METHOD OF FUNDAMENTAL SOLUTIONS FOR NONLINEAR SKIN BIOHEAT MODEL

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In this paper, the method of fundamental solution (MFS) coupling with the dual reciprocity method (DRM) is developed to solve nonlinear steady state bioheat transfer problems. A two-dimensional nonlinear skin model with temperature-dependent blood perfusion rate is studied. Firstly, the original bioheat transfer governing equation with nonlinear term induced by temperature-dependent blood perfusion rate is linearized with the Taylor's expansion technique. Then, the linearized governing equation with specified boundary conditions is solved using a meshless approach, in which the DRM and the MFS are employed to obtain particular and homogeneous solutions, respectively. Several numerical examples involving linear, quadratic and exponential relations between temperature and blood perfusion rate are tested to verify the efficiency and accuracy of the proposed meshless model in solving nonlinear steady state bioheat transfer problems, and also the sensitivity of coefficients in the expression of temperature-dependent blood perfusion rate is analyzed for investigating the influence of blood perfusion rate to temperature distribution in skin tissues.

Keywords: Nonlinear bioheat transfer; blood perfusion rate; method of fundamental solution; dual reciprocity method.

Symbols

\( a_1 \) : Coefficient in blood perfusion rate relation
\( a_2 \) : Coefficient in blood perfusion rate relation
\( a_3 \) : Coefficient in blood perfusion rate relation
\( c_b \) : Specific heat of blood (Jkg\(^{-1}\)K\(^{-1}\))
\( k \) : Thermal conductivity of tissue (Wm\(^{-1}\)K\(^{-1}\))
\( q \) : Heat flux (Wm\(^{-2}\))
\( Q_m \): Metabolic heat of tissue (Wm\(^{-3}\))
\( Q_r \): Spatial heat (Wm\(^{-3}\))
\( T \): Temperature of tissue (\(^{\circ}\)C)
\( T_b \): Artery blood temperature (\(^{\circ}\)C)
\( T_c \): Temperature of body core (\(^{\circ}\)C)
\( T_s \): Temperature of skin surface boundary (\(^{\circ}\)C)
\( \rho_b \): Density of blood (kgm\(^{-3}\))
\( \omega_b \): Blood perfusion rate (m\(^3\)s\(^{-1}\)m\(^{-3}\))
\( \theta \): Temperature variable for convenience (\(^{\circ}\)C)
\( \theta_s \): Surface temperature at boundary (\(^{\circ}\)C)
\( \theta_c \): Body core temperature at boundary (\(^{\circ}\)C)

1. Introduction

With the increasing of clinical research on human skin, numerical methods are widely developed to simulate and analyze bioheat transfer behaviors of various skin materials. Xu et al. reviewed the mathematical models and experimental methods to detail the progress of the thermal damage in skin tissue. For simplicity, some parameters in a complicated biological system, such as blood perfusion rate and thermal conductivity, are generally assumed to be constants. As a result, the biological heat conduction system in skin tissues is usually approximated by a linear bioheat governing equation. Currently, the linear bioheat model has been well developed and simulated successfully using various numerical methods. However, these parameters actually change with temperature in the bioheat system, rather than keeping constant. From the view point of material, the skin can be viewed as one kind of functionally graded materials (FGM) for changed material properties with different location. For this case, the commonly used linear biological system should be replaced with a nonlinear governing equation in order to obtain more accurate and reliable results. In this work, nonlinear effects of temperature-dependent blood perfusion rate on bioheat transfer are considered.

Regarding nonlinear bioheat transfer, some research works have been conducted to simulate the effect of temperature-dependent blood perfusion rate on temperature distribution in the biological system using numerical methods, instead of analytical methods, because of the complication of the nonlinear bioheat system. For example, Liu et al. used the dual reciprocity boundary element method (DRBEM) to give a simple investigation of plane nonlinear bioheat skin model with linear and exponential case of temperature-dependent blood perfusion rate for tumor hyperthermia diagnostics. Deng et al. also employed the DRBEM to study the response of temperature and heat flux in the transient nonlinear biological model. In their research work, three linear cases of temperature-dependent blood perfusion rates with different constants were involved. Besides, the finite element method (FEM) was
used by Kim et al. to investigate the nonlinear temperature behavior by introducing the various blood perfusion rates in a laser coagulation of human tissue model.\textsuperscript{7} Their research indicated that the tissue temperature could be overestimated significantly by ignoring the temperature dependence of blood perfusion rate. Similar conclusions were drawn by Drizdal et al. in their research of three-dimensional temperature distribution prediction for superficial hyperthermia using the commercial finite element software, COMSOL multi-physics package.\textsuperscript{8} Among the numerical methods mentioned above, the DRBEM\textsuperscript{9} can be viewed as the mixed boundary-type element method, which integrates the domain and boundary discretization by the boundary element method (BEM) and the domain interior collocation implemented by simple basis function interpolation. Thus, only boundary integrals are included in the procedure of DRBEM. Different from the DRBEM, the FEM\textsuperscript{10–12} is a classical domain-type element method, which employed the domain discretization by large number of elements, based on weak energy integral functional. So the domain integral is involved in the procedure of FEM.

Besides the element-type BEM and FEM, the meshless method like the method of fundamental solutions coupling with the dual reciprocity method (DRM-MFS) has been well developed to predict the temperature distribution for the linear or nonlinear heat transfer problems.\textsuperscript{13–16} The kernel functions, that is, fundamental solutions, in the conventional MFS can theoretically be viewed as one type of Trefftz basis.\textsuperscript{17} The meshless DRM-MFS is a type of collocation method and is usually performed by allocating internal and boundary points in the solution domain to achieve the proper particular and homogeneous solutions, respectively. The particular solutions are usually approximated by the radial basis function (RBF) interpolation at interior points, while the homogeneous solutions are approximated by constructing an explicit solution with the superposition of finite number of source points on artificial boundary, in terms of the fundamental solutions of the homogeneous problems. Since no mesh generation process and integral are involved in the procedure of DRM-MFS, and thus it is purely meshless or meshfree. Additionally, it can easily be implemented and programmed, because of the ease of collocation. These are advantages of the meshless DRM-MFS over the element-type methods like DRBEM and FEM. Moreover, to deal with those problems in which complicated governing equations are encountered and thus no explicit fundamental solutions are available, the DRM-MFS has been improved by introducing the analog equation method (AEM)\textsuperscript{18} for the solutions of nonlinear steady state heat conduction problems in anisotropic and isotropic inhomogeneous systems.\textsuperscript{14,15} Furthermore, Wang and Qin discussed some potential problems including the location of the virtual boundary, the differential and integrating strategies and the effect of shape parameter in multiquadric basis in the usage of the DRM-MFS.\textsuperscript{19}

To our knowledge, the application of the DRM-MFS to nonlinear bioheat problems has not, as yet, been investigated, thus in this paper, the meshless DRM-MFS is developed to determine temperature distribution in the nonlinear skin system in which the blood perfusion rate is assumed to be a function of temperature. The
paper is organized as follows: In Sec. 2, a two-dimensional skin tissue model with temperature-dependent blood perfusion is presented. Then solution procedures including AEM, DRM and MFS are described in Sec. 3. Section 4 presents numerical results from the proposed DRM-MFS which are compared with those from MATLAB PDE Toolbox. Additionally the sensitivity analysis for various blood perfusions is also included in Sec. 4. At the end of this paper, conclusions are made.

2. Mathematical Model of Skin Tissue

The steady-state heat transfer in a biological tissue is usually governed by the well-known Pennes’s bioheat equation\(^{20,21}\)

\[
k \nabla^2 T + \rho_b c_b \omega_b (T_b - T) + Q_r + Q_m = 0,
\]

in which \(k\) is the thermal conductivity, \(T\) temperature change of the tissue, \(\nabla^2\) the Laplace operator, \(\rho_b\), \(c_b\) and \(\omega_b\) are, respectively, density, specific heat and perfusion rate of blood, \(T_b\) is the temperature of arterial blood, \(Q_m\) and \(Q_r\) are, respectively, metabolic heat generation and heat deposition in tissues caused by outer heating factor such as laser and microwave.

The bioheat transfer Eq. (1) is a statement of the law of conservation of energy. The first term on the left represents conduction of heat in the tissue caused by the temperature gradient, and the second term describes the heat transport between the tissue and microcirculatory blood perfusion. The third and last terms are internal heat generation due to tissue metabolism and outer heating sources.

According to physiology of biological tissues containing blood vessels, the blood vessels will expand along with an increase in the temperature to allow more blood flow to dissipate the heat accumulated in the body. So, the blood perfusion varies practically with the tissue temperature. As a result, the governing Eq. (1) can be rewritten as follows

\[
k \nabla^2 T + \rho_b c_b \omega_b (T_b - T) + Q_r + Q_m = 0.
\]

In hyperthermia treatment, the blood perfusion varies linearly with tissue temperature \(T\) as\(^{6,22}\)

\[
\omega_b(T) = a_1 + a_2 T,
\]

or exponentially with \(T\) as\(^{2,7}\)

\[
\omega_b(T) = a_1 e^{a_2 T},
\]

or quadratically in terms of \(T\) as\(^2\)

\[
\omega_b(T) = a_1 + a_2 T + a_3 T^2,
\]

where \(a_1\), \(a_2\) and \(a_3\) are positive constants.
For the sake of convenience, we introduce a new temperature variable $\theta$ as

$$\theta = T - T_b.$$  \hspace{1cm} (6)

Then, the governing Eq. (2) can be rewritten in terms of the new variable as

$$k\nabla^2 \theta - \rho_b c_b \omega_b (\theta + T_b) \theta + Q_r + Q_m = 0.$$  \hspace{1cm} (7)

To deal with the nonlinearity caused by the temperature-dependent blood perfusion, following linearized strategy is introduced using the first-order Taylor-series expansion, i.e.,

$$\rho_b c_b \omega_b (\theta + T_b) \theta = \theta f_1(\theta^n) + f_2(\theta^n),$$  \hspace{1cm} (8)

where $\theta^n$ is the solution at $n$th iteration and

$$f_1(\theta^n) = \rho_b c_b \left( \theta \frac{\partial \omega_b}{\partial \theta} + \omega_b \right) \bigg|_{\theta = \theta^n},$$

$$f_2(\theta^n) = \rho_b c_b \omega_b (\theta^n + T_b) - f_1(\theta^n) \theta^n.$$  \hspace{1cm} (9)

Making use of the three types of blood perfusion rate defined by Eqs. (3)–(5), the term $f_1$ can be written as

$$f_1(\theta^n) = \begin{cases} 
\rho_b c_b (a_2 \theta^n + \omega_b) & \text{linear case,} \\
\rho_b c_b [a_1 a_2 e^{a_2 (\theta^n + T_b)} + \omega_b] & \text{exponential case,}
\end{cases}$$

$$f_1(\theta^n) = \rho_b c_b [2a_3 (\theta^n)^2 + (a_2 + 2a_3 T_b) \theta^n + \omega_b] \quad \text{quadratic case.}$$  \hspace{1cm} (10)

Substituting Eq. (8) into Eq. (7), we have

$$k\nabla^2 \theta - f_1(\theta^n) \theta - f_2(\theta^n) + Q_r + Q_m = 0$$  \hspace{1cm} (11)

or

$$\nabla^2 \theta - \frac{f_1(\theta^n)}{k} \theta = \frac{f_2(\theta^n) - Q_r - Q_m}{k}.$$  \hspace{1cm} (12)

Equation (11) is a nonhomogeneous potential equation and the coefficient $f_1(\theta^n)$ changes with spatial position, because the iteration temperature $\theta$ is generally a function of spatial coordinates. Equation (11) will be completed by considering following boundary conditions.

In this study, the single rectangular domain model\textsuperscript{5, 6} is considered (see Fig. 1). The boundary $\Gamma_1$ represents the right most surface of the skin, thus the temperature on $\Gamma_1$ can be approximately assumed to be the body core temperature $\theta_c$, that is

$$\theta = \theta_c \quad \text{at boundary } \Gamma_1.$$  \hspace{1cm} (13)

At the upper and bottom surfaces, no heat flow occurred along these two edges by assuming that the tissue far from the area of interest is not affected by the imposed thermal disturbance,\textsuperscript{23–25} so the conditions at these two surfaces are given by

$$-k \frac{\partial T}{\partial n} = 0 \quad \text{at boundaries } \Gamma_2 \text{ and } \Gamma_3.$$  \hspace{1cm} (14)
At the left surface of the tissue, the temperature is assumed to be constant which is approximately equal to the temperature of the contact heating body, \( 23-25 \) i.e.,
\[
\theta = \theta_s \quad \text{at boundary } \Gamma_4.
\] (15)

3. Solution Procedure

3.1. The AEM

According to the basic theory of the AEM \(18\), if the temperature \( \theta \) is twice differential with respect to spatial variable \( x \), we can apply the Laplace operator to the sought solution \( \theta \) leading to the following equivalent system \(15,18\)
\[
\nabla^2 \theta(x) = b(x),
\] (16)
in which the right-hand side term \( b \) contains generally the unknown temperature.

Due to the linear feature of the Laplace operator, the solution to Eq. (16) can be divided into two parts:
\[
\theta(x) = \theta_h(x) + \theta_p(x),
\] (17)
where \( \theta_h(x) \) is the homogeneous solution satisfying
\[
\nabla^2 \theta_h(x) = 0,
\] (18)
which is subjected to a modified boundary conditions, and \( \theta_p(x) \) stands for the particular solution satisfying
\[
\nabla^2 \theta_p(x) = b(x).
\] (19)

Equations (18) and (19), respectively represent the Laplace equation and Poisson’s equation. Their solution can be obtained separately using the following DRM and MFS.
3.2. **DRM for particular solutions**

In the dual reciprocity technique, it is essential to approximate the source term $b(x)$ by a series of RBFs.\(^{26}\) Let $\phi$ be a RBF, the source term $b$ in Eq. (19) can be approximated as follows:

$$ b(x) = \sum_{i=1}^{M} \alpha_i \phi_i(r), $$

where $r = \|x - x_i\|$ and $\{x_i\}_1^M$ is a set of points for interpolation in the domain of interest.

Then, the particular solution of Eq. (19) can be obtained in the following way

$$ \theta_p(x) = \sum_{i=1}^{M} \alpha_i \Phi_i(r), $$

where

$$ \nabla^2 \Phi_i(r) = \phi_i(r). $$

If we employ the following thin plate spline (TPS)\(^{26}\) to approximate $b$ in Eq. (20)

$$ \phi_i(r) = r^{2n} \ln r, \quad n = 1, 2, 3, \ldots, $$

then the particular solution $\Phi_i(r)$ can be obtained directly

$$ \Phi_i(r) = \frac{(n + 1) \ln r - 1}{4(n + 1)^3} r^{2+2n}. \quad n = 1, 2, 3, \ldots $$

3.3. **MFS for homogeneous solutions**

In the proposed MFS, $N$ virtual source points outside the domain are used. According to the basic definition of the fundamental solution for the Laplace operator,\(^{14}\) we have

$$ \nabla^2 G(x, s_j) + \delta(x, s_j) = 0, $$

where $x(x, y)$ is a field point inside the domain of interest or on its boundary, $s_j(x_j, y_j)$ ($j = 1, 2, \ldots, N$) is the fictitious source points outside the domain and $G(x, s_j)$ is the fundamental solution for the Laplacian operator, i.e.,

$$ G_j(x) = -\frac{1}{2\pi} \ln \sqrt{(x - x_j)^2 + (y - y_j)^2}, $$

then the linear combination of the fundamental solution at different source points

$$ \theta_{h}(x) = \sum_{j=1}^{N} \beta_j G_j(x), \quad x \neq s_j, $$

obviously satisfy the Laplace Eq. (18). Here we denote $G_j(x) = G(x, s_j)$ for simplicity.
In the conventional MFS, the coordinates of source points are prescribed in advance to simplify the computation and usually the source points can locate on a pseudo boundary whose shape is similar to the physical domain boundary. The distance of pseudo and physical boundaries has been studied by many researchers and typically it can be selected to be 0.8–3 times the characteristic length of the real boundary to achieve stable results.\textsuperscript{27–29} Besides, to avoid putting source points outside the domain, Chen \textit{et al.} developed a regularized meshless method by using the double-layer potential, and thus the source points can be located on the physical boundary.\textsuperscript{30,31}

### 3.4. Construction of complete solution

Based on the process above, the final complete solution can be expressed below

\[
\theta(x) = \sum_{i=1}^{M} \alpha_i \Phi_i(x) + \sum_{j=1}^{N} \beta_j G_j(x),
\]

\[
q(x) = -\sum_{i=1}^{M} \alpha_i \frac{\partial \Phi_i(x)}{\partial n} - \sum_{j=1}^{N} \beta_j \frac{\partial G_j(x)}{\partial n}.
\]

For simplicity, Eqs. (28) and (29) are rewritten in matrix form

\[
\theta(x) = U(x)c,
\]

\[
q(x) = Q(x)c,
\]

where

\[
U(x) = [\Phi_1(x) \cdots \Phi_M(x) \ G_1(x) \cdots G_N(x)],
\]

\[
Q(x) = \begin{bmatrix}
-\frac{\partial \Phi_1(x)}{\partial n} & \cdots & -\frac{\partial \Phi_M(x)}{\partial n} & -\frac{\partial G_1(x)}{\partial n} & \cdots & -\frac{\partial G_N(x)}{\partial n}
\end{bmatrix},
\]

\[
c^T = [\alpha_1 \cdots \alpha_M \ \beta_1 \cdots \beta_N].
\]

As can be seen from Fig. 1, the complete boundary of two-dimensional skin model $\Gamma$ is composed of four boundaries $\Gamma_1$, $\Gamma_2$, $\Gamma_3$ and $\Gamma_4$. A set of points $\{P_i\}_{i=1}^{N}$ are selected on the boundary $\Gamma$. There are $N_1$, $N_2$, $N_3$ and $N_4$ points uniformly distributed on boundaries $\Gamma_1$, $\Gamma_2$, $\Gamma_3$ and $\Gamma_4$, respectively. Therefore, $N = N_1 + N_2 + N_3 + N_4$. Similarly, a set of fictitious source points $\{Q_i\}_{i=1}^{N}$ outside the solution domain are placed on the pseudo boundary $\Gamma_{ps}$. Correspondingly, $N_1$, $N_2$, $N_3$ and $N_4$ fictitious source points are uniformly distributed on pseudo boundaries segments parallel to $\Gamma_1$, $\Gamma_2$, $\Gamma_3$ and $\Gamma_4$. In the following numerical examples, we choose $N_2 = N_3 = 7$ and $N_1 = N_4 = 9$. Thus, $N = N_1 + N_2 + N_3 + N_4 = 32$. Therefore, a linear equations system is required for uniquely determining in the 32 unknowns. Further, considering that $M$ unknown coefficients occurred in the process of DRM for particular solutions, the final linear equation system consists of $M + N = M + 32$ unknowns. These unknowns can be uniquely determined by imposing the temperature $\theta$ to
satisfy the governing Eq. (12) at \( M \) internal points and the boundary conditions Eqs. (13), (14) and (15) at \( N \) boundary points. The resulting equation system can then be written as

\[
\begin{align*}
[B(x_i) - A(x_i)U(x_i)]c &= F(x_i), & i = 1 \ldots M, \\
U(x_j)c &= \theta_e, & j = 1 \ldots N_1, \\
Q(x_k)c &= 0, & k = 1 \ldots N_2, \\
Q(x_l)c &= 0, & l = 1 \ldots N_3, \\
U(x_m)c &= \theta_s, & m = 1 \ldots N_4,
\end{align*}
\] (35)

in which

\[
A(x_i) = \left. \frac{f_1(\theta^\mu)}{k} \right|_{x_i}, \quad F(x_i) = \left. \frac{f_2(\theta^\mu) - Q_r - Q_m}{k} \right|_{x_i}
\] (36)

and

\[
B(x_i) = \nabla^2 U(x_i) = \begin{bmatrix} \nabla^2 \Phi_1(x_i) & \cdots & \nabla^2 \Phi_M(x_i) & \nabla^2 G_1(x_i) & \cdots & \nabla^2 G_N(x_i) \end{bmatrix} = \begin{bmatrix} \phi_1(x_i) & \cdots & \phi_M(x_i) & 0 & \cdots & 0 \end{bmatrix}.
\] (37)

Solving the linear equation system yields the unknown coefficient vector \( c \) and then the temperature field can be determined using Eq. (30).

4. Numerical Results and Discussion

In this section, all calculations are based on the rectangular domain illustrated in Fig. 1. Only the upper half of the domain is used in our calculation due to symmetry of this model. The thermal parameters used in the calculation are listed in Table 1.

### Table 1. Thermal parameters of the skin tissue.

<table>
<thead>
<tr>
<th>Thermal properties of skin</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermal conductivity ( k ) (Wm(^{-1})K(^{-1}))</td>
<td>0.5</td>
</tr>
<tr>
<td>Density of blood ( \rho_b ) (kgm(^{-3}))</td>
<td>1000</td>
</tr>
<tr>
<td>Specific heat of blood ( c_b ) (Jkg(^{-1})K(^{-1}))</td>
<td>4200</td>
</tr>
<tr>
<td>Spatial heat ( Q_r ) (Wm(^{-3}))</td>
<td>30000</td>
</tr>
<tr>
<td>Metabolic heat ( Q_m ) (Wm(^{-3}))</td>
<td>4200</td>
</tr>
<tr>
<td>Temperature of body core ( T_c ) (°C)</td>
<td>37</td>
</tr>
<tr>
<td>Temperature of skin surface ( T_s ) (°C)</td>
<td>25</td>
</tr>
</tbody>
</table>

4.1. Verification of the proposed method

In order to validate the efficiency and accuracy of the proposed DRM-MFS method for analyzing the steady state bioheat transfer in skin tissues with metabolic heat...
and temperature dependent blood perfusion rate, MATLAB partial differential equation (PDE) Toolbox is employed to simulate bioheat transfer in the 2D skin tissue model. The results from DRM-MFS and PDE Toolbox are compared for the three cases of temperature dependent blood perfusion rate defined by Eqs. (3)–(5). In all the calculations, 63 interpolation points inside the rectangular domain are used for modeling the particular solutions. While 32 boundary nodes along each of the four boundaries $\Gamma_1$, $\Gamma_2$, $\Gamma_3$ and $\Gamma_4$ and the same number of nodes (32) along the pseudo boundary are, respectively, used to determine the homogeneous solutions. To investigate the convergence of the present algorithm with respect to the interpolation points, the results obtained using 486 interpolation points are compared with those using 63 points. The collocation scheme with the random 486 interpolation points is displayed in Fig. 2.

For the purpose of comparison, the finite element scheme embedded in the MATLAB PDE Toolbox is employed to produce the corresponding results. In the calculation, the solution domain is modeled with 1044 triangular elements and 560 nodes whose results can be viewed as a reference for comparison. Consider first the case $\omega_b(T) = a_1 + a_2T$ with $a_1 = 0.0005$ and $a_2 = 0.0001$.

It can be seen from Fig. 3 that, results from the proposed DRM-MFS algorithm (with 63 interpolation points inside the rectangular domain) has negligible difference from the finite element results obtained using MATLAB PDE Toolbox, the results from the proposed DRM-MFS algorithm can converge to those from the MATLAB PDE Toolbox when the interpolation points inside the rectangular domain increases to 486. Moreover, the relative error of the results from the DRM-MFS algorithm with respect to those from the PDE Toolbox is listed in Fig. 4. It is

![Collocation scheme with random 486 interpolation points.](image)
observed from the figure that the maximum relative error for the DRM-MFS with 63 interpolation points is roughly 0.34%. In contrast, the maximum relative error for the DRM-MFS with 486 interpolation points is about 0.06%.

Secondly, for the case of blood perfusion rate

$$\omega_b(T) = a_1 + a_2 T + a_3 T^2,$$

the positive constants $a_1 = 0.0005$, $a_2 = 0.0002$ and $a_3 = 0.000001$ are adopted.\(^2\)
Numerical results from the present DRM-MFS with 63 and 486 interpolation points are presented in Fig. 5 and the relative error is shown in Fig. 6. From these two figures, it is clearly seen that the more the number of interpolation points are, the more accurate the numerical results are. So the convergence and accuracy of the present meshless method for the case of quadratic form is verified.

Fig. 5. Validation with the quadratic case blood perfusion rate.

Fig. 6. Relative error of quadratic case.
Thirdly, consider the exponential case \( \omega_b(T) = a_1 e^{a_2 T} \) with \( a_1 = 0.0005 \) and \( a_2 = 0.01 \). It can be seen from Figs. 7 and 8 that both the DRM-MFS with 63 and 486 interpolation points inside the solution domain have almost the same numerical results, although the results from DRM-MFS with 486 interpolation points are still more accurate than those from DRM-MFS with 63 internal points.

![Fig. 7. Validation with exponential case of blood perfusion rate.](image1)

![Fig. 8. Relative error of exponential case.](image2)
Finally, to investigate the effect of blood perfusion rate on temperature distribution, numerical results for problems with the blood perfusion rate as a function of temperature (linear, quadratic and exponential relations) are displayed in Fig. 9, from which it can be seen that there are relatively larger temperature gradient in the region nearby the skin surface for all three blood perfusion forms, due to the significant temperature difference between the skin surface and the body core or artery blood. Besides, the three temperature curves have an intersect point which locates at about $x = 12.18$ mm. In the region $x < 12.18$ mm, the quadratic form produces the highest tissue temperature and the exponential form produces lowest tissue temperature, while in the region $x > 12.18$ mm, the temperature distribution changes inversely. The highest temperature is found to exceed 39°C, due to the nonlinearity of the bioheat equation.

Furthermore, to investigate the influence of TPS type of RBFs on temperature distribution, results for the order $n = 1$ and $n = 2$ are presented in Fig. 10. In Fig. 10, the quadratic case of temperature-dependent blood perfusion rate is only used. We see that curve of the skin tissue temperature obtained by the proposed DRM-MFS with order $n = 2$ matches much better with those from the PDE Toolbox simulation results than the DRM-MFS with order $n = 1$. It indicates that increasing the order of TPS type can improve the accuracy of the DRM-MFS, without increasing the number of interpolation points inside the domain.

Figure 11 shows the relative errors of the proposed DRM-MFS with different order of TPS RBFs and 63 interpolation points. The maximum error for $n = 1$ is 0.56% which is slightly higher than that for $n = 2$ (0.17%).
It is concluded from the analysis above that numerical experiments from DRM-MFS can converge to the reference value when increasing the internal interpolation points. The method seems to be a promising and simple method in solving the nonlinear steady state bioheat transfer problems. In the next section, the sensitivity analysis is conducted using the proposed meshless method.

![Fig. 10. Validation of quadratic case with different TPS orders.](image1)

![Fig. 11. Relative error of quadratic case with different TPS order and interpolation points.](image2)
4.2. Sensitivity to variation of constants $a_i$ in quadratic case of blood perfusion rate

In this section, the quadratic case of temperature dependent blood perfusion rate $\omega_b(T) = a_1 + a_2T + a_3T^2$ is considered to investigate the sensitivity of tissue temperature to the variation of constants $a_i$. In the sensitivity analysis, 486 interpolation points and first-order TPS type RBFs are employed. First of all, the constant term $a_1$ in Eq. (5) is assumed to be 0.00005, 0.0005 and 0.005 when the first order term constant $a_2$ and quadratic term constant $a_3$ are set to be 0.0002 and 0.000001, respectively. It is noticed from Fig. 12 that, when the constant $a_1$ is changed from 0.0005 to 0.00005, the variation of the skin tissue temperature curve is negligibly small. However, when the constant $a_2$ is changed from 0.0005 to 0.005, the variation of the skin tissue temperature curve is relatively larger. The location at about (12.2 mm, 0) is the crossing point of three curves with different constant $a_1$. It means that from location (1.875 mm, 0) to (12.2 mm, 0), the skin tissue temperature increases with an increase in the constant $a_1$. But between the location (12.2 mm, 0) and (28.125 mm, 0), the skin tissue temperature decreases along with an increase in the constant $a_1$. The main reason is that before the location (12.2 mm, 0), the skin tissue temperature is lower than the blood temperature. Therefore, the heat flow is transferred from blood to skin tissue. Larger blood perfusion rate means more heat flows from blood to skin tissue. In contrast, after the rough location (12.2 mm, 0) the skin tissue temperature is larger than the blood temperature. Thus the heat flow is transferred from skin tissue to blood. Therefore, larger blood perfusion rate allows more heat flow loss from skin tissue to blood.

![Fig. 12. Sensitivity to constant $a_1$ in quadratic case of blood perfusion rate.](image)
Next, the constant $a_1$ is set to be 0.0005 and quadratic term $a_3$ is set to be 0.000001 while the first-order term coefficient $a_2$ is changed from 0.00002 to 0.002. Compared with the situation in Fig. 12, the variation of skin tissue temperature curve caused by the changing of first-order coefficient $a_2$ is relatively larger than changing constant $a_1$ (see Fig. 13). From Fig. 13, it can also be found that increasing the first-order coefficient $a_2$ makes the skin tissue temperature increase fast to the

![Fig. 13. Sensitivity to constant $a_2$ in quadratic case of blood perfusion rate.](image1)

![Fig. 14. Sensitivity to constant $a_3$ in quadratic case of blood perfusion rate.](image2)
same temperature as blood temperature and keep around 37°C which is equal to the blood temperature. Thus a strong heat flow protection or regulation effect of blood to the skin tissue is shown in Fig. 13 especially when the first-order term coefficient $a_2$ is equal to 0.002.

As we can see from Fig. 14, if the constant $a_1$ and $a_2$ are kept to be constant at 0.0005 and 0.0002, respectively, the variation of $a_3$ makes the crossing point move to the location roughly at (11.25 mm, 0). From the skin surface boundary to the location (11.25 mm, 0), larger quadratic term coefficient $a_3$ produces larger skin tissue temperature. In contrast, from the crossing point location (11.25 mm, 0) to the body core boundary, smaller skin tissue temperature is induced by the larger quadratic term coefficient $a_3$. It is presented that when the quadratic term coefficient of the temperature dependent blood perfusion $a_3$ is equal to 0.00001, the skin tissue temperature keeps quite stable rate from location (11.25 mm, 0) to (26.25 mm, 0) at 37°C, which is the same as the blood temperature.

5. Conclusion

In this work, a meshless DRM-MFS algorithm is developed for analyzing the nonlinear bioheat transfer in a 2D skin model. The nonlinearity is due to the temperature dependence of the blood perfusion rate. The Taylor expansion technology is first employed to linearize the nonlinear bioheat equation and then the DRM and the MFS coupling with the analogy equation technique are respectively used to derive the particular and homogeneous solutions. The satisfaction of the governing equations and boundary conditions at interpolation points and boundary collocation points can determine all unknowns. Next, numerical experiments are performed to verify the developed meshless algorithm and numerical results show that the accurate and convergent results can be obtained by using proposed meshless method in solving the nonlinear bioheat transfer problems considered in the paper. Also, results obtained from the proposed meshless model show that the change of the blood perfusion rate in terms of temperature variable plays a significant role in altering the temperature distribution inside the tissue body. Finally, the sensitivities of the three positive constants in the quadratic form of the blood perfusion rate are evaluated to investigate the caused temperature change in the tissue due to various parameters. It is found that the variations of second and third coefficients in the expression of quadratic blood perfusion rate can cause evident temperature change.

References


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