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EXACT SOLUTION OF COUPLED HEAT AND MASS TRANSFER WITH DOUBLE MOVING INTERFACES IN A POROUS HALF-SPACE

PART 1: MASS TRANSFER CONTROLLED BY THE FICK LAW

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SUMMARY

Coupled heat and mass transfer with double moving interfaces, taking place in a porous half-space, is defined, and exact solutions for the temperature and moisture distributions as well as the positions of the two moving fronts are obtained. The effects of the ratio of the thermal diffusivity in the sublimated region to that in the frozen region, the ratio of the steady heat flux in the frozen region to that in the sublimated region with the same heat conduction distance, the ratio of the steady heat flux in the desorbed region to that in the sublimated region, and the volume fractions of the free water and absorbed water in the pore of the porous body on the sublimation and desorption are analysed. Conclusions are drawn on the basis of the results.

KEY WORDS Double moving fronts Sublimation Desorption Porous media Fick law

INTRODUCTION

Coupled heat and mass transfer with a moving interface may be found in the freeze-drying process, which has wide applications in separation processes and in food technology. The primary object of freeze-drying is to preserve biological materials without injury by freezing the contained water and then removing the ice by sublimation. Thus, one can expect to combine the advantages of both freezing and desiccation to obtain a more propitious state of preservation. Systematic freeze-drying requires several successive steps as follows

(i) *Prefreezing*: The materials are prepared and frozen at a very low temperature.
(ii) *Primary drying, or sublimation drying*: The ice crystals formed on freezing are sublimated by vigorous and gentle heating, usually under vacuum.
(iii) *Secondary drying, or desorption drying*: With the disappearance of the ice, the residual moisture is desorbed at around ambient temperature under high vacuum.

Considerable research on sublimation drying has been made over the past few decades. Heat transfer was studied by Dyer and Sunderland (1967), and the transient temperature distribution during sublimation dehydration was developed. Later, mass transfer and the role of convection were also considered (Dyer and Sunderland, 1968). McCulloch and Sunderland (1970) applied integral techniques to sublimation drying with radiation boundary conditions, and quasisteady solutions of freezing-drying of slabs, cylinders and spheres were also developed by Dyer and Sunderland (1971). The same problem was investigated by Cho and

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Sunderland (1970). None of these researches, however, considered the interaction of heat and mass transfer. Hill and Sunderland (1971) developed the quasisteady solution of coupled heat and mass transfer in sublimation dehydration in the continuum, transition and free molecule flow regions. Some experimental results were presented by Katayama and Hayashi (1972).

Mathematical formulation of the heat and mass transfer in capillary porous bodies has been established by Luikov (1975). To solve the problem of evaporation of liquid moisture from a porous medium, Gupta (1974) made an assumption that the effect of the temperature gradient on the mass transfer is small and may be neglected. His approximate solution of the problem includes the effect of the moisture variation on the heat transfer. An exact solution of the same problem described by Gupta (1974) was later given by Cho (1975). Another exact solution of the problem, without Gupta's assumption and with two more complicated models, was presented by Mikhailov (1975). For the problem of freezing (desublimation) of a humid porous half-space, Mikhailov (1976) also presented an exact solution. An exact solution was developed by Boles and Özisik (1983) for freezing in cylindrically symmetric porous moist media. Some recent development has been done on the coupled heat and mass transfer in sublimation dehydration by Lin (1981, 1982) and by Fey and Boles (1987, 1988). Lin (1981, 1982) presented the exact solution of coupled heat and mass transfer of the sublimation and desublimation problems in a porous medium, assuming that mass transfer was controlled by the Fick law alone. Then he extended his solution to the case with unknown temperature and vapour concentration at the moving sublimation front. Fey and Boles (1987) analysed the effect of convection heat transfer on the sublimation, and found that convection heat transfer is significant only when the combination of boundary conditions, initial condition and thermal properties cause the rate of sublimation without convection to be very high. Fey and Boles (1988) also presented an analytical study of vacuum sublimation in an initially partially filled frozen porous medium with recondensation.

Yao (1988) studied the heat and mass transfer in freeze-drying by the use of an integral method, and the effect of convection in the tunnel between the heating plate and the surface of the product on sublimation was considered for several cases. Peng and Cheng (1990) and Peng (1991) applied the perturbation method to the investigation of heat and mass transfer in freeze-drying, and proposed that the perturbation of the moving boundary conditions must be made incorporating a Taylor expansion. Peng (1991) also studied the freeze-drying problem in a corner, and applied the exact solution of one-dimensional freeze-drying to build the approximate two-dimensional solution directly. The effect of vacuum pressure on the rate of freeze-drying was studied experimentally by Peng and Cheng (1990) and by Peng (1991).

Compared with sublimation, there has been relatively little research on desorption. King (1968) considered that desorption in a porous dry semi-infinite solid involves moisture diffusion in the vapour phase and possibly surface diffusion across moisture adsorption sites as well. The interaction of desorption with heat transfer must also be considered, because of the large heat sorption and the likelihood of heat transfer control. Most authors have derived the theory of desorption based on Fick's law. Yao (1988) developed some solutions for the constant effective diffusivity of the vapour in the dry product.

All the previous researches on freeze-drying considered the primary drying and secondary drying as separate, i.e. the desorption drying takes place only after the sublimation drying comes to the end. For sublimation drying, there is a maximum safe surface temperature, which varies according to properties of the product and insures the dried region against being scorched or denatured, and the frozen region against melting. The ordinary freeze-drying process is designed to raise the temperature for desorption to a higher level, accompanied by a high vacuum after the sublimation ends. However, there are instances where the maximum safe surface temperature is higher than the temperature at which the desorption drying takes place, such as freeze-drying for some rigid products. These instances also exist in accelerated freeze-drying (AFD) techniques (see Mellor, 1978, p. 73). The faster drying obtained outweighed the disadvantages of high product surface temperature (up to 60 °C). In this case, not only sublimation drying, but also desorption drying takes place.

In what follows, coupled heat and mass transfer with both sublimation and desorption in a porous medium is studied, an analytical model of the sublimation and desorption processes taking place in a porous half-space is defined, and exact solutions for temperature and moisture distributions, as well as the positions of moving sublimation and desorption fronts, are obtained.

STATEMENT OF THE PROBLEM

We consider a rigid porous half-space containing uniformly frozen moisture. The porous body exists in an environment having a very low pressure, so that there is practically only the vapour pressure of the moisture acting on the frozen phase. It is therefore assumed that, during the sublimation process, the vapour pressure acting on the frozen phase is equal to the environmental pressure, which remains unchanged. Under this assumption, the sublimation process will take place at a fixed sublimation temperature corresponding to the vapour pressure acting on the frozen phase (Lin, 1981). At a temperature higher than the desorption temperature, not only sublimation drying but also desorption drying takes place. Furthermore, it is assumed that desorption drying is present at a certain temperature; as a result, following the sublimation moving interface, a desorption moving interface appears. The whole porous medium is divided into three parts by these two interfaces, namely the frozen region, the sublimated region, and the desorbed region.

For the purpose of formulation of the freeze-drying problem as shown in Figure 1, the following assumptions are made:

- (i) The vapour pressure of the moisture in the sublimation process is very low, so that the vapour can be considered as an ideal gas.
- (ii) The frozen moisture is uniformly distributed in the porous half-space, which is initially at a uniform moisture mass concentration C_i.
- (iii) The frozen humid body is initially at a uniform temperature T_0, which is lower than the sublimation temperature T_{v1}. The sublimation process takes place when the temperature of the frozen body reaches the sublimation temperature T_{v1}. Similarly, the desorption process takes place when the temperature of the sublimated body reaches the desorption temperature T_{v2}. The sublimation temperature T_{v1} and the desorption temperature T_{v2} are taken to be known constants.
- (iv) The frozen porous body is dried by maintaining the surface at $x = 0$ at a constant moisture mass concentration C_s, which is lower than the initial concentration, and at a constant temperature T_s, which is higher than the sublimation and desorption temperatures. Therefore there are two moving interfaces in the porous body, namely the sublimation moving interface and the desorption moving interface. The whole porous body is divided into three parts, namely the frozen region, the sublimated region and the desorbed region. The concentrations at the interfaces are taken to be unknown constants, but they vary according to the interfaces.

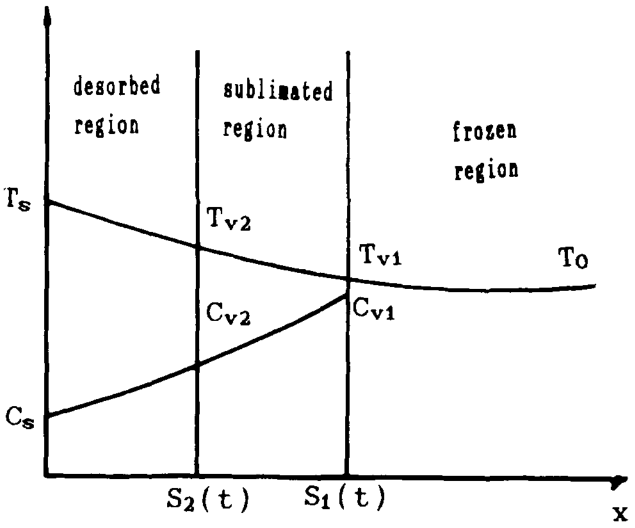

Figure 1. Physical model

(v) In the frozen region, $S_1(t) < x < \infty$, and there is no moisture movement. In the sublimated region $S_2(t) < x < S_1(t)$, and in the desorbed region $0 < x < S_2(t)$; there are heat and moisture flows, where $S_1(t)$ locates the sublimation front and $S_2(t)$ locates the desorption front.
(vi) The convective terms in the dried regions are small and may be neglected.
(vii) The thermophysical properties of each phase remain constant, but may differ for different phases.
(viii) The Soret effect, or the thermal diffusion, gives rise to a mass flux which is normally very small relative to the normal Fickian flux, and may be neglected.

Based on the equation system of Luikov (1975), the three-regions problem with two moving interfaces can be formulated by the following differential equations:

$$\partial T_1(x, t)/\partial t = \alpha_1 \partial^2 T_1(x, t)/\partial x^2 \qquad (S_1(t) < x < \infty) \tag{1}$$

$$\partial T_2(x, t)/\partial t = \alpha_2 \partial^2 T_2(x, t)/\partial x^2 \qquad (S_2(t) < x < S_1(t)) \tag{2}$$

$$\partial C_2(x, t)/\partial t = D_2 \partial^2 C_2(x, t)/\partial x^2 \qquad (S_2(t) < x < S_1(t)) \tag{3}$$

$$\partial T_3(x, t)/\partial t = \alpha_3 \partial^2 T_3(x, t)/\partial x^2 \qquad (0 < x < S_2(t)) \tag{4}$$

$$\partial C_3(x, t)/\partial t = D_3 \partial^2 C_3(x, t)/\partial x^2 \qquad (0 < x < S_2(t)) \tag{5}$$

where equation (1) represents the heat transfer in the frozen region, equations (2) and (3) represent the heat and mass transfer in the sublimated region, and equations (4) and (5) represent the heat and mass transfer in the desorbed region. The thermal diffusivities α_1, α_2 and α_3 and the mass diffusivities D_2 and D_3 take the effective values of the porous body, which can be obtained by calculation or experiment.

The initial and boundary conditions can be described as

$$T_1(x, 0) = T_0 \tag{6}$$

$$T_3(0, t) = T_s, \; C_3(0, t) = C_s \tag{7}$$

$$T_1(\infty, t) = T_0 \tag{8}$$

On the sublimation front, the matching condition can be expressed as

$$T_1(S_1, t) = T_2(S_1, t) = T_{v1} \tag{9}$$

$$C_2(S_1, t) = C_{v1} \tag{10}$$

$$-k_2 \frac{\partial T_2(S_1, t)}{\partial x} + k_1 \frac{\partial T_1(S_1, t)}{\partial x} = \omega C_i L \frac{\mathrm{d}S_1(t)}{\mathrm{d}t} \tag{11}$$

$$D_2 \frac{\partial C_2(S_1, t)}{\partial x} = \omega C_i \frac{\mathrm{d}S_1(t)}{\mathrm{d}t} \tag{12}$$

On the desorption front, the matching condition can be expressed as

$$T_2(S_2, t) = T_3(S_2, t) = T_{v2} \tag{13}$$

$$C_2(S_2, t) = C_3(S_2, t) = C_{v2} \tag{14}$$

$$-k_3 \frac{\partial T_3(S_2, t)}{\partial x} + k_2 \frac{\partial T_2(S_2, t)}{\partial x} = (\varepsilon - \omega)\rho \Delta H \frac{\mathrm{d}S_2(t)}{\mathrm{d}t} \tag{15}$$

$$D_3 \frac{\partial C_3(S_2, t)}{\partial x} - D_2 \frac{\partial C_2(S_2, t)}{\partial x} = (\varepsilon - \omega)\rho \frac{\mathrm{d}S_2(t)}{\mathrm{d}t} \tag{16}$$

where C_i is the density of ice in the pores of a frozen porous body, ρ is the density of absorbed water on the wall of a pore, L is the latent heat of sublimation, and ΔH is the absorption heat. Equations (1)–(5), together with the boundary conditions, equations (6)–(16), constitute the completed expression of the three-regions problem.

SOLUTION

The system of equations describing the temperature and concentration distributions in the frozen, sublimated and absorbed regions in the drying process is similar to that describing the temperature distribution in the solid and liquid regions in a melting process for a pure substance. Therefore, referring to the Neumann solution of the Stefan problem (Özisik, 1980), the following similarity transformation can be introduced:

$$\eta = x/2\sqrt{\alpha_2 t} \tag{17}$$

let

$$S_1(t) = 2\lambda_1\sqrt{\alpha_2 t} \tag{18}$$

$$S_2(t) = 2\lambda_2\sqrt{\alpha_2 t} \tag{19}$$

thereby, the partial differential equations can be transformed into the following ordinary differential equations:

$$\frac{d^2T_1(\eta)}{d\eta^2} + 2\eta\frac{dT_1(\eta)}{d\eta}\frac{\alpha_2}{\alpha_1} = 0 \qquad (\lambda_1 < \eta < +\infty) \tag{20}$$

$$\frac{d^2T_2(\eta)}{d\eta^2} + 2\eta\frac{dT_2(\eta)}{d\eta} = 0 \qquad (\lambda_2 < \eta < \lambda_1) \tag{21}$$

$$\frac{d^2C_2(\eta)}{d\eta^2} + 2\eta\frac{dC_2(\eta)}{d\eta}\frac{\alpha_2}{D_2} = 0 \qquad (\lambda_2 < \eta < \lambda_1) \tag{22}$$

$$\frac{d^2T_3(\eta)}{d\eta^2} + 2\eta\frac{dT_3(\eta)}{d\eta}\frac{\alpha_2}{\alpha_3} = 0 \qquad (0 < \eta < \lambda_2) \tag{23}$$

$$\frac{d^2C_3(\eta)}{d\eta^2} + 2\eta\frac{dC_3(\eta)}{d\eta}\frac{\alpha_2}{D_3} = 0 \qquad (0 < \eta < \lambda_2) \tag{24}$$

the initial and boundary conditions become

$$T_1(\eta = \infty) = T_0 \tag{25}$$

$$T_3(\eta = 0) = T_s \tag{26}$$

$$C_3(\eta = 0) = C_s \tag{27}$$

on the sublimation front, the matching conditions become

$$T_1(\eta = \lambda_1) = T_2(\eta = \lambda_1) = T_{v1} \tag{28}$$

$$C_2(\eta = \lambda_1) = C_{v1} \tag{29}$$

$$-k_2\frac{dT_2(\lambda_1)}{d\eta} + k_1\frac{dT_1(\lambda_1)}{d\eta} = 2\omega C_i L\alpha_2\lambda_1 \tag{30}$$

$$D_2\frac{dC_2(\lambda_1)}{d\eta} = 2\omega C_i\alpha_2\lambda_1 \tag{31}$$

On the desorption front, the matching conditions become

$$T_2(\eta = \lambda_2) = T_3(\eta = \lambda_2) = T_{v2} \tag{32}$$

$$C_2(\eta = \lambda_2) = C_3(\eta = \lambda_2) = C_{v2} \tag{33}$$

$$-k_3\frac{dT_3(\lambda_2)}{d\eta} + k_2\frac{dT_2(\lambda_2)}{d\eta} = 2(\varepsilon - \omega)\rho\Delta H\alpha_2\lambda_2 \tag{34}$$

$$D_3\frac{dC_3(\lambda_2)}{d\eta} - D_2\frac{dC_2(\lambda_2)}{d\eta} = 2(\varepsilon - \omega)\rho\alpha_2\lambda_2 \tag{35}$$

The solution of equations (20)–(24) which satisfies the initial and boundary conditions, equations (25)–(29) and (32)–(33), can be obtained as

$$T_1(\eta) = T_0 + (T_{v1} - T_0)\frac{\operatorname{erfc}(\eta\sqrt{\alpha_2/\alpha_1})}{\operatorname{erfc}(\lambda_1\sqrt{\alpha_2/\alpha_1})} \tag{36}$$

$$T_2(\eta) = T_{v1} + (T_{v2} - T_{v1})\frac{\operatorname{erf}(\eta) - \operatorname{erf}(\lambda_1)}{\operatorname{erf}(\lambda_2) - \operatorname{erf}(\lambda_1)} \tag{37}$$

$$T_3(\eta) = T_s + (T_{v2} - T_s)\frac{\operatorname{erf}(\eta\sqrt{\alpha_2/\alpha_3})}{\operatorname{erf}(\lambda_2\sqrt{\alpha_2/\alpha_3})} \tag{38}$$

$$C_2(\eta) = C_{v2} + (C_{v1} - C_{v2})\frac{\operatorname{erf}(\lambda_2\sqrt{\alpha_2/D_2}) - \operatorname{erf}(\eta\sqrt{\alpha_2/D_2})}{\operatorname{erf}(\lambda_2\sqrt{\alpha_2/D_2}) - \operatorname{erf}(\lambda_1\sqrt{\alpha_2/D_2})} \tag{39}$$

$$C_3(\eta) = C_s + (C_{v2} - C_s)\frac{\operatorname{erf}(\eta\sqrt{\alpha_2/D_3})}{\operatorname{erf}(\lambda_2\sqrt{\alpha_2/D_3})} \tag{40}$$

where there are four unknown parameters λ_1, λ_2, C_{v1} and C_{v2} to be determined. The determination of λ_1 and λ_2 may be achieved by substituting the temperature distribution equations (36), (37) and (39) into the heat balance equations (30) and (34) at the sublimation and the desorption front. Then the following equations can be derived:

$$\frac{e^{-\lambda_1^2}}{\operatorname{erf}(\lambda_1) - \operatorname{erf}(\lambda_2)} - \sqrt{\frac{\alpha_2}{\alpha_1}}\,\frac{k_1(T_{v1} - T_0)}{k_2(T_{v2} - T_{v1})}\,\frac{e^{-\lambda_1^2\alpha_2/\alpha_1}}{\operatorname{erfc}(\lambda_1\sqrt{(\alpha_2/\alpha_1)})} = \frac{\sqrt{\pi}\,\omega C_i L\alpha_2\lambda_1}{k_2(T_{v2} - T_{v1})} \tag{41}$$

$$\frac{-e^{-\lambda_2^2}}{\operatorname{erf}(\lambda_1) - \operatorname{erf}(\lambda_2)} + \sqrt{\frac{\alpha_2}{\alpha_3}}\,\frac{k_3(T_s - T_{v2})}{k_2(T_{v2} - T_{v1})}\,\frac{e^{-\lambda_2^2\alpha_2/\alpha_3}}{\operatorname{erf}(\lambda_2\sqrt{(\alpha_2/\alpha_3)})} = \frac{\sqrt{\pi}(\varepsilon - \omega)\rho\Delta H\alpha_2\lambda_2}{k_2(T_{v2} - T_{v1})} \tag{42}$$

the interface parameters λ_1 and λ_2 can be determined by the above coupled nonlinear algebraical equations. Then, on substituting the mass concentration distribution equations (39) and (40) into the moisture balance

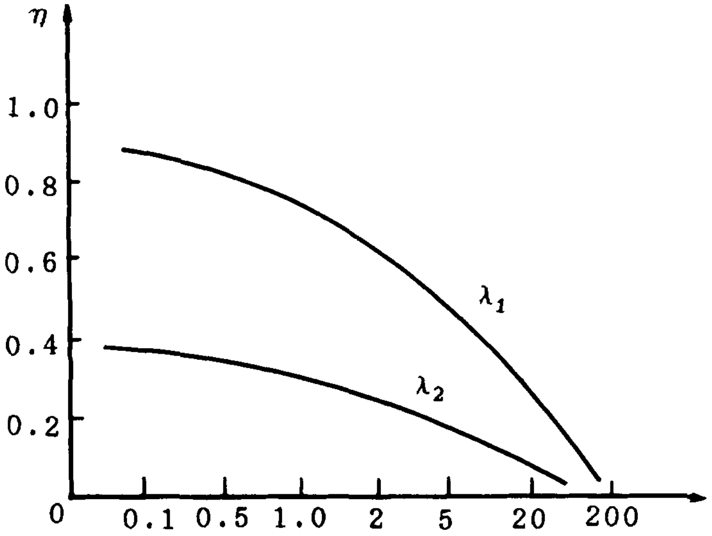

Figure 2. Distribution of three regions: $kT_1 = kT_3 = 1$; $\omega = 0{\cdot}7$; $\alpha_{21} = 0{\cdot}4$; $\alpha_{23} = 1$; $\varepsilon = 0{\cdot}8$

equations (31) and (35) at the sublimation and desorption fronts, C_{v1} and C_{v2} can be determined from the following coupled nonlinear algebraical equations:

$$\frac{e^{-\lambda_1^2\alpha_2/D_2}}{\operatorname{erf}(\lambda_1\sqrt{(\alpha_2/D_2)})-\operatorname{erf}(\lambda_2\sqrt{(\alpha_2/D_2)})}=\frac{\sqrt{\pi}\,\omega C_i\alpha_2\lambda_1}{D_2(C_{v1}-C_{v2})}\sqrt{\frac{D_2}{\alpha_2}} \tag{43}$$

$$\frac{-e^{-\lambda_2^2\alpha_2/D_2}}{\operatorname{erf}(\lambda_1\sqrt{(\alpha_2/D_2)})-\operatorname{erf}(\lambda_2\sqrt{(\alpha_2/D_2)})}\sqrt{\frac{\alpha_2}{D_2}}+\sqrt{\frac{\alpha_2}{D_3}}\,\frac{D_3(C_{v2}-C_s)}{D_2(C_{v1}-C_{v2})}\,\frac{e^{-\lambda_2^2\alpha_2/D_3}}{\operatorname{erf}(\lambda_2\sqrt{(\alpha_2/D_3)})}=\frac{\sqrt{\pi}(\varepsilon-\omega)\rho\lambda_2\alpha_2}{D_2(C_{v1}-C_{v2})} \tag{44}$$

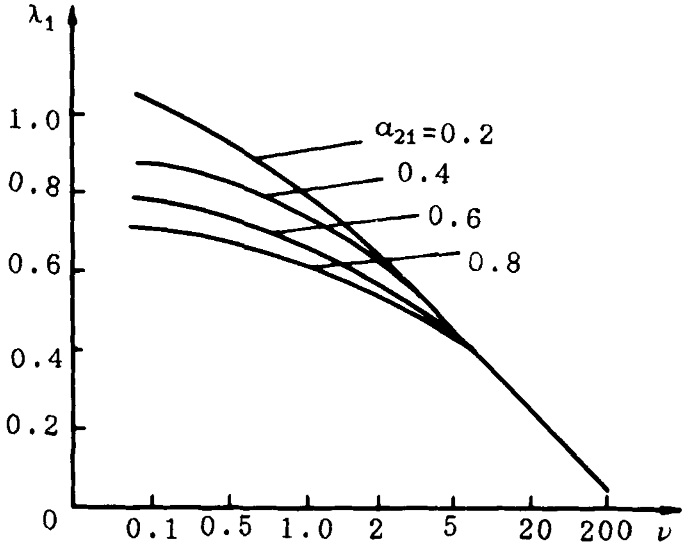

Figure 3. Effect of α_{21} on sublimation: $kT_1 = kT_3 = 1$; $\omega = 0{\cdot}7$; $\alpha_{23} = 1$; $\varepsilon = 0{\cdot}8$

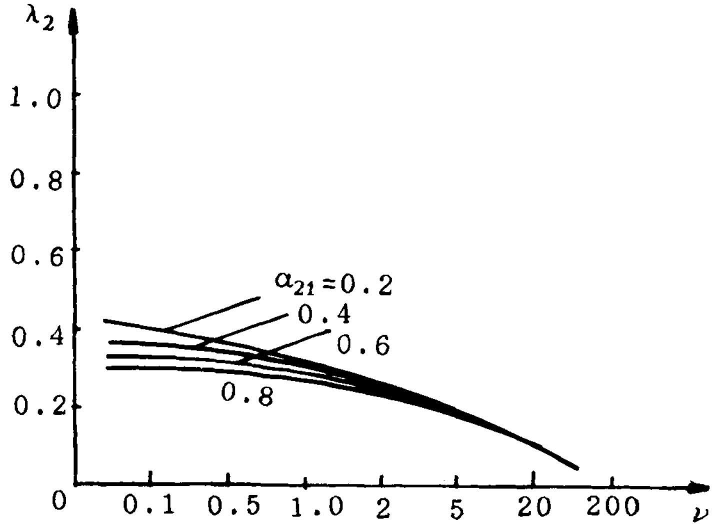

Figure 4. Effect of α_{21} on desorption: $kT_1 = kT_3 = 1$; $\omega = 0{\cdot}7$; $\alpha_{23} = 1$; $\varepsilon = 0{\cdot}8$

ANALYSIS AND DISCUSSION

Using the dimensionless parameters defined in the nomenclature, the solution of the equations may be rearranged as follows:

$$\vartheta_1(\eta) = \frac{\operatorname{erfc}(\eta\sqrt{\alpha_{21}})}{\operatorname{erfc}(\lambda_1\sqrt{\alpha_{21}})} \tag{45}$$

$$\vartheta_2(\eta) = \frac{\operatorname{erf}(\eta) - \operatorname{erf}(\lambda_1)}{\operatorname{erf}(\lambda_2) - \operatorname{erf}(\lambda_1)} \tag{46}$$

$$\vartheta_3(\eta) = \frac{\operatorname{erf}(\eta\sqrt{\alpha_{23}})}{\operatorname{erf}(\lambda_2\sqrt{\alpha_{23}})} \tag{47}$$

$$\theta_2(\eta) = \frac{\operatorname{erf}(\lambda_2/\sqrt{\mathrm{Lu}_2}) - \operatorname{erf}(\eta/\sqrt{\mathrm{Lu}_2})}{\operatorname{erf}(\lambda_2/\sqrt{\mathrm{Lu}_2}) - \operatorname{erf}(\lambda_1/\sqrt{\mathrm{Lu}_2})} \tag{48}$$

$$\theta_3(\eta) = \frac{\operatorname{erf}(\eta\sqrt{D_{23}/\mathrm{Lu}_2})}{\operatorname{erf}(\lambda_2\sqrt{D_{23}/\mathrm{Lu}_2})} \tag{49}$$

$$\frac{e^{-\lambda_1^2}}{\operatorname{erf}(\lambda_1) - \operatorname{erf}(\lambda_2)} - \sqrt{\alpha_{21}}\,kT_1\frac{e^{-\lambda_1^2\alpha_{21}}}{\operatorname{erfc}(\lambda_1\sqrt{\alpha_{21}})} = \sqrt{\pi}\,\omega v_1\lambda_1 \tag{50}$$

$$\frac{-e^{-\lambda_2^2}}{\operatorname{erf}(\lambda_1) - \operatorname{erf}(\lambda_2)} + \sqrt{\alpha_{23}}\,kT_3\frac{e^{-\lambda_2^2\alpha_{23}}}{\operatorname{erf}(\lambda_2\sqrt{\alpha_{23}})} = \sqrt{\pi}(\varepsilon - \omega)v_2\lambda_2 \tag{51}$$

$$\frac{e^{-\lambda_1^2/\mathrm{Lu}_2}}{\operatorname{erf}(\lambda_1/\sqrt{\mathrm{Lu}_2}) - \operatorname{erf}(\lambda_2/\sqrt{\mathrm{Lu}_2})} = \frac{\sqrt{\pi}\,\omega C_i\lambda_1}{(C_{v1} - C_{v2})\sqrt{\mathrm{Lu}_2}} \tag{52}$$

$$\frac{-e^{-\lambda_2^2/\mathrm{Lu}_2}}{\operatorname{erf}(\lambda_1/\sqrt{\mathrm{Lu}_2}) - \operatorname{erf}(\lambda_2/\sqrt{\mathrm{Lu}_2})} + \frac{C_{v2} - C_s}{(C_{v1} - C_{v2})\sqrt{D_{23}}}\frac{e^{-\lambda_2^2 D_{23}/\mathrm{Lu}_2}}{\operatorname{erf}(\lambda_2\sqrt{(D_{23}/\mathrm{Lu}_2)})} = \frac{\sqrt{\pi}(\varepsilon - \omega)\rho\lambda_2}{(C_{v1} - C_{v2})\sqrt{\mathrm{Lu}_2}} \tag{53}$$

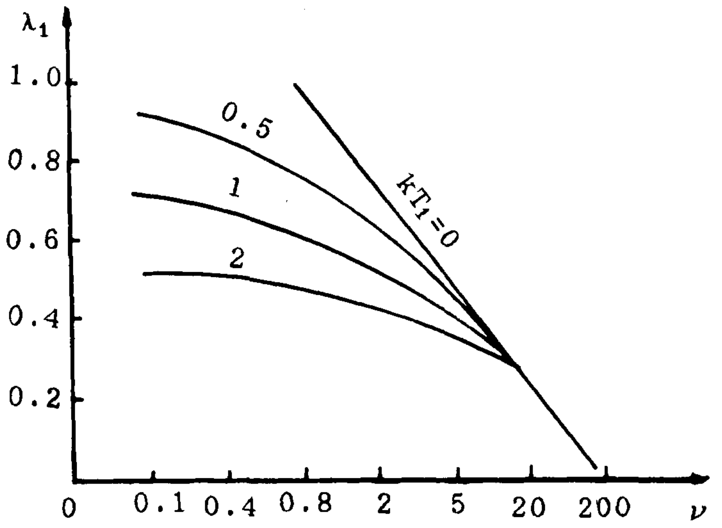

Figure 5. Effect of kT_1 on sublimation: $kT_3 = 1$; $\omega = 0{\cdot}7$; $\alpha_{21} = 0{\cdot}8$; $\alpha_{23} = 1$; $\varepsilon = 0{\cdot}8$

For the exact solutions of coupled heat and mass transfer with double moving interfaces in a porous body derived, the effects of several parameters on the sublimation and desorption process are analysed with the help of a computer. From equations (18) and (19), it can be seen that the motions of the sublimation and desorption fronts are proportional to the constants λ_1 and λ_2, respectively, and these can be determined from equations (50) and (51). The higher the values of λ_1 and λ_2, the faster are the movements of the sublimation and desorption fronts. Because the latent heat of sublimation is almost equal to the heat of desorption, as are the densities of ice and water (C_i and ρ), from the standpoint of convenience for analysis, the assumption is made that the dimensionless latent heat of sublimation ν_1 is equal to the dimensionless heat of desorption ν_2, i.e. $\nu_1 = \nu_2 = \nu$, called the phase-change heat. Figure 2 shows the illustration of the three regions for a special case; it can be seen that the whole half porous body is divided into three parts, namely the frozen region ($\lambda_1 < \eta < +\infty$), the sublimated region ($\lambda_2 < \eta < \lambda_1$), and the desorbed region ($0 < \eta < \lambda_2$). Figures 3–12

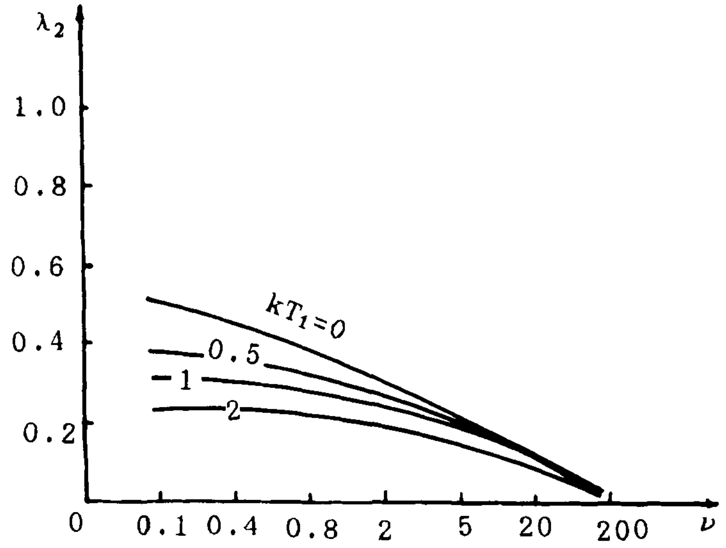

Figure 6. Effect of kT_1 on desorption: $kT_3 = 1$; $\omega = 0{\cdot}7$; $\alpha_{21} = 0{\cdot}8$; $\alpha_{23} = 1$; $\varepsilon = 0{\cdot}8$

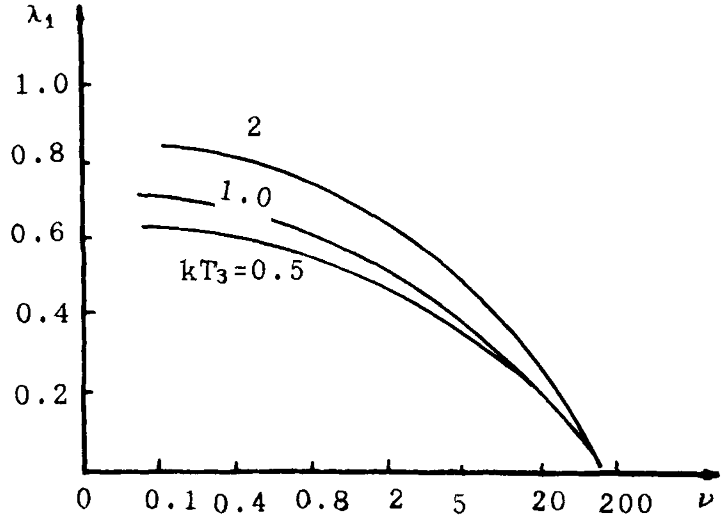

Figure 7. Effect of kT_3 on sublimation: $kT_1 = 1$; $\omega = 0{\cdot}7$; $\alpha_{21} = 0{\cdot}8$; $\alpha_{23} = 1$; $\varepsilon = 0{\cdot}8$

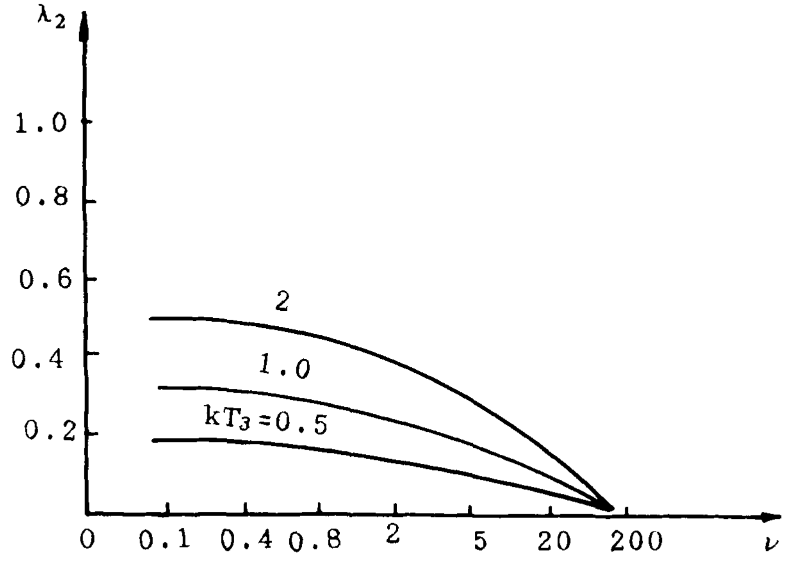

Figure 8. Effect of kT_3 on desorption: $kT_1 = 1$; $\omega = 0{\cdot}7$; $\alpha_{21} = 0{\cdot}8$; $\alpha_{23} = 1$; $\varepsilon = 0{\cdot}8$

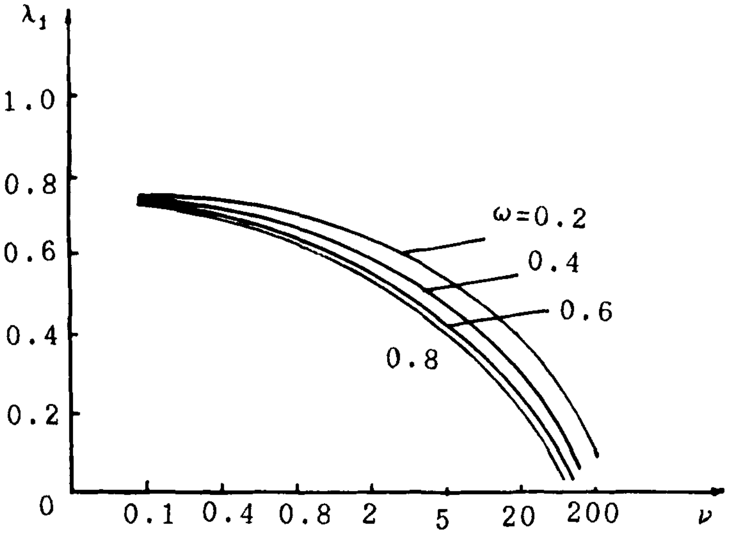

Figure 9. Effect of ω on sublimation: $kT_1 = kT_3 = 1$; $\alpha_{21} = 0{\cdot}8$; $\alpha_{23} = 1$; $\varepsilon - \omega = 0{\cdot}05$

show that λ_1 and λ_2 are functions of the dimensionless phase change heat v with α_{21}, kT_1, kT_3, ω and $\varepsilon - \omega$ as parameters, respectively. A larger phase-change heat v yields smaller λ_1 and λ_2 and thus lowers the sublimation and desorption. When the phase-change heat is very large, the sublimation and desorption fronts approach each other. Figures 3 and 4 show the effect of the ratio α_{21} of the thermal diffusivity in the sublimated region to that in the frozen region on the sublimation and desorption; a smaller α_{21} results not only in faster sublimation but in faster desorption. Figures 5 and 6 indicate the effect of kT_1 on the sublimation and desorption speeds. kT_1 represents the ratio of the steady heat flux in the frozen region to that in the sublimated region with the same heat conduction distance. It is shown that the lower the value of kT_1, the faster the sublimation and desorption. The top curve in each of Figures 5 and 6 with $kT_1 = 0$ represents the case in which the initial temperature in the front region is equal to the sublimation temperature. Figures 7 and 8 show the effect of kT_3 on the sublimation and desorption speeds. kT_3 represents the ratio of the steady

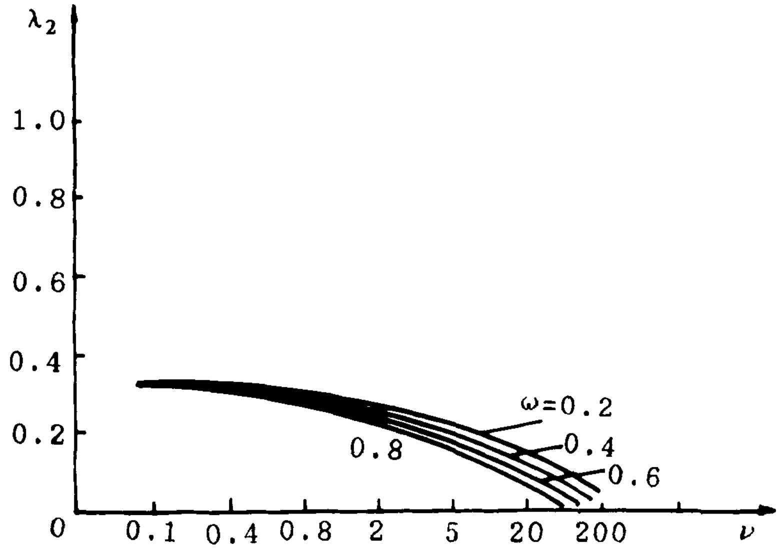

Figure 10. Effect of ω on desorption: $kT_1 = kT_3 = 1$; $\alpha_{21} = 0{\cdot}8$; $\alpha_{23} = 1$; $\varepsilon - \omega = 0{\cdot}05$

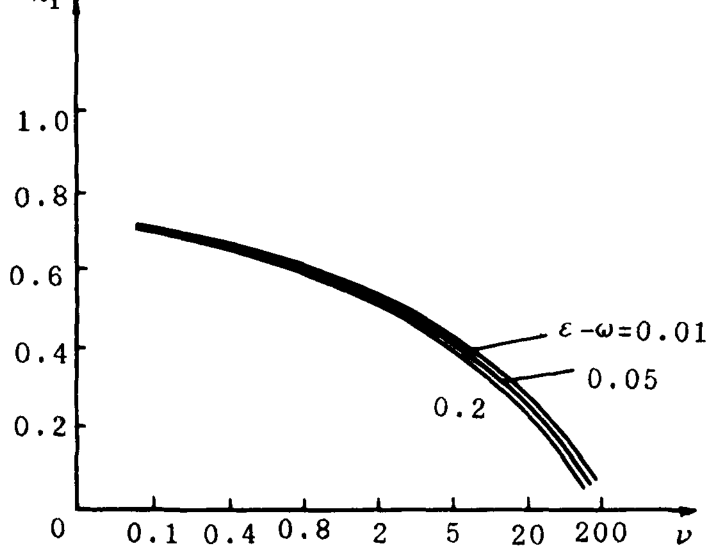

Figure 11. Effect of $\varepsilon - \omega$ on sublimation: $kT_1 = kT_3 = 1$; $\alpha_{21} = 0{\cdot}8$; $\alpha_{23} = 1$; $\omega = 0{\cdot}8$

heat flux in the desorbed region to that in the sublimated region with the same heat conduction distance. It is shown that the lower is the value of kT_3, the slower are the sublimation and desorption. Figures 9 and 10 demonstrate the effect of the volume fraction ω of free water in a pore of the porous body on the sublimation and desorption speeds. It can be seen that a smaller ω results in faster sublimation and desorption. Figures 11 and 12 indicate the effect of the volume fraction of the absorbed water in a pore of the porous body, $\varepsilon - \omega$, on sublimation and desorption; a smaller $\varepsilon - \omega$ also results in faster the sublimation and desorption.

CONCLUSION

For the freeze-drying of a rigid porous medium containing moisture, the maximum safe surface temperature is high enough to allow the surface heating temperature to be higher than the desorption temperature of the

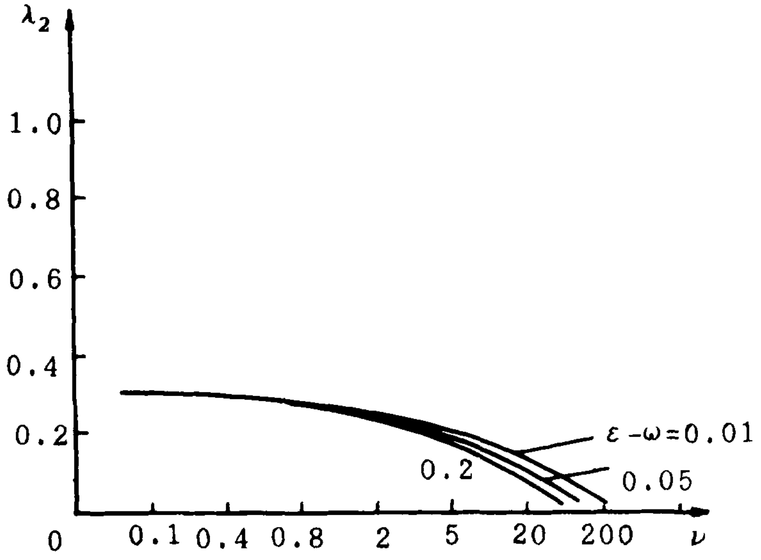

Figure 12. Effect of $\varepsilon - \omega$ on desorption: $kT_1 = kT_3 = 1$; $\alpha_{21} = 0.8$; $\alpha_{23} = 1$; $\omega = 0.8$

residual moisture after sublimation. Especially in the AFD technique, not only sublimation but also desorption takes place in the porous body. By considering the desorption as the second moving interface, coupled heat and mass transfer with double moving interfaces in a porous half-space was studied theoretically, and an exact solution was developed, where the mass transfer is controlled by the Fick law. Then the effects of the ratio of thermal diffusivity in the sublimated region to that in the frozen region, the ratio of steady heat flux in the frozen region to that in the sublimated region with the same heat conduction distance, the ratio of steady heat flux in the desorbed region to that in the sublimated region, and the volume fraction of free water and absorbed water in the pores of the porous body on sublimation and desorption have been analysed. The results show that, except for the ratio of steady heat flux in the desorbed region to that in the sublimated region, a larger value of which results in faster sublimation and desorption, a lower value of all other parameters results in faster sublimation and desorption. The dimensionless phase-change heat also has a prominent effect on sublimation and desorption; a larger heat of phase change yields slower sublimation and desorption, and also closer sublimation and desorption fronts.

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NOMENCLATURE

C = vapour concentration, kg/m^3
C_i = density of ice in pore, kg/m^3
C_p = specific heat, J/kgK
D = effective mass diffusivity, m^2/s
D_{23} = D_2/D_3
ΔH = desorption heat, J/kg
k = heat conductivity, W/mK
L = latent heat of sublimation, J/kg
Lu = Luikov number = D/α
$S_1(t)$ = position of sublimation front, m

$S_2(t)$ = position of desorption front, m
t = time, s
T = temperature, K
x = displacement, m
kT_1 $= k_1(T_{v1} - T_0)/\{k_2(T_{v2} - T_{v1})\}$
kT_3 $= k_3(T_s - T_{v2})/\{k_2(T_{v2} - T_{v1})\}$
α = thermal diffusivity, m^2/s
ε = porosity
ω = volume fraction of free water
$\vartheta_1(\eta)$ $= (T_1(\eta) - T_0)/(T_{v1} - T_0)$
$\vartheta_2(\eta)$ $= (T_2(\eta) - T_{v1})/(T_{v2} - T_{v1})$
$\vartheta_3(\eta)$ $= (T_3(\eta) - T_s)/(T_{v2} - T_s)$
$\theta_2(\eta)$ $= (C_2(\eta) - C_{v2})/(C_{v1} - C_{v2})$
$\theta_3(\eta)$ $= (C_3(\eta) - C_s)/(C_{v2} - C_s)$
ρ = density of absorbed water, kg/m^3
λ_1, λ_2 = position parameters at sublimation and desorption fronts
η $= x/2\sqrt{\alpha_2 t}$
ν_1 $= \alpha_2 C_i L/\{k_2(T_{v2} - T_{v1})\}$
ν_2 $= \alpha_2 \rho \Delta H/\{k_2(T_{v2} - T_{v1})\}$
ν $= \nu_1 = \nu_2$ = dimensionless phase change heat
α_{21} $= \alpha_2/\alpha_1$

Subscripts
1 = frozen region
2 = sublimated region
3 = desorbed region
s = surface
$v1$ = sublimation front
$v2$ = desorption front

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