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Analytical and numerical investigation of interfacial stresses of FRP–concrete hybrid structure

Qing-Sheng Yang [a,*], Qing-Hua Qin [b,c], Dai-Hua Zheng [d]

[a] Department of Engineering Mechanics, Numerical Simulation Center for Engineering, Beijing Polytechnic University, Beijing 100022, China
[b] Department of Mechanics, Tianjin University, Tianjin 300072, China
[c] School of AMME, University of Sydney, Sydney, NSW 2006, Australia
[d] Central Research Institute of Building and Construction, Ministry of Metallurgical Industry, Beijing 100088, China

Abstract

This paper presents a study on interfacial stresses of a concrete column confined by fiber-reinforced plastic (FRP) plate, which has been widely applied in the civil engineering for rehabilitation and retrofitting of conventional structures. It is assumed that both the FRP plate and concrete structures are elastic and the interface between them is perfectly bonded. An analytical model for analysis of the interfacial stresses is developed and the finite element modeling is carried out for an axisymmetric FRP–concrete hybrid column. Components of the FRP plate with different geometric and material properties are considered to study their effects on the interfacial stresses. The study shows that the interfacial stresses are influenced by several factors, such as modulus ratio of FRP and concrete and thickness of FRP. This work provides a comprehensive investigation on the mechanical behavior of the interface in hybrid structures.

Keywords: Hybrid structures; Interfacial shear stress; Concrete; FRP; Mechanical model; Finite element method

1. Introduction

Due to its high strength, low weight, corrosion-resistance, and electromagnetic neutrality the fiber-reinforced plastic (FRP) becomes a popular composite recently and suitable for many structural applications such as rehabilitation and strengthening of concrete, steel or wood members, which results in some new structures like concrete–FRP, steel–FRP or wood–FRP hybrid structures.

External confinement of concrete by high-strength fiber composites can significantly enhance its strength and ductility as well as gain large energy absorption capacity. The confinement mechanism may occur by including fiber-wrapping of existing columns as a retrofitting measure or encasement of concrete in a FRP tube for new construction. The enhancement performance of such hybrid columns, however, depends on bonding quality and property of interface between concrete and FRP.

The high performance of hybrid structures results from the mechanism of stress transfer from concrete to FRP. Much progress in this field has been made in past decade. The progress includes the results and conclusions on overall properties and failure modes of hybrid structures, mechanical behavior of interface between concrete and FRP [1–5].

Column, which is able to carry axial compressive stress, is one of most important members in civil engineering. Concrete columns often suffer from the striking or corrosion and other local damage so that their performance degenerates. The concrete column confined by FRP can enhance the load-carrying capacity considerably and impact- and corrosion-resistance. The usual confinement method in practical engineering is that a concrete column is wrapped by carbon or glass fiber impregnated by resin. A laminate will be formed around the concrete column after the resin solidifies. Discussions on the performance of the concrete–FRP hybrid columns can be found in Ref. [4,5].

Apparently, performance of the interface between concrete and FRP plays an important role in hybrid columns due to the stress transfer from concrete to FRP laminate through the interface. FRP loses its protect

* Corresponding author. Tel.: +86-10-6739-2760; fax: +86-10-6739-1617.
E-mail address: qsyang@bjpu.edu.cn (Q.-S. Yang).

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function and reinforcement effect when the interface become failure such as debonding or slipping. The numerical simulation and experimental investigation have been carried out for properties and failure modes of the interface of the hybrid columns [6–10]. The experimental techniques [11,12] and theoretical models [13] were developed and the prestressed fibers was applied in the hybrid structures [14].

Present research focuses on the stress distribution at the interface of axisymmetric hybrid columns. The simple analytical and finite element models are proposed. The effect of property and size of FRP on the interfacial stresses are investigated. The numerical results show that there is a good agreement between analytical and finite element solutions.

2. Analytical model for interfacial shear stress

The common reinforcement method of hybrid column is such that a concrete column, which sustains axial pressure, is wrapped by FRP. The hybrid column consists of a concrete column core and a FRP shell (see Fig. 1). The FRP is not directly subjected to the external axial load. The load is transferred from concrete column to FRP shell through the interfaces between them. It is assumed that the hybrid column is cylindrical and its concrete core imposes the axial uniformed pressure, as shown in Fig. 1. In the figure, L is the half-length of the column, r_1 the radius of the column, and r_0 the radius of concrete core. Then the thickness of the FRP is $t = r_1 - r_0$.

It is assumed that the interface between the concrete and FRP is perfectly bonded and then the load transfer is carried out by way of the shear stress on the interface. An element with infinitesimal length, dx, of the column is considered here, as shown in Fig. 2. The axial traction of the axisymmetric element has an infinitesimal difference at the sections of x and $x + \mathrm{d}x$. In order to find the stresses of the element, the equilibrium conditions of the core and FRP parts will be considered, respectively.

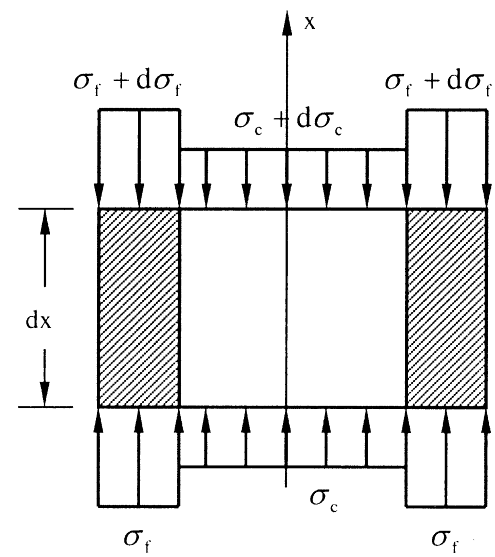

Fig. 2. Unit cell with length dx.

Firstly, consider a part of the concrete core with varied radius r, $0 \leqslant r \geqslant r_0$. The traction is shown in Fig. 3. The equilibrium condition along axis gives

$$\frac{\mathrm{d}\sigma_c}{\mathrm{d}x} - \frac{2\tau}{r} = 0 \tag{1}$$

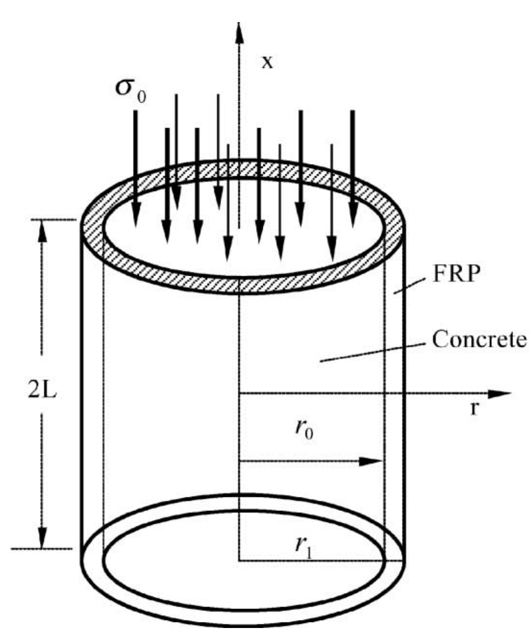

Fig. 1. Axisymmetric hybrid column.

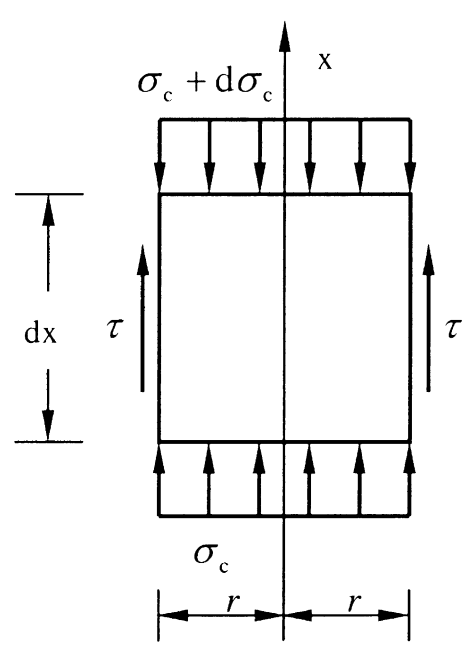

Fig. 3. Cylindrical concrete core.

where σ_c is axial average stress and τ shear stress in concrete. Note that $\tau = \tau_i$ when $r = r_0$, then we have

$$\frac{d\sigma_c}{dx} - \frac{2\tau_i}{r_0} = 0 \tag{2}$$

where τ_i is the shear stress at interface between concrete and FRP plate. Combining Eqs. (1) and (2) gives

$$\tau = \frac{r}{r_0}\tau_i \tag{3}$$

Let $u(x, r)$ be the axial displacement undergone in concrete and G_c be the shear modulus of concrete. Apply Hook's law to the elastic concrete, we have

$$\tau = G_c\frac{du}{dr} \tag{4}$$

Substituting Eq. (4) into Eq. (3), leads to

$$G_c\,du = \frac{r}{r_0}\tau_i\,dr \tag{5}$$

It is assumed that $u = u_c$ when $r = \bar{r}$ $(0 \leqslant \bar{r} \leqslant r_0)$ where u_c is the axial average displacement and $\bar{r}$ is an unknown parameter to be determined. Then the displacement condition can be written as

$$\begin{aligned} r = r_0 &: u = u_f \\ r = \bar{r} &: u = u_c \end{aligned} \tag{6}$$

where u_f is the axial displacement of FRP plate and is assumed to be a constant throughout the thickness considering the FRP plate being very thin. Intergrating of Eq. (5) reads

$$\int_{\bar{r}}^{r_0} r\frac{\tau_i}{r_0}\,dr = G_c\int_{u_c}^{u_f} du \tag{7}$$

From Eq. (7), the interfacial shear stress can be obtained as

$$\tau_i = \frac{2r_0G_c(u_f - u_c)}{r_0^2 - \bar{r}^2} \tag{8}$$

Substituting Eq. (8) into Eq. (2) we have

$$\frac{d\sigma_c}{dx} - \frac{4G_c(u_f - u_c)}{r_0^2 - \bar{r}^2} = 0 \tag{9}$$

Next, let us consider equilibrium condition of FRP element as shown in Fig. 4. The cylindrical shell FRP undergoes the shear τ_i on the interface and is free of traction on the outside of the shell. The equilibrium equation in axial direction of the FRP element provides

$$\frac{d\sigma_f}{dx} = \frac{2r_0\tau_i}{r_1^2 - r_0^2} \tag{10}$$

where σ_f is axial average stress in FRP shell.

Substituting Eq. (8) into Eq. (10), one has

$$\frac{d\sigma_f}{dx} - \frac{4r_0^2G_c(u_f - u_c)}{(r_1^2 - r_0^2)(r_0^2 - \bar{r}^2)} = 0 \tag{11}$$

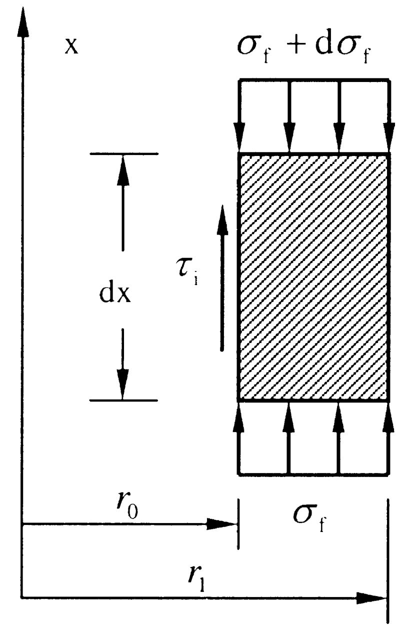

Fig. 4. FRP shell.

It is assumed that Hook's law is satisfied in both the concrete and the FRP shell, i.e.,

$$\sigma_c = E_c\frac{du_c}{dx}, \quad \sigma_f = E_f\frac{du_f}{dx} \tag{12}$$

where E_c and E_f are Young's moduli of the concrete and the FRP, respectively. Substituting Eq. (12) into Eqs. (9) and (11), a set of differential equations can be obtained in terms of average axial displacements u_c and u_f,

$$\begin{cases} \frac{d^2u_c}{dx^2} - \frac{4G_c}{(r_0^2 - \bar{r}^2)E_c}(u_f - u_c) = 0 \\ \frac{d^2u_f}{dx^2} - \frac{4r_0^2G_c}{(r_0^2 - \bar{r}^2)(r_1^2 - r_0^2)E_f}(u_f - u_c) = 0 \end{cases} \tag{13}$$

Eliminating u_c in Eq. (13) gives

$$\frac{d^4u_f}{dx^4} + (b - a)\frac{d^2u_f}{dx^2} = 0 \tag{14}$$

where $a = -((4G_c)/((r_0^2 - \bar{r}^2)E_c))$, $b = -((4r_0^2G_c)/((r_0^2 - \bar{r}^2) \times (r_1^2 - r_0^2)E_f))$.

Eq. (14) is a fourth order differential equation and its eigenequation is

$$\lambda^4 + (b - a)\lambda^2 = 0 \tag{15}$$

Its eigenvalues are obviously 0 and $\pm\sqrt{a - b}$. Then the solution of Eq. (14) can be written as

$$u_f = c_1 + c_2x + c_3\exp\left(\sqrt{a - b}x\right) + c_4\exp\left(-\sqrt{a - b}x\right) \tag{16}$$

Then the displacement u_c can be found by substituting Eq. (16) into the second part of Eq. (13):

$$u_c = c_1 + c_2 x + \frac{a}{b} c_3 \exp\left(\sqrt{a-b}x\right) + \frac{a}{b} c_4 \exp\left(-\sqrt{a-b}x\right) \quad (17)$$

where c_1, c_2, c_3, c_4 are unknown constants which can be determined by boundary conditions of the element.

For simplification, the half of the element is considered only due to the symmetry of the model (see also Fig. 1). The boundary conditions of the hybrid column are

$$\begin{cases} u_c|_{x=0} = u_f|_{x=0} = 0 \\ \sigma_c|_{x=L} = \sigma_0 \\ \sigma_f|_{x=L} = 0 \end{cases} \quad (18)$$

Having determined the unknown constants c_1, c_2, c_3, c_4, and the average axial displacements u_c and u_f from Eqs. (16)–(18)we can write the axial stresses of concrete and FRP shell, respectively, in the form

$$\sigma_c = \sigma_0 \frac{b}{a-b}\left[-1 + \frac{a}{b}\frac{1}{\cosh\left(\sqrt{a-b}L\right)}\cosh\left(\sqrt{a-b}x\right)\right] \quad (19)$$

$$\sigma_f = E_f \frac{\sigma_0}{E_c}\frac{b}{a-b}\left[-1 + \frac{1}{\cosh\left(\sqrt{a-b}L\right)}\cosh\left(\sqrt{a-b}x\right)\right] \quad (20)$$

Then the interfacial shear stress can be obtained by substituting Eq. (19) into Eq. (2) as

$$\tau_i = \sigma_0 \frac{r_0}{2}\frac{a}{\sqrt{a-b}}\frac{1}{\cosh\left(\sqrt{a-b}L\right)}\sinh\left(\sqrt{a-b}x\right) \quad (21)$$

When $x = L$, which is the end of hybrid column, the maximum of interfacial stress τ_i will be reached.

$$\tau_{i\,max} = \sigma_0 \frac{r_0}{2}\frac{a}{\sqrt{a-b}}\mathrm{th}\left(\sqrt{a-b}L\right) \quad (22)$$

3. Finite element simulation

A finite element analysis for the concrete–FRP is considered in this section. The commercial FE software ANSYS was used. The 8-node quadrilateral elements are chosen and smooth transition of the mesh was applied so that there are enough numbers of elements near the interface and the end of column. A typical FE mesh used in the analysis is shown in Fig. 5. The material and geometrical constants used in the calculation are as follows:

$$r_0 = 75 \text{ mm}, \quad r_1 = 79 \text{ mm}, \quad L = 750 \text{ mm},$$
$$E_c = 12.6 \text{ GPa}, \quad v_c = 0.1, \quad E_f = 10.0 \text{ GPa}, \quad v_f = 0.26 \quad (23)$$

where v_c and v_f are Poisson ratio of concrete and FRP, respectively.

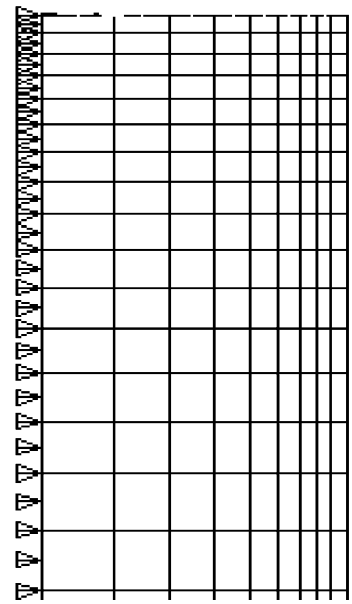

Fig. 5. An axisymmetric FE mesh.

Note that the interfacial stress in Eq. (19) is related to the unknown parameter $\bar{r}$. It is determined by a numerical experiment so that the results from Eq. (19) agree with FE results. The numerical relations of the interfacial shear stress distribution vs the distance from the end of column for different $\bar{r}$ in Eq. (19) and FE results are shown in Fig. 6 in which $h = L - x$. An agreement can be achieved between the formula and FE requires if $\bar{r} \cong r_0/2$. In other word, the displacement at $r = \bar{r} \cong r_0/2$ is equal to the average displacement in the concrete.

Fig. 7 shows interfacial shear stress distribution obtained by analytical solution and FE method, respectively, where different values of modulus of FRP is used, and other parameters are assumed to be the same as in

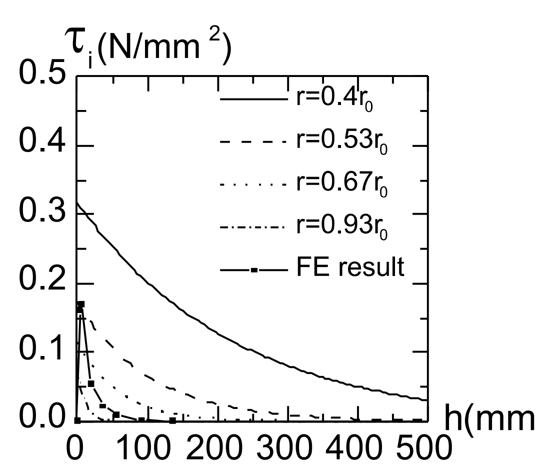

Fig. 6. Interfacial shear stresses.

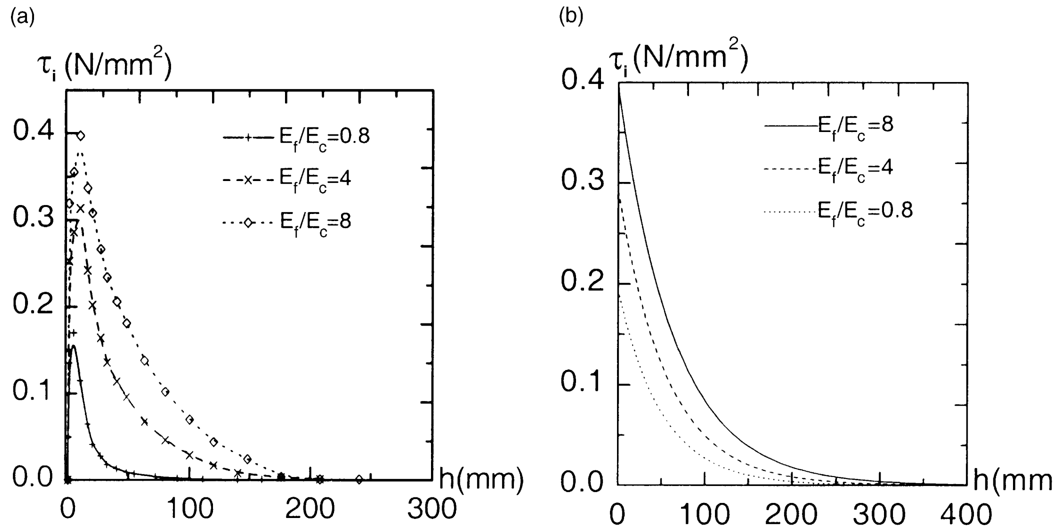

Fig. 7. The interfacial shear stress for different modulus ratios, (a) FE results, (b) analytical solutions.

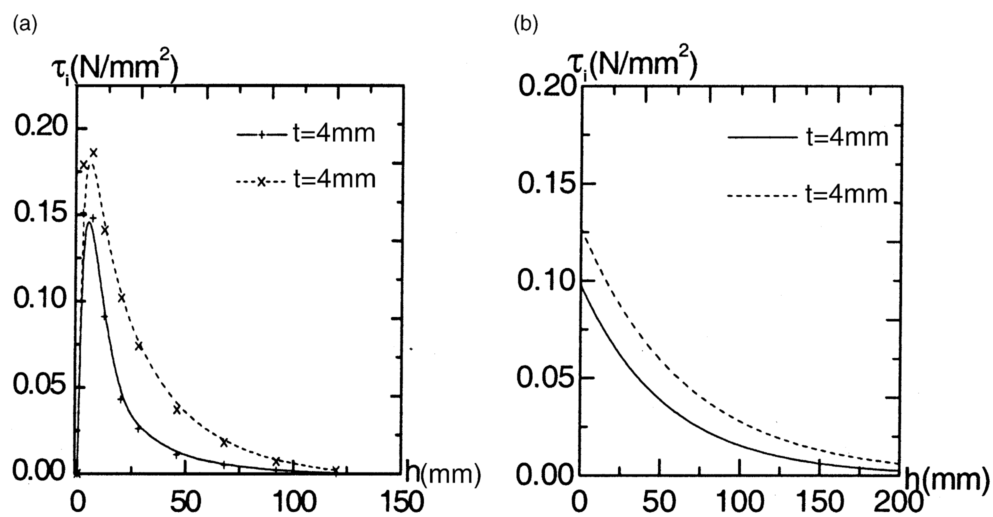

Fig. 8. The interfacial shear stress for different FRP thickness, (a) FE results, (b) analytical solutions.

Eq. (23). Three cases $E_f/E_c = 0.8, 4.0, 8.0$ are considered in order to investigate the effect of the modulus ratio of FRP and concrete on the interfacial shear stress. It is shown that interfacial shear stress increases as the modulus ratio increases. Accordingly, the maximum of interfacial shear stress at the end and stress transfer distance of the shear stress along the interface enhances.

Fig. 8 shows the effect of thickness of FRP on the interfacial shear stress for different values of thickness of FRP: $t = 4$ and 8 mm. The numerical results show that the interfacial shear stress and the maximum of the shear stress at the end considerably enhances along with the increase of the FRP thickness. The maximum of the shear stress at the end is a main cause for interfacial debonding between the concrete and FRP plate.

4. Conclusions

The conclusions of this work are following threefold:

(1) A theoretical model for analysis of interfacial shear stress in hybrid column is presented. The finite element simulation for different material and geometry parameters is carried out. The numerical examples show that the theoretical solution is agreeable well with the FE analysis.
(2) There is a strong stress concentration at the end of hybrid column. The maximum of the shear stress at the end is a main cause for interfacial debonding between the concrete and FRP plate.
(3) The distribution and maximum of interfacial shear stress are observably influenced by the modulus

ratio of concrete and FRP and thickness of FRP. The interfacial shear stress and maximum stress at the end enhance as the modulus ratio and FRP thickness increase.

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