},
\]

\( (17d) \)

which can also be written in a more compact form as

\[
[\dot{E}](\dot{X}) = \{\dot{R}_3\},
\]

\( (18a) \)

\[
[\dot{F}](\dot{Y}) = \{\dot{R}_4\},
\]

\( (18b) \)

where \( \{\dot{X}\} = \{\dot{U} \quad \dot{N}\} \) contains the unknown in-plane displacements and tractions, and \( \{\dot{Y}\} = \{\dot{W} \quad \dot{M}\} \) the unknown bending deflections and internal forces.
4. Iterative solution

By virtue of the property of (17a,b) mentioned above, a reasonable iterative solution scheme is developed. In the process of iteration, only an initial value of lateral deflection of the plate is required because \{\dot{R}_1\} in (17a) and \{\dot{R}_3\} in (18a) are functions independent of \dot{U}_1 and \dot{U}_2. Suppose that \dot{U}_1^{(k)}, \dot{U}_2^{(k)} and \dot{W}^{(k)} express the kth approximations which can be obtained from the preceding cycle of iteration. For the purpose of the \((k+1)\)th solution, the iterative procedure is illustrated as follows:

1. Assume the initial value \dot{W}_0 in \Omega.
2. Calculate \{\dot{R}_1\} on the right-hand side of (17a) by means of (4) and (6).
3. Calculate \{\dot{R}_3\} on the right-hand side of (18a) by using the given boundary conditions.
4. Solve (18a) for the boundary unknown vector \{\dot{X}\} and then determine the values of \dot{U}_1 and \dot{U}_2 in \Omega.
5. Calculate \{\dot{R}_4\} on the right-hand side of (18b) by means of (4), (16) and (17b) and the given boundary conditions.
6. Solve (18b) for the boundary unknown vector \{\dot{Y}\} and then determine the values of \dot{W} in \Omega.
7. If \left[\frac{\dot{W}(i) - \dot{W}_0(i)}{\dot{W}(i)}\right]_{\max} \leq \varepsilon (\varepsilon is a convergence tolerance), proceed to the next loading step and \dot{W}^{(k+1)} = \dot{W}^{(k)} + \dot{W}. Otherwise, modify the initial value \dot{W}_0 and then continue the iteration.

It is important to note that once the matrices \[E\] and \[F\] in (18a,b) have been formed, they can be stored in the core and used in each cycle of iteration without any change. That is because these matrices depend only upon the geometric and material parameters of the plates. Obviously, this can save a large amount of computing time.

5. Numerical examples

In order to illustrate the efficiency and feasibility of the proposed method, two examples of circular plates with various boundary conditions resting on an elastic foundation subjected to an external radial uniform compressive load at the boundary are considered. The geometric and material parameters are

\[E = 2 \times 10^6 \text{ kg/cm}^2, \quad \nu = 0.3, \quad h = 1 \text{ cm}, \quad R = 50 \text{ cm}, \quad k = 10D/R^4,\]

where \(R\) is the radius of a plate.

In these examples, we divided the boundary of each plate into 20 constant elements and the load step \(p_0\) is taken as 0.02\(p_{cr}\) (\(p_{cr}\) is the linear buckling load of the plate under consideration). The convergence tolerance \(\varepsilon = 0.0001\).

**EXAMPLE 1: Simply supported circular plate.** In this case \(p_{cr} = 5.92D/R^2\). The numerical results describing the relationship between the maximum deflection \(W_{\max}\) occurring at the center and the compressive load \(p_0 (p_0 > p_{cr})\) are shown in Table 1. Comparison is made with a finite element method solution [5].
Table 1
Postbuckling path for a circular plate on an elastic foundation with simply supported boundary

<table>
<thead>
<tr>
<th>$W_{max}/h$ for $P_0/P_{cr}$ =</th>
<th>1.02</th>
<th>1.04</th>
<th>1.06</th>
<th>1.08</th>
<th>1.10</th>
</tr>
</thead>
<tbody>
<tr>
<td>BEM</td>
<td>0</td>
<td>0.3512</td>
<td>0.4636</td>
<td>0.5710</td>
<td>0.6612</td>
</tr>
<tr>
<td>FEM [5]</td>
<td>0</td>
<td>0.3443</td>
<td>0.4590</td>
<td>0.5627</td>
<td>0.6503</td>
</tr>
</tbody>
</table>

Table 2
Postbuckling path for a circular plate on an elastic foundation with clamped boundary

<table>
<thead>
<tr>
<th>$W_{max}/h$ for $P_0/P_{cr}$ =</th>
<th>1.02</th>
<th>1.04</th>
<th>1.06</th>
<th>1.08</th>
<th>1.10</th>
</tr>
</thead>
<tbody>
<tr>
<td>BEM</td>
<td>0</td>
<td>0.3314</td>
<td>0.4698</td>
<td>0.5782</td>
<td>0.6677</td>
</tr>
<tr>
<td>FEM [5]</td>
<td>0</td>
<td>0.3260</td>
<td>0.4618</td>
<td>0.5665</td>
<td>0.6552</td>
</tr>
</tbody>
</table>

EXAMPLE 2: Clamped circular plate. In this case $P_{cr} = 16.04D/R^2$. The numerical results obtained and the FEM solution [5] are shown in Table 2 for comparison.

It can be seen from the results that the maximum error is less than 3%. Due to the fact that the results given in [5] are presented in the form of empirical formulae for the radial load ratio and central deflection, the solution obtained in this paper gives more accurate results.

6. Concluding remarks

The achievement of the fundamental solution $W^*$ presented here makes it possible to shorten the time for calculating the pseudo-distributed normal load, and in our iterative calculations only an initial value of lateral deflection is required. Two sample computations have shown that the postbuckling behavior of plates on an elastic foundation can be successfully investigated by the proposed method. Therefore, this method appears to be very promising.

References