

EXACT SOLUTION OF COUPLED HEAT AND MASS TRANSFER WITH DOUBLE MOVING INTERFACES IN A POROUS HALF-SPACE

PART 2: MASS TRANSFER CONTROLLED BY THE FICK AND DARCY LAWS

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SUMMARY

Coupled heat and mass transfer with double moving interfaces, taking place in a porous half-space, is defined, and exact solutions for the temperature and moisture distributions as well as the positions of two moving fronts are obtained. As in Part 1 of the paper, the interest is focused on the case of moderate vacuum environment pressure where the moisture transfer is due to the result of the vapour concentration and pressure gradients. The effect of the surface pressure and the introduced parameter β_2 on the moisture transfer is analysed. Conclusions are drawn on the basis of the results.

KEY WORDS Double moving fronts Sublimation Desorption Porous media Fick and Darcy Laws

INTRODUCTION

There are many applications of vacuum sublimation and desorption in the freeze-drying of products. For instance, the food, medical, and chemical industries use the freeze-drying process to maintain the shape and quality of heat sensitive products during the drying process. Ordinary freeze-drying consists of prefreezing, primary drying or sublimation drying and secondary drying or desorption drying. The phenomenon of vacuum sublimation has received considerable attention over the past 20 years. Lin (1981, 1982a, b) analysed the sublimation and desublimation processes in a semi-infinite porous medium by considering the transient heat equation in both the dried and frozen regions and by assuming that the vacuum pressure was very low, so that there was practically only the vapour pressure of the moisture acting on the frozen phase. During the freeze-drying process, the vapour pressure acting on the frozen phase was therefore equal to the environmental pressure, which remained unchanged, so that the vapour transfer in the dried region was the result of the concentration gradient only. In addition, by relating the conditions of the interface to the Clapeyron equation coupled with the heat and mass balances at the interface, Lin obtained the values of temperature and concentration at the interface and the exact solution for the problem. Luikov (1975) formulated a system of equations for heat and mass transfer in the capillary porous medium that included the effect of the pressure gradients and equation of pressure field. Fey and Boles (1987, 1988) presented a more complete description of the vacuum sublimation process for which an exact analytical solution was obtained. Their formulation was based on the Luikov (1975) system, and the vapour flow in the dried region was considered as the result of both moisture concentration and pressure gradients in the porous medium.

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Theoretically, for moderate environmental vacuum pressure, the mass transfer condition is not so good that the pressure gradient exists in the dried region, thus vapour movement in porous media is due to the concentration and/or pressure gradients. Fey and Boles also analysed the effect of convection heat transfer on sublimation and vacuum sublimation in an initially partially filled frozen porous medium with recondensation.

King (1968) considered that desorption in a porous dry semi-infinite solid involves moisture diffusion in the vapour phase and possibly surface diffusion across moisture adsorption sites as well. The theory of desorption was derived from Fick's law. Yao (1988) developed some solution for the constant effective diffusivity of the vapour in the dry product.

All the previous authors have considered primary drying and secondary drying as separate cases, i.e. the desorption drying takes place only after the sublimation drying comes to an end. Peng *et al.* (1992) presented an exact solution of coupled heat and mass transfer with double moving interfaces in a porous half-space for the freeze-drying of some rigid products and accelerated freeze-drying (AFD) technique, where the heating temperature at the surface is higher than the desorption temperature of the residual moisture of the sublimated region, so that the sublimation and desorption take place in a porous body at the same time. Just as for Lin, mass transfer in the dried region is assumed to be the result of the concentration gradient only, and so its result is only applicable to the case of very low vacuum pressure.

The present formulation is based on the Luikov system and the fact the vapour flow in the dried region results from both moisture concentration and pressure gradients in the porous medium for moderate environmental vacuum pressure.

STATEMENT OF PROBLEM

A semi-infinite frozen porous medium is exposed to an environment where the pressure and the vapour concentration are maintained below the triple point of the bounded substance, and the temperature is higher than the desorption temperature of the residual moisture in the sublimated region. The porous medium is assumed to be composed of very small solid particles of equal size. Also, the medium is isotropic, homogeneous, and rigid. Figure 1 shows a semi-infinite frozen porous medium where the temperature and mass content are initially constant throughout the medium. At times greater than zero, the temperature, vapour concentration, and pressure of the surface at $x = 0$ are maintained at constant values and the sublimation and desorption processes begin. Since the vapour is transferred outwardly, the frozen region, the sublimated region and the desorbed region are formed and are separated by two distinct, sharply thin,

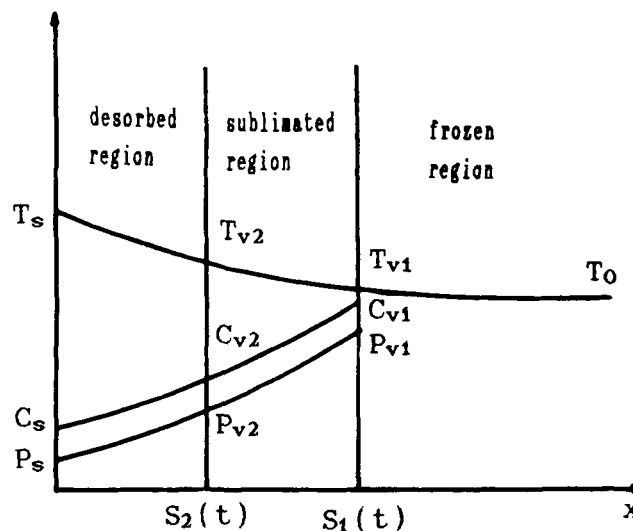


Figure 1. Physical model

moving interfaces defined by $x_1 = S_1(t)$ and $x_2 = S_2(t)$, respectively. $S_1(t)$ refers to the sublimation front, and $S_2(t)$ refers to the desorption front. The temperatures at the interfaces are assumed to be known constants, and the vapour concentrations and pressures at the interfaces are assumed to be in equilibrium and to have constant values which will be determined later. There are no moisture concentrations or pressure gradients in the frozen region. In the sublimated and desorbed regions, vapour concentration flows occur as a result of the interaction of temperature, vapour concentration and pressure gradients.

To formulate the theoretical model of the coupled heat and mass transfer with double moving interfaces in a porous body, additional assumption are made as follows:

- (i) One-dimensional heat and mass transfer is considered. Both the Darcy and Fick laws for vapour transfer are valid through the sublimated and desorbed regions.
- (ii) The heat radiation, heat convection and thermal expansion of the medium, condensation, and Soret and Dufour effects are assumed to be small and negligible.
- (iii) The sublimated and desorbed regions contain the vapour resulting from the sublimation process and a small amount of residual air in relation to the vapour mass.
- (iv) The thermophysical properties remain constant, but they may be different for different regions.
- (v) The ideal gas law is assumed to be valid at the interfaces.

From the above assumptions, the freeze-drying process can be formulated by the following equations:

$$\partial T_1(x, t)/\partial t = \alpha_1 \partial^2 T_1(x, t)/\partial x^2 \quad (S_1(t) < x < \infty) \quad (1)$$

$$\partial T_2(x, t)/\partial t = \alpha_2 \partial^2 T_2(x, t)/\partial x^2 \quad (S_2(t) < x < S_1(t)) \quad (2)$$

$$\partial C_2(x, t)/\partial t = D_2 \partial^2 C_2(x, t)/\partial x^2 + \kappa_2 \partial^2 P_2(x, t)/\partial x^2 \quad (S_2(t) < x < S_1(t)) \quad (3)$$

$$\partial P_2(x, t)/\partial t = \alpha_{p2} \partial^2 P_2(x, t)/\partial x^2 \quad (S_2(t) < x < S_1(t)) \quad (4)$$

$$\partial T_3(x, t)/\partial t = \alpha_3 \partial^2 T_3(x, t)/\partial x^2 \quad (0 < x < S_2(t)) \quad (5)$$

$$\partial C_3(x, t)/\partial t = D_3 \partial^2 C_3(x, t)/\partial x^2 + \kappa_3 \partial^2 P_3(x, t)/\partial x^2 \quad (0 < x < S_2(t)) \quad (6)$$

$$\partial P_3(x, t)/\partial t = \alpha_{p3} \partial^2 P_3(x, t)/\partial x^2 \quad (0 < x < S_2(t)) \quad (7)$$

where equations (1), (2) and (5) describe the temperature distributions in the frozen, sublimated and desorbed regions, respectively. Equations (3), (4), (6) and (7), based on the theories described above and the Luikov system (Luikov, 1975), describe the concentration and pressure fields in the sublimated and desorbed regions, respectively. It should be noted that the first and second terms on the right-hand sides of equations (3) and (6) represent the Fick and Darcy Laws, respectively.

The initial and boundary conditions can be described as

$$T_1(x, 0) = T_0 \quad (8)$$

$$T_3(0, t) = T_s, \quad C_3(0, t) = C_s, \quad P_3(0, t) = P_s \quad (9)$$

$$T_1(\infty, t) = T_0 \quad (10)$$

On the sublimation front, the matching conditions can be expressed as

$$T_1(S_1, t) = T_2(S_1, t) = T_{v1} \quad (11)$$

$$C_2(S_1, t) = C_{v1} \quad (12)$$

$$P_2(S_1, t) = P_{v1} \quad (13)$$

where T_{v1} , C_{v1} and P_{v1} are the sublimation interface temperature, concentration, and pressure, respectively. C_{v1} and P_{v1} are assumed to be constant but unknown. T_{v1} is assumed to be known and constant.

On applying the ideal gas law at the sublimation interface, we have

$$P_{v1} = C_{v1} R T_{v1} \quad (14)$$

The energy and moisture balances at the sublimation interface yield

$$-k_2 \frac{\partial T_2(S_1, t)}{\partial x} + k_1 \frac{\partial T_1(S_1, t)}{\partial x} = \omega C_i L \frac{dS_1(t)}{dt} \quad (15)$$

$$D_2 \frac{\partial C_2(S_1, t)}{\partial x} + \kappa_2 \frac{\partial P_2(S_1, t)}{\partial x} = \omega C_i \frac{dS_1(t)}{dt} \quad (16)$$

On the desorption front, the matching conditions can be expressed as

$$T_2(S_2, t) = T_3(S_2, t) = T_{v2} \quad (17)$$

$$C_2(S_2, t) = C_3(S_2, t) = C_{v2} \quad (18)$$

$$P_2(S_2, t) = P_3(S_2, t) = P_{v2} \quad (19)$$

where T_{v2} , C_{v2} and P_{v2} are the desorption interface temperature, concentration and pressure, respectively. C_{v2} and P_{v2} are assumed to be constant but unknown, and T_{v2} is assumed to be a known constant.

On applying the ideal gas law at the desorption interface, we have

$$P_{v2} = C_{v2} R T_{v2} \quad (20)$$

The energy and moisture balances at the desorption interface yield

$$-k_3 \frac{\partial T_3(S_2, t)}{\partial x} + k_2 \frac{\partial T_2(S_2, t)}{\partial x} = (\varepsilon - \omega) \rho \Delta H \frac{dS_2(t)}{dt} \quad (21)$$

$$D_3 \frac{\partial C_3(S_2, t)}{\partial x} + \kappa_3 \frac{\partial P_3(S_2, t)}{\partial x} - D_2 \frac{\partial C_2(S_2, t)}{\partial x} - \kappa_2 \frac{\partial P_2(S_2, t)}{\partial x} = (\varepsilon - \omega) \rho \frac{dS_2(t)}{dt} \quad (22)$$

SOLUTION

We introduce the nondimensional similarity variable as given by Özisik (1980):

$$\eta = x/2\sqrt{\alpha_2 t} \quad (23)$$

into equations (1)–(22), and define two new variables $Z_1(\eta)$ and $Z_2(\eta)$ as given by Fey and Boles (1987, 1988) which decouple equations (3) and (4) and equations (6) and (7), respectively:

$$Z_2(\eta) = P_2(\eta) + \beta_2 C_2(\eta) \quad (24)$$

$$Z_3(\eta) = P_3(\eta) + \beta_3 C_3(\eta) \quad (25)$$

where

$$\beta_2 = (D_2 - \alpha_{p2})/\kappa_2 \quad (26)$$

$$\beta_3 = (D_3 - \alpha_{p3})/\kappa_3 \quad (27)$$

The location of the sublimation and desorption interfaces is assumed to be given by

$$S_1(t) = 2\lambda_1 \sqrt{\alpha_2 t} \quad (28)$$

$$S_2(t) = 2\lambda_2 \sqrt{\alpha_2 t} \quad (29)$$

respectively, where λ_1 and λ_2 are two unknown constants to be determined during the solution.

With the introduction of the new variables η and λ , we note that the desorbed region corresponds to $0 < \eta < \lambda_2$, the sublimated region corresponds to $\lambda_2 < \eta < \lambda_1$, and the frozen region corresponds to $\lambda_1 < \eta < \infty$. The problem is transformed to a system of ordinary differential equations for T_1 , T_2 , T_3 , P_2 , P_3 , Z_2 and Z_3 with variable coefficients subject to the transformed boundary and interface conditions.

Basic equations:

$$\frac{d^2 T_1(\eta)}{d\eta^2} + 2\eta \frac{dT_1(\eta)}{d\eta} \frac{\alpha_2}{\alpha_1} = 0 \quad (\lambda_1 < \eta < +\infty) \quad (30)$$

$$\frac{d^2 T_2(\eta)}{d\eta^2} + 2\eta \frac{dT_2(\eta)}{d\eta} = 0 \quad (\lambda_2 < \eta < \lambda_1) \quad (31)$$

$$\frac{d^2 P_2(\eta)}{d\eta^2} + 2\eta \frac{dP_2(\eta)}{d\eta} \frac{\alpha_2}{\alpha_{p2}} = 0 \quad (\lambda_2 < \eta < \lambda_1) \quad (32)$$

$$\frac{d^2 Z_2(\eta)}{d\eta^2} + 2\eta \frac{dZ_2(\eta)}{d\eta} \frac{\alpha_2}{D_2} = 0 \quad (\lambda_2 < \eta < \lambda_1) \quad (33)$$

$$\frac{d^2 T_3(\eta)}{d\eta^2} + 2\eta \frac{dT_3(\eta)}{d\eta} \frac{\alpha_2}{\alpha_3} = 0 \quad (0 < \eta < \lambda_2) \quad (34)$$

$$\frac{d^2 P_3(\eta)}{d\eta^2} + 2\eta \frac{dP_3(\eta)}{d\eta} \frac{\alpha_2}{\alpha_{p3}} = 0 \quad (0 < \eta < \lambda_2) \quad (35)$$

$$\frac{dZ_3(\eta)}{d\eta^2} + 2\eta \frac{dZ_3(\eta)}{d\eta} \frac{\alpha_2}{D_3} = 0 \quad (0 < \eta < \lambda_2) \quad (36)$$

the initial and boundary conditions become

$$T_1(\eta = \infty) = T_0 \quad (37)$$

$$T_3(\eta = 0) = T_s \quad (38)$$

$$C_3(\eta = 0) = C_s \quad (39)$$

$$P_3(\eta = 0) = P_s \quad (40)$$

$$Z_3(\eta = 0) = Z_s \quad (41)$$

on the sublimation front, the matching conditions become

$$T_1(\eta = \lambda_1) = T_2(\eta = \lambda_1) = T_{v1} \quad (42)$$

$$C_2(\eta = \lambda_1) = C_{v1} \quad (43)$$

$$P_2(\eta = \lambda_1) = P_{v1} \quad (44)$$

$$Z_2(\eta = \lambda_1) = P_{v1} + \beta_2 C_{v1} \quad (45)$$

$$-k_2 \frac{dT_2(\lambda_1)}{d\eta} + k_1 \frac{dT_1(\lambda_1)}{d\eta} = 2\omega C_i L \alpha_2 \lambda_1 \quad (46)$$

$$D_2 \frac{dC_2(\lambda_1)}{d\eta} + \kappa_2 \frac{dP_2(\lambda_1)}{d\eta} = 2\omega C_i \alpha_2 \lambda_1 \quad (47)$$

On the desorption front, the matching conditions become

$$T_2(\eta = \lambda_2) = T_3(\eta = \lambda_2) = T_{v2} \quad (48)$$

$$C_2(\eta = \lambda_2) = C_3(\eta = \lambda_2) = C_{v2} \quad (49)$$

$$P_2(\eta = \lambda_2) = P_3(\eta = \lambda_2) = P_{v2} \quad (50)$$

$$Z_2(\eta = \lambda_2) = P_{v2} + \beta_2 C_{v2} \quad (51)$$

$$Z_3(\eta = \lambda_2) = P_{v2} + \beta_3 C_{v2} \quad (52)$$

$$-k_3 \frac{dT_3(\lambda_2)}{d\eta} + k_2 \frac{dT_2(\lambda_2)}{d\eta} = 2(\varepsilon - \omega)\rho\Delta H\alpha_2\lambda_2 \quad (53)$$

$$D_3 \frac{dC_3(\lambda_2)}{d\eta} + \kappa_3 \frac{dP_3(\lambda_2)}{d\eta} - D_2 \frac{dC_2(\lambda_2)}{d\eta} - \kappa_2 \frac{dP_2(\lambda_2)}{d\eta} = 2(\varepsilon - \omega)\rho\alpha_2\lambda_2 \quad (54)$$

The above system can be solved exactly, and we obtain

$$T_1(\eta) = T_0 + (T_{v1} - T_0) \frac{\operatorname{erfc}(\eta\sqrt{\alpha_2/\alpha_1})}{\operatorname{erfc}(\lambda_1\sqrt{\alpha_2/\alpha_1})} \quad (55)$$

$$T_2(\eta) = T_{v1} + (T_{v2} - T_{v1}) \frac{\operatorname{erf}(\eta) - \operatorname{erf}(\lambda_1)}{\operatorname{erf}(\lambda_2) - \operatorname{erf}(\lambda_1)} \quad (56)$$

$$T_3(\eta) = T_s + (T_{v2} - T_s) \frac{\operatorname{erf}(\eta\sqrt{\alpha_2/\alpha_3})}{\operatorname{erf}(\lambda_2\sqrt{\alpha_2/\alpha_3})} \quad (57)$$

$$P_2(\eta) = P_{v2} + (P_{v1} - P_{v2}) \frac{\operatorname{erf}(\lambda_2\sqrt{\alpha_2/\alpha_{p2}}) - \operatorname{erf}(\eta\sqrt{\alpha_2/\alpha_{p2}})}{\operatorname{erf}(\lambda_2\sqrt{\alpha_2/\alpha_{p2}}) - \operatorname{erf}(\lambda_1\sqrt{\alpha_2/\alpha_{p2}})} \quad (58)$$

$$P_3(\eta) = P_s + (P_{v2} - P_s) \frac{\operatorname{erf}(\eta\sqrt{\alpha_2/\alpha_{p3}})}{\operatorname{erf}(\lambda_2\sqrt{\alpha_2/\alpha_{p3}})} \quad (59)$$

$$Z_2(\eta) = (P_{v2} + \beta_2 C_{v2}) + [(P_{v1} + \beta_2 C_{v1}) - (P_{v2} + \beta_2 C_{v2})] \frac{\operatorname{erf}(\lambda_2\sqrt{\alpha_2/D_2}) - \operatorname{erf}(\eta\sqrt{\alpha_2/D_2})}{\operatorname{erf}(\lambda_2\sqrt{\alpha_2/D_2}) - \operatorname{erf}(\lambda_1\sqrt{\alpha_2/D_2})} \quad (60)$$

$$Z_3(\eta) = (P_s + \beta_3 C_s) + [(P_{v2} + \beta_3 C_{v2}) - (P_s + \beta_3 C_s)] \frac{\operatorname{erf}(\eta\sqrt{\alpha_2/D_3})}{\operatorname{erf}(\lambda_2\sqrt{\alpha_2/D_3})} \quad (61)$$

and the solution for $C_2(\eta)$ and $C_3(\eta)$ is obtained from equations (24) and (25):

$$C_2(\eta) = C_{v2} - \frac{(P_{v1} - P_{v2})}{\beta_2} \frac{\operatorname{erf}(\lambda_2\sqrt{\alpha_2/\alpha_{p2}}) - \operatorname{erf}(\eta\sqrt{\alpha_2/\alpha_{p2}})}{\operatorname{erf}(\lambda_2\sqrt{\alpha_2/\alpha_{p2}}) - \operatorname{erf}(\lambda_1\sqrt{\alpha_2/\alpha_{p2}})} + \left[\frac{(P_{v1} - P_{v2})}{\beta_2} + (C_{v1} - C_{v2}) \right] \frac{\operatorname{erf}(\lambda_2\sqrt{\alpha_2/D_2}) - \operatorname{erf}(\eta\sqrt{\alpha_2/D_2})}{\operatorname{erf}(\lambda_2\sqrt{\alpha_2/D_2}) - \operatorname{erf}(\lambda_1\sqrt{\alpha_2/D_2})} \quad (62)$$

$$C_3(\eta) = C_s - \frac{(P_{v2} - P_s)}{\beta_3} \frac{\operatorname{erf}(\eta\sqrt{\alpha_2/\alpha_{p3}})}{\operatorname{erf}(\lambda_2\sqrt{\alpha_2/\alpha_{p3}})} + \left[\frac{(P_{v2} - P_s)}{\beta_3} + (C_{v2} - C_s) \right] \frac{\operatorname{erf}(\eta\sqrt{\alpha_2/D_3})}{\operatorname{erf}(\lambda_2\sqrt{\alpha_2/D_3})} \quad (63)$$

There are six unknown parameters λ_1 , λ_2 , C_{v1} , C_{v2} , P_{v1} and P_{v2} in the course of solutions to be determined. After substituting the above solutions into the interface equations, we obtain the following four transcendental interface equations:

$$\frac{e^{-\lambda_1^2}}{\operatorname{erf}(\lambda_1) - \operatorname{erf}(\lambda_2)} - \sqrt{\frac{\alpha_2}{\alpha_1}} \frac{k_1(T_{v1} - T_0)}{k_2(T_{v2} - T_{v1})} \frac{e^{-\lambda_1^2 \alpha_2/\alpha_1}}{\operatorname{erfc}(\lambda_1\sqrt{\alpha_2/\alpha_1})} = \frac{\sqrt{\pi}\omega C_i L \alpha_2 \lambda_1}{k_2(T_{v2} - T_{v1})} \quad (64)$$

$$\frac{-e^{-\lambda_2^2}}{\operatorname{erf}(\lambda_1) - \operatorname{erf}(\lambda_2)} + \sqrt{\frac{\alpha_2}{\alpha_3}} \frac{k_3(T_s - T_{v2})}{k_2(T_{v2} - T_{v1})} \frac{e^{-\lambda_2^2 \alpha_2/\alpha_3}}{\operatorname{erf}(\lambda_2\sqrt{\alpha_2/\alpha_3})} = \frac{\sqrt{\pi}(\varepsilon - \omega)\rho\Delta H\alpha_2\lambda_2}{k_2(T_{v2} - T_{v1})} \quad (65)$$

$$- \sqrt{\frac{\alpha_{p2}}{D_2}} \frac{e^{-\lambda_1^2 \alpha_2/\alpha_{p2}}}{\operatorname{erf}(\lambda_1\sqrt{\alpha_2/\alpha_{p2}}) - \operatorname{erf}(\lambda_2\sqrt{\alpha_2/\alpha_{p2}})} + \left[\frac{C_{v1} - C_{v2}}{P_{v1} - P_{v2}} \beta_2 + 1 \right] \frac{e^{-\lambda_1^2 \alpha_2/D_2}}{\operatorname{erf}(\lambda_1\sqrt{\alpha_2/D_2}) - \operatorname{erf}(\lambda_2\sqrt{\alpha_2/D_2})} = \sqrt{\frac{\alpha_2}{D_2}} \frac{\sqrt{\pi}\omega C_i \beta_2 \lambda_1}{(P_{v1} - P_{v2})} \quad (66)$$

$$\begin{aligned}
& - \sqrt{\frac{\alpha_{p3}}{D_2}} \frac{\beta_2}{\beta_3} \frac{P_{v2} - P_s}{P_{v1} - P_{v2}} \frac{e^{-\lambda_2^2 \alpha_2 / \alpha_{p3}}}{\text{erf}(\lambda_2 \sqrt{\alpha_2 / \alpha_{p3}})} + \sqrt{\frac{D_3}{D_2}} \frac{\beta_2}{\beta_3} \frac{P_{v2} - P_s}{P_{v1} - P_{v2}} \left[\frac{C_{v2} - C_s}{P_{v2} - P_s} \beta_3 + 1 \right] \\
& \times \frac{e^{-\lambda_2^2 \alpha_2 / D_3}}{\text{erf}(\lambda_2 \sqrt{\alpha_2 / D_3})} + \sqrt{\frac{\alpha_{p2}}{D_2}} \frac{e^{-\lambda_1^2 \alpha_2 / \alpha_{p2}}}{\text{erf}(\lambda_1 \sqrt{\alpha_2 / \alpha_{p2}}) - \text{erf}(\lambda_2 \sqrt{\alpha_2 / \alpha_{p2}})} \\
& - \frac{e^{-\lambda_2^2 \alpha_2 / D_2}}{\text{erf}(\lambda_1 \sqrt{\alpha_2 / D_2}) - \text{erf}(\lambda_2 \sqrt{\alpha_2 / D_2})} \left[\frac{C_{v1} - C_{v2}}{P_{v1} - P_{v2}} \beta_2 + 1 \right] \\
& = \sqrt{\frac{\alpha_2}{D_2}} \frac{\sqrt{\pi}(\varepsilon - \omega) \rho \beta_2 \lambda_2}{(P_{v1} - P_{v2})} \quad (67)
\end{aligned}$$

which can determine the six unknown parameters when coupled with the ideal gas laws at the interfaces, equations (14) and (20).

ANALYSIS AND DISCUSSION

Using the dimensionless parameters defined in the nomenclature, the solutions of the equations may be represented as follows:

$$\vartheta_1(\eta) = \frac{\text{erfc}(\eta \sqrt{\alpha_{21}})}{\text{erfc}(\lambda_1 \sqrt{\alpha_{21}})} \quad (68)$$

$$\vartheta_2(\eta) = \frac{\text{erf}(\eta) - \text{erf}(\lambda_1)}{\text{erf}(\lambda_2) - \text{erf}(\lambda_1)} \quad (69)$$

$$\vartheta_3(\eta) = \frac{\text{erf}(\eta \sqrt{\alpha_{23}})}{\text{erf}(\lambda_2 \sqrt{\alpha_{23}})} \quad (70)$$

$$\Omega_2(\eta) = \frac{\text{erf}(\lambda_2 / \sqrt{\text{Lu}_{p2}}) - \text{erf}(\eta / \sqrt{\text{Lu}_{p2}})}{\text{erf}(\lambda_2 / \sqrt{\text{Lu}_{p2}}) - \text{erf}(\lambda_1 / \sqrt{\text{Lu}_{p2}})} \quad (71)$$

$$\Omega_3(\eta) = \frac{\text{erf}(\eta \sqrt{\alpha_{p23} / \text{Lu}_{p2}})}{\text{erf}(\lambda_2 \sqrt{\alpha_{p23} / \text{Lu}_{p2}})} \quad (72)$$

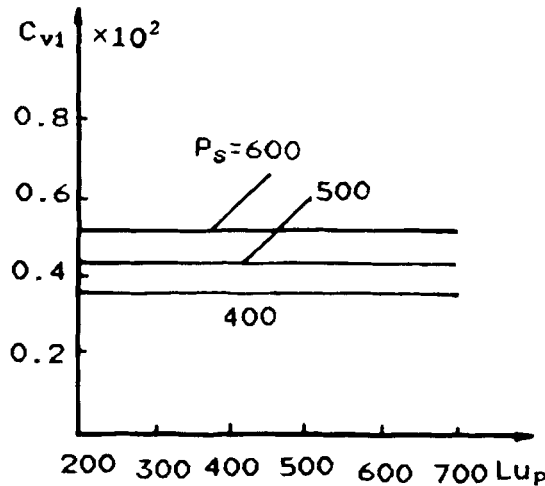


Figure 2. Effect of surface pressure on C_{v1} for $\lambda_1 = 0.8$; $\lambda_2 = 0.2$; $\alpha_{21} = 0.8$; $\alpha_{23} = 1$; $D_{23} = 1$; $\omega = 0.75$; $\varepsilon = 0.8$; $\beta_{23} = 1$; $\beta_2 = -1$; $k_1 = k_2 = k_3$; $C_s = 0.0005$; $T_s = 50^\circ\text{C}$; $T_{v1} = -25^\circ\text{C}$; $T_{v2} = 40^\circ\text{C}$

$$\Theta_2(\eta) = -Z_1 \frac{\operatorname{erf}(\lambda_2/\sqrt{\operatorname{Lu}_{p2}}) - \operatorname{erf}(\eta/\sqrt{\operatorname{Lu}_{p2}})}{\operatorname{erf}(\lambda_2/\sqrt{\operatorname{Lu}_{p2}}) - \operatorname{erf}(\lambda_1/\sqrt{\operatorname{Lu}_{p2}})} + (1 + Z_1) \frac{\operatorname{erf}(\lambda_2/\sqrt{\operatorname{Lu}_2}) - \operatorname{erf}(\eta/\sqrt{\operatorname{Lu}_2})}{\operatorname{erf}(\lambda_2/\sqrt{\operatorname{Lu}_2}) - \operatorname{erf}(\lambda_1/\sqrt{\operatorname{Lu}_2})} \quad (73)$$

$$\Theta_3(\eta) = -Z_2 \frac{\operatorname{erf}(\eta\sqrt{\alpha_{p23}/\operatorname{Lu}_{p2}})}{\operatorname{erf}(\lambda_2\sqrt{\alpha_{p23}/\operatorname{Lu}_{p2}})} + (1 + Z_2) \frac{\operatorname{erf}(\eta\sqrt{D_{23}/\operatorname{Lu}_2})}{\operatorname{erf}(\lambda_2\sqrt{D_{23}/\operatorname{Lu}_2})} \quad (74)$$

energy and mass balances at the sublimation and desorption fronts are

$$\frac{e^{-\lambda_1^2}}{\operatorname{erf}(\lambda_1) - \operatorname{erf}(\lambda_2)} - \sqrt{\alpha_{21}} k T_1 \frac{e^{-\lambda_1^2 \alpha_{21}}}{\operatorname{erfc}(\lambda_1 \sqrt{\alpha_{21}})} = \sqrt{\pi} \omega v_1 \lambda_1 \quad (75)$$

$$\frac{-e^{-\lambda_2^2}}{\operatorname{erf}(\lambda_1) - \operatorname{erf}(\lambda_2)} + \sqrt{\alpha_{23}} k T_3 \frac{e^{-\lambda_2^2 \alpha_{23}}}{\operatorname{erf}(\lambda_2 \sqrt{\alpha_{23}})} = \sqrt{\pi} (\varepsilon - \omega) v_2 \lambda_2 \quad (76)$$

$$\begin{aligned} & - \sqrt{\frac{\operatorname{Lu}_{p2}}{\operatorname{Lu}_2}} \frac{e^{-\lambda_1^2/\operatorname{Lu}_{p2}}}{\operatorname{erf}(\lambda_1/\sqrt{\operatorname{Lu}_{p2}}) - \operatorname{erf}(\lambda_2/\sqrt{\operatorname{Lu}_{p2}})} + \left[\frac{C_{v1} - C_{v2}}{P_{v1} - P_{v2}} \beta_2 + 1 \right] \\ & \times \frac{e^{-\lambda_1^2/\operatorname{Lu}_2}}{\operatorname{erf}(\lambda_1/\sqrt{\operatorname{Lu}_2}) - \operatorname{erf}(\lambda_2/\sqrt{\operatorname{Lu}_2})} = \frac{\sqrt{\pi} \omega C_i \beta_2 \lambda_1}{(P_{v1} - P_{v2}) \sqrt{\operatorname{Lu}_2}} \quad (77) \\ & - \sqrt{\frac{\operatorname{Lu}_{p3}}{\operatorname{Lu}_2 \alpha_{23}}} \beta_{23} \frac{P_{v2} - P_s}{P_{v1} - P_{v2}} \frac{e^{-\lambda_2^2 \alpha_{23}/\operatorname{Lu}_{p3}}}{\operatorname{erf}(\lambda_2 \sqrt{\alpha_{23}/\operatorname{Lu}_{p3}})} + \beta_{23} \sqrt{D_{32}} \frac{P_{v2} - P_s}{P_{v1} - P_{v2}} \left[\frac{C_{v2} - C_s}{P_{v2} - P_s} \beta_3 + 1 \right] \\ & \times \frac{e^{-\lambda_2^2 \alpha_{23}/\operatorname{Lu}_3}}{\operatorname{erf}(\lambda_2 \sqrt{\alpha_{23}/\operatorname{Lu}_3})} + \sqrt{\frac{\operatorname{Lu}_{p2}}{\operatorname{Lu}_2}} \frac{e^{-\lambda_2^2/\operatorname{Lu}_{p2}}}{\operatorname{erf}(\lambda_1/\sqrt{\operatorname{Lu}_{p2}}) - \operatorname{erf}(\lambda_2/\sqrt{\operatorname{Lu}_{p2}})} \\ & - \frac{e^{-\lambda_2^2/\operatorname{Lu}_2}}{\operatorname{erf}(\lambda_1/\sqrt{\operatorname{Lu}_2}) - \operatorname{erf}(\lambda_2/\sqrt{\operatorname{Lu}_2})} \left[\frac{C_{v1} - C_{v2}}{P_{v1} - P_{v2}} \beta_2 + 1 \right] = \frac{\sqrt{\pi} (\varepsilon - \omega) \rho \beta_2 \lambda_2}{(P_{v1} - P_{v2}) \sqrt{\operatorname{Lu}_2}} \quad (78) \end{aligned}$$

since the temperatures at the sublimation and desorption interfaces are assumed to be known and constant. Equations (75) and (76) are independent of equations (77) and (78) and can be solved independently. Equations (75) and (76), which are used to determine the interface position parameters λ_1 and λ_2 , are the same as those in Peng *et al.* (1992), and a detailed analysis has been performed. The present work analyses the

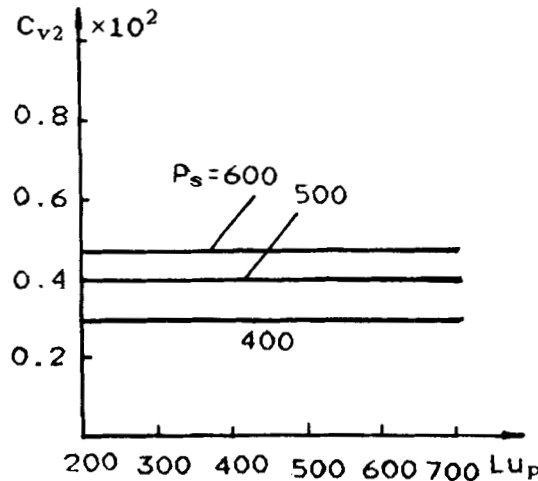


Figure 3. Effect of surface pressure on C_{v2} for all parameters as in Figure 1

effect of the surface pressure and the parameter β_2 on the mass transfer in the sublimated and desorbed regions, which is reflected in the vapour concentration at the two interfaces. Figures 2, 3, 4 and 5 show that C_{v1} and C_{v2} are functions of the Luikov number Lu_p of pressure diffusion, with the surface pressure and the β_2 as parameters, respectively. As shown in the Figures, Lu_p has little effect on mass transfer. Figures 2 and 3 show the effect of the surface pressure on the vapour concentration at the sublimation and desorption interfaces, respectively. A smaller surface pressure results in smaller values of vapour concentration at the two interfaces. Figures 4 and 5 indicate the effect of the parameter β_2 on the vapour concentration at the sublimation and desorption interfaces, respectively. It can be seen that β_2 has no effect on mass transfer.

CONCLUSION

Coupled heat and mass transfer with double moving interfaces in the porous half-space that exists in the freeze-drying of a rigid product, especially in AFD techniques, has been studied and an exact solution has been developed for the case of moderate vacuum environmental pressure where moisture transfer in the

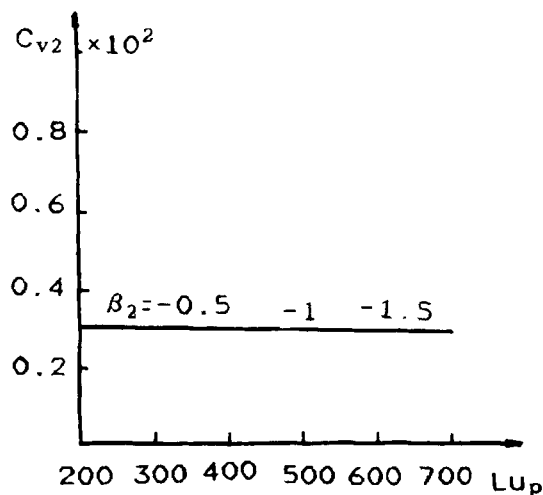


Figure 4. Effect of β_2 on C_{v1} for $P_s = 400$ and all other parameters as for Figure 1

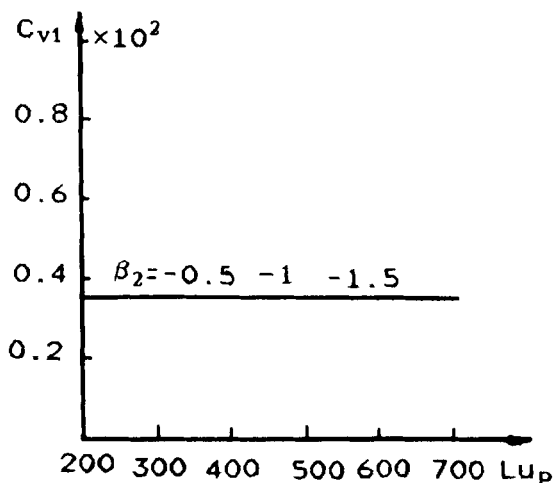


Figure 5. Effect of β_2 on C_{v2} for $P_s = 400$ and all other parameters as for Figure 1

sublimated and desorbed region is taken to be the result of both vapour concentration and pressure gradients. The effect of the surface pressure and the introduced parameter β_2 on the mass transfer has been analysed. The results show that the surface pressure has a prominent effect on mass transfer, but, relatively, the parameter β_2 has little effect on the moisture transfer.

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NOMENCLATURE

C	= vapour concentration, kg/m ³
C_i	= density of ice in a pore
C_p	= specific heat, J/kgK
D	= effective mass diffusivity, m ² /s
D_{23}	= $D_2/D_3 = 1/D_{32}$
ΔH	= desorption heat, J/kg
k	= heat conductivity, W/mK
L	= latent heat of sublimation, J/kg
Lu	= Luikov number = D/α
Lu_p	= Luikov number of pressure diffusion = α_p/α
P	= pressure
$S_1(t)$	= position of sublimation front, m
$S_2(t)$	= position of desorption front, m
t	= time, s
T	= temperature, K; T_0 = initial frozen temperature, K
x	= displacement, m
kT_1	= $k_1(T_{v1} - T_0)/\{k_2(T_{v2} - T_{v1})\}$
kT_3	= $k_3(T_s - T_{v2})/\{k_2(T_{v2} - T_{v1})\}$
Z_1	= $(P_{v1} - P_{v2})/[\beta_2(C_{v1} - C_{v2})]$
Z_2	= $(P_{v2} - P_s)/[\beta_3(C_{v2} - C_s)]$
$Z_3(\eta)$	= $P_3(\eta) + \beta_3 C_3(\eta)$
α	= thermal diffusivity, m ² /s
α_p	= pressure diffusivity, m ² /s
ε	= porosity
κ	= permeability in Luikov (1975) system sec
ω	= volume fraction of free water
$\vartheta_1(\eta)$	= $(T_1(\eta) - T_0)/(T_{v1} - T_0)$
$\vartheta_2(\eta)$	= $(T_2(\eta) - T_{v1})/(T_{v2} - T_{v1})$
$\vartheta_3(\eta)$	= $(T_3(\eta) - T_s)/(T_{v2} - T_s)$
$\Theta_2(\eta)$	= $(C_2(\eta) - C_{v2})/(C_{v1} - C_{v2})$
$\Theta_3(\eta)$	= $(C_3(\eta) - C_s)/(C_{v2} - C_s)$
ρ	= density of absorbed water, kg/m ³
λ_1, λ_2	= position parameters at sublimation and desorption fronts
η	= $x/2\sqrt{\alpha_2 t}$
β	= $(D - \alpha_p)/\kappa$
v_1	= $\alpha_2 C_i L/\{k_2(T_{v2} - T_{v1})\}$
v_2	= $\alpha_2 \rho \Delta H/\{k_2(T_{v2} - T_{v1})\}$
v	= $v_1 = v_2$ dimensionless phase change heat
α_{21}	= α_2/α_1
α_{23}	= α_2/α_3
α_{p23}	= α_{p2}/α_{p3}
β_{23}	= β_2/β_3

Subscripts

- 1 = frozen region
 2 = sublimated region
 3 = desorbed region
 s = surface
 v1 = sublimation front
 v2 = desorption front

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