Simulation of ellipsoidal particle-reinforced materials with eigenstrain formulation of 3D BIE

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1. Introduction

Determination of the elastic behavior of an inclusion embedded in a matrix is of considerable importance in a wide variety of physical and engineering problems. Following the pioneering work of Eshelby [1,2], inclusion and inhomogeneity problems have been a focus of solid mechanics for several decades. Due to Eshelby's work on an equivalent inclusion and eigenstrain solution, numerous investigations both analytical [3–9] and numerical [10–17] have been reported in the literature. In various physical problems, the eigenstrain can represent thermal mismatch, lattice mismatch, phase transformation, microstructural evolution, and intrinsic strains in residual stress problems [18]. Eshelby's solution is of great versatility and has been employed to address a wide range of physical problems in materials science, mechanics, and physics.

The analytical equivalent inclusion models available in the literature can be taken as the basis for predicting stress/strain distributions either within or outside the inhomogeneity and for further study of the mechanical performance of heterogeneous materials. However, the available solutions apply generally to only simple geometries such as single ellipsoidal, cylindrical and spherical inclusions in an infinite domain, because of the complexity of the mathematical expression and difficulty in solving the corresponding governing equations in 3D systems. Therefore, numerical methods including finite element methods (FEM), volume integral methods (VIM) and boundary element methods (BEM) have been used in the analysis of inhomogeneity problems involving various shapes and materials. The FEM may yield results for the entire composite materials, including results within the inhomogeneity [11], but the solution scale would be very large since both matrix and inhomogeneities must be discretized. The VIM and the BEM seem more suitable than the FEM for the solution of inhomogeneity problems. However, in the VIM [12–14], the domains of inhomogeneity are represented by volume integrals, essentially simplifying the construction of the final matrix of the linear algebraic system to which the problem is reduced to some extent after discretization. However, as the interfaces between matrix and inclusion need to be discretized in the VIM, it is suitable for small scale problems with only a few inhomogeneities. The situation in the application of the BEM to inclusion problems, often coupled with VIM [15,16], is much the same as that of the VIM in which problems of simple arrays of inclusions are solved on a small scale, for a similar reason to that in the VIM, i.e., unknowns appearing in
the interfaces. For large-scale problems of inhomogeneity with the BEM [17], special techniques of fast multipole expansions [19] must be employed, which leads to complexity of the solution procedure.

To the authors’ knowledge, the potential engineering applications of Eshelby’s idea of equivalent inclusion and eigenstrain solution have not yet been fully explored in the area of computational treatment of materials with inhomogeneities [20]. With Eshelby’s idea as the basis, Ma et al. [21] recently proposed the eigenstrain formulation of the BIE for modeling elliptical particle-reinforced materials.

In two-dimensional elasticity. In the present work, that computational model is extended to the three-dimensional case by incorporating the corresponding BEM for analyzing the stress/strain behavior of ellipsoidal particle-reinforced/void-weakened materials.

2. Eigenstrain formulations of BIE

In the present model, perfect adhesion between the particle and the matrix, both being isotropic materials, is assumed, so that the displacement continuity and the traction equilibrium still hold in the matrix, both being isotropic materials, is assumed, so that the tractions and stresses, respectively.

It should be mentioned that the coefficients in the system matrix, \( \mathbf{B} \), are the shear modulus, \( \mu \), and Poisson’s ratio, \( \nu \).

3. Solution procedures

The present computational model for ellipsoidal particle-reinforced materials is solved numerically by way of the BEM [22]. In order to avoid domain discretization, the domain integrals in Eqs. (1) and (2) need to be transformed into boundary-type integrals before discretization, as [23]

\[
\int_{\Omega} \sigma_{ijkl} \, d\Omega = \int_{\Gamma} x_i \tau_{ijk} \, d\Gamma
\]

where \( \mathbf{A} \) is the system matrix, \( \mathbf{B} \) is the coefficient matrix for eigenstrains, \( \mathbf{b} \) the right vector related to the known quantities applied on the outer boundary, \( \mathbf{x} \) the vector unknowns to be solved, \( \kappa \) is the eigenstrain vector of all the particles to be corrected in the iteration. It should be mentioned that the coefficients in \( \mathbf{A} \), \( \mathbf{B} \) and \( \mathbf{b} \) are all constants, and thus need to be computed only once. At the starting point, the vector \( \kappa \) is assigned by initial values for the applied
strains via Eq. (2) at each position of the particles in the elastic state, computed irrespective of particles (i.e., the whole solution domain contains the homogeneous matrix only). Then the unknown vector \( \mathbf{x} \) can be computed by the following iterative formulae:

\[
\mathbf{x}(k+1) = \mathbf{A}^{-1} (\mathbf{b} + \mathbf{B} \mathbf{e}(k)) \tag{11}
\]

where \( k \) is the iteration count. Define the maximum iteration error as

\[
\varepsilon_{\text{max}} = \max \left| \{ \mathbf{e}^{(k)} - \mathbf{e}^{(k-1)} \} \right| \tag{12}
\]

which is the maximum difference of the eigenstrain components between two consecutive iterations. The convergent criterion in the present study is chosen as follows:

\[
E_{\text{M}}\varepsilon_{\text{max}} \leq 10^{-3} \tag{13}
\]

where \( E_{\text{M}}\varepsilon_{\text{max}} \) corresponds to the maximum difference of the 'stress' components among total equivalent inclusions between two consecutive iterations and \( E_{\text{M}} \) is the Young's modulus of the matrix material. If the criterion (13) is not met, then the stress states at each particle, \( \mathbf{\Omega}_i \), are computed using a modified Eq. (2), as follows, with the renewed vector \( \mathbf{x} \) and then the applied strains to update the values of the eigenstrains:

\[
\begin{align*}
\sigma_{ij}(p) &= \int_{R} \tau_{ij}(q) u_{ik}(p, q) d\Gamma(q) - \int_{R} u_{ij}(q) \tau_{ik}(p, q) d\Gamma(q) \\
&\quad + \sum_{p' \in R_i} \mathbf{e}_{\text{e}}(q) \int_{\Gamma} \mathbf{x}_{ij}(q) \tau_{ik}(p, q) d\Gamma(q), \\
&\quad p \in \mathbf{\Omega}_i, \quad i = 1, \ldots, N_i
\end{align*} \tag{14}
\]

It should be noted here that in Eq. (14), the current particle has been excluded because the stress state at the due place is generated, in addition to the applied load, by the disturbances of all other particles in the solution domain. The principal steps in the solution procedures can be summarized as follows:

(a) Compute the constant coefficients in \( \mathbf{A}, \mathbf{B} \) and \( \mathbf{b} \) in Eq. (1).

(b) Assign the vector \( \mathbf{e} \) an initial value which is the solution to the applied strains via Eq. (2) at the corresponding position of each particle without the presence of particles (i.e., the whole solution domain contains the homogeneous matrix only).

(c) Compute the unknown vector \( \mathbf{x} \) using the iterative formulae (11) with the current eigenstrain vector \( \mathbf{e} \).

(d) Check the convergent criterion (12).

(e) If the criterion (13) is not met, perform the following:

(i) Compute the current stresses in each particle using Eq. (14) with the current particle absent at the corresponding place.

(ii) Compute the current applied strains in each particle using Hooke’s Law.

(iii) Renew the eigenstrain vector \( \mathbf{e} \) using Eq. (6).

and then return to the step (c). Otherwise proceed to the next step (f).

(f) Compute the stresses of interest or the overall properties, etc.

It can be seen that the determination of eigenstrains in each particle is the crucial step where the Eshelby tensor plays an essential role. Although some explicit expressions exist in the literature...
for simple cases, in general the Eshelby tensor can always be obtained conveniently via the numerical means. The flow chart of the algorithm is shown in Fig. 1.

### 4. Numerical examples

In this section, the technique of one-point computing [24] is first assessed and used for evaluating domain integrals in Eqs. (1) and (2), in order to improve the efficiency of the proposed algorithm further. As mentioned at the beginning of Section 2, perfect adhesion between particle and matrix is assumed in the present computational model, which is solved by way of the BEM, so that the displacement continuity and the traction equilibrium hold true.

Fig. 3. Comparison of values of kernel integrals between one-point computing and Gauss quadrature.

Fig. 4. Comparison of absolute errors between one-point computing and Gauss quadrature.

Fig. 5. A single ellipsoidal particle in a RVE.

Fig. 6. Comparison of the computed stresses across the interface of a single soft particle in tension with those using BEM with the sub-domain technique.

Fig. 7. Comparison of the computed stresses across the interface of a single hard particle in tension with those using BEM with the sub-domain technique.

Fig. 8. Comparison of the computed stresses across the interface of a single soft particle in pure shear with those using BEM with the sub-domain technique.
The element mesh used is shown in Fig. 2b for the outer boundary of the RVE and in Fig. 2c for the interface between matrix and particle (an octant of particle surface). It should be pointed out that the domain cells are no longer required as the domain integrals in Eqs. (1) and (2) and considering the boundary-type integrals in Eqs. (8) and (9), when the distances between \( p \) and \( q \) are relatively small. In all the examples, the Poisson’s ratios are taken to be \( \nu_0 = 0.3 \).

4.1. One-point computing

Instead of using Eqs. (8) and (9) by Gauss quadrature to evaluate the domain integrals in Eqs. (1) and (2) and considering the properties of kernel functions explained in [24], the efficiency of the algorithm can be further improved by introducing the technique of one-point computing under certain appropriate conditions as follows:

\[
\int_{\Omega_0} \sigma_{ik} d\Omega \approx V_i \sigma_{ik}
\]

(15)

\[
\int_{\Omega_0} \sigma_{ik} d\Omega \approx V_i \sigma_{ik}
\]

(16)

where \( V_i \) stands for the volume of \( \Omega_i \) and \( \Omega_{\text{ex}} = 0 \) in Eq. (9) for \( p \in \Omega_i \setminus (\Omega_0 \cup \Gamma) \) (the source point \( p \) is outside the \( \Omega_0 \)). The integral values and the absolute errors as a function of relative distance, \( r/\langle 2r_0 \rangle \), are compared in Figs. 3 and 4, respectively, using the two computing methods, one-point computing and Gauss quadrature, over a spheroidal domain with a radius \( r_0 \), where \( r \) represents the distance from \( p \) to the center of the spheroidal domain. It is evident from both Figs. 3 and 4 that the one-point computing can achieve acceptable accuracy if the distances, \( r/\langle 2r_0 \rangle \), are relatively large, which is the most cases when \( p \) is outside the current \( \Omega_i \), since the difference in the integral values between the two methods becomes negligibly small with the increase of the relative distance, \( r/\langle 2r_0 \rangle \). In the present work, \( r/\langle 2r_0 \rangle = 5 \) serves as the criterion to switch the methods between one-point computing and Gauss quadrature around closed interfaces.

4.2. Stress distributions in a cube with a single particle

In order to assess the model with the eigenstrain formulation, the stresses across the interface of a single particle in the RVE as shown in Fig. 5 are computed using the present algorithm and compared with the exact solutions for a spheroidal particle and with the numerical solutions for ellipsoidal particles using

![Comparison of the computed stresses along a quarter arc outside a hole in pure shear.](image1)

![Comparison of the computed stresses as a function of modulus ratio, \( E_i/E_m \), for a spheroidal particle in triaxial tension.](image2)

Fig. 11. Comparison of the computed stresses along a quarter arc outside a hole in single tension.

![The particle spacing \( 2s \) and the equivalent radius \( r_0 \).](image3)

Fig. 10. The particle spacing \( 2s \) and the equivalent radius \( r_0 \).

along their interfaces. To demonstrate the accuracy and efficiency of the proposed algorithm, the results of stresses across the interfaces for a simple example are obtained using the present algorithm and compared with those from the conventional BEM. Further, the overall properties and the stress distributions of a representative volume element (RVE) in a particle-reinforced material are identified using the proposed model. The convergent performance of the algorithm is assessed through investigating the effect of various factors on the number of iterations.

In the following analysis, a cube as shown in Fig. 2a is chosen as the RVE containing triply periodically spaced ellipsoidal particles. The element mesh used is shown in Fig. 2b for the outer boundary of the RVE and in Fig. 2c for the interface between matrix and particle (an octant of particle surface). It should be pointed out that the domain cells are no longer required as the domain integrals have been transformed into the boundary-type. It should also be mentioned that the interface discretization makes no contribution to the degrees of freedom of the problem for the present algorithm. The purpose of the discretization is only for numerical evaluation of the boundary-type integrals in Eqs. (8) and (9), when the distances between \( p \) and \( q \) are relatively small. In all the examples, the Poisson’s ratios are taken to be \( \nu_0 = 0.3 \).
conventional BEM with sub-domain techniques. As shown in Fig. 5, a, b, and c are three half radii of the ellipsoidal particle, which is placed at the center of the cube. In the present work, the shape of the ellipsoidal particle is chosen as oblate \((a = b < c)\) or prolate \((a = b > c)\). However, for the spheroidal particle, \(a = b = c = r_0\). Define the equivalent radius \(r_0\) for ellipsoidal particles as

\[ r_0 = \sqrt[3]{abc} \quad (17) \]

for the purpose of comparing the relative sizes of different particles with the RVE, \(r_0/l = 0.001\) is used in the computation, where \(l\) stands for the side length of the cube RVE. The stresses are computed across the interface of the particle along the \(x_3\) axis as shown Fig. 2c, where \(\xi_1\) in Fig. 5. On the interface, \(\xi_1/a = 1\). To show the stress behavior when the computing point is very close to the interface, the technique of distance transformation [25] is utilized to handle the nearly singularity of integral kernels. The stresses very close to the interface are evaluated at the locations \(\xi_1/a = 0.998\) and \(\xi_1/a = 1.002\), respectively.

The effect of particle stiffness on stress distribution is studied by computing the stresses across the interface of a soft and a hard particle in single tension in \(x_3\) direction with unit traction load \(\sigma_{x3} = 1\), compared in Figs. 6 and 7, respectively, and the stresses of a soft particle in pure shear with unit shear load \(\sigma_{x3} = \sigma_{z3} = 1\) are presented in Fig. 8 for the sake of comparison. It can be seen

### Table 1

Comparison of the degrees of freedom and CPU times for the two algorithms.

<table>
<thead>
<tr>
<th>(N_i)</th>
<th>Degree of freedom</th>
<th>CPU time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BEM</td>
<td>Eigenstrain</td>
</tr>
<tr>
<td>1</td>
<td>1362</td>
<td>492</td>
</tr>
<tr>
<td>2</td>
<td>2232</td>
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</tr>
<tr>
<td>8</td>
<td>7452</td>
<td>492</td>
</tr>
</tbody>
</table>

### Fig. 13

Overall properties of the RVE with spheroidal particles as a function of total particle number, \(N_i\), where the equivalent sizes, \(r_0/s\), are kept constant.

### Fig. 14

Young’s modulus of the RVE as a function of modulus ratio, \(E/E_{\text{m}}\).

### Fig. 15

Shear modulus of the RVE as a function of modulus ratio, \(E/E_{\text{m}}\).

### Fig. 16

Poisson’s ratio of the RVE as a function of modulus ratio, \(E/E_{\text{m}}\).

### Fig. 17

Young’s modulus of the RVE as a function of equivalent particle size, \(r_0/s\).
from the three figures that the stresses are constant inside the particle as expected. It is found that the computed tangential stresses in tension and the shear stress in shear load all have a jump across the interface using the traditional BEM with the sub-domain technique computed on the interface, or more definitely, at the two corresponding places in boundaries of the two sub-domains. However, these computed stress components using the eigenstrain formulation show almost exactly the same behavior. It is interesting to see from Figs. 6–8 that the stress components which have jumps on the interface computed using the eigenstrain formulation take just the average values of the two sides, the particle and the matrix.

Nevertheless, it is observed that the stress components which have jumps computed with the present algorithm at the places in the matrix very close to the interface have tiny differences from those just on the interface computed with the sub-domain method.
Fig. 9 gives the comparison of stresses in both the spheroidal particle and the matrix as a function of the modulus ratio, $E_I/E_M$, in triaxial tension load $r_{11} = r_{22} = r_{33} = 1$ with the exact solutions [14]. The stresses in the matrix are computed at the place $r_0/s = 1.002$, very close to the interface. The computed results show the validity and accuracy of the present algorithm.

4.3. Stress distributions in a cube with multiple empty holes

In order to check the model further with the eigenstrain formulation, the stress distributions in the matrix of the RVE with $N_t = 8$ spheroidal empty holes are computed using the present algorithm.

Fig. 24. Stress distributions across the interface of a soft ellipsoidal particle placed at the center of the RVE with various aspect ratios $c/a$.

Fig. 25. Stress distributions across the interface of a hard ellipsoidal particle placed at the center of the RVE with various aspect ratios, $c/a$.

Fig. 26. Required number of iterations of the algorithm in tension with respect to the modulus ratio, $E_I/E_M$.

Fig. 27. Required number of iterations of the algorithm in shear with respect to the modulus ratio, $E_I/E_M$.

Fig. 28. Required number of iterations of the algorithm in tension with respect to the equivalent particle size, $r_0/s$, with hard particles.

Fig. 29. Required number of iterations of the algorithm in tension with respect to the equivalent particle size, $r_0/s$, with soft particles.
and compared with the numerical solutions using the conventional BEM. In the computation, the RVE used is loaded with unit load in either single tension or pure shear. The equivalent particle or hole size, \( r_0/s \), is defined dimensionlessly by the ratio of the equivalent radius and the half spacing between particles, as shown in Fig. 10. The equivalent size is set as \( r_0/s = 0.4 \) in this example for the RVE with 8 holes.

The stresses are computed along a quarter arc with the radius \( r = 1.1 r_0 \) outside a hole in the matrix of the RVE while the local coordinate \( \xi_k \) (Fig. 1a and c, \( \xi_k/x_0 \), \( k = 1,2,3 \)) is kept constant, as shown in Figs. 11 and 12, respectively, under single tension and pure shear. It is shown from Figs. 11 and 12 that the computed results with the proposed algorithm using the eigenstrain formulation are in well agreement with those of the conventional BEM. As the degree of freedom increases very fast with the number of particles, the computing program using the BEM does not work on the present desktop computer (Intel Pentium Dual CPU, 1.60GHz), the comparison between the two algorithms cannot be performed for the RVE containing more particles. However, there is no such limitation for the proposed algorithm using the eigenstrain formulation.

The degree of freedom and CPU time of the two algorithms, the eigenstrain formulation and the BEM, are listed in Table 1 by run-time of the algorithm with respect to the aspect ratio,\( c/a \).

4.4. Overall properties

In the computation, the RVE used is loaded with unit load in either single tension or pure shear. The results for the overall properties of the RVE with triply periodically spaced spherical particles as a function of the total particle number, \( N_p \), are shown in Fig. 13, where the equivalent sizes, \( r_0/s \), are kept constant, which means that the volume fraction of particles in the RVE remains constant. In this case, the overall properties are isotropic. It can be seen that the values of the computed overall properties become stable when \( N_p \geq 1000 \), showing that the boundary effect can be neglected, so that \( N_p = 1000 \) is chosen in the following examples.

The Young’s modulus, shear modulus and Poisson’s ratio of the RVE as a function of the modulus ratio, \( E_1/E_m \), are presented in Figs. 14–16 with \( N_p = 1000 \), where \( c/a \) is the aspect ratio of particles. If \( c/a = 1 \) the particles are spheroidal and the overall properties become isotropic. It should be pointed out that \( E_1/E_m = 10^{-3} \) behaves almost like holes whereas \( E_1/E_m = 10^8 \) corresponds to rigid particles in the matrix. It can be seen from Figs. 14 and 15 that for either Young’s or shear moduli, the most effective range of the modulus ratio to the overall properties is between \( E_1/E_m = 0.1 \) and \( E_1/E_m = 10 \), while the stagnancy of the properties can be observed when \( E_1/E_m \) is very small or very large, similar to observations in the two-dimensional case [21]. However, the Poisson’s ratio behaves in a very complex relationship with the modulus ratio, \( E_1/E_m \), as shown in Fig. 16.

The study of the effect of equivalent particle size on the overall properties of the RVE, the Young’s moduli and the Poisson’s ratios of the RVE are presented in Figs. 17 and 18, respectively, as a function of equivalent particle size \( r_0/s \). The two figures show monotonic variations as expected since the equivalent particle size corresponds directly to the volume fraction of particles in the RVE. With the increase of \( r_0/s \), the Young’s modulus of the RVE decreases with soft particles but increases with hard particles. However, the Poisson’s ratio of the RVE shows diverse behaviors with \( r_0/s \), as shown in Fig. 18.

Figs. 19–21 show the effects of aspect ratio, \( c/a \), on Young’s modulus, the shear modulus and Poisson’s ratio of the RVE when the equivalent particle size, \( r_0/s \), is kept constant. It is seen from Figs. 19 and 20 that the value of \( E_{33} \) increases monotonically with an increase in the value of the aspect ratio for the RVE with both hard and soft particles. It is interpreted that with the increase of the aspect ratio, \( c/a \), the strengthening effect increases for hard particles but the weakening effect decreases for soft particles. In contrary, the value of \( E_{11} \) shows exactly the opposite behavior. The values of the shear moduli, \( E_{12} \) and \( E_{13} \), behave very similarly to those of \( E_{33} \) and \( E_{11} \), respectively, owing to the similar effects. It is evident from Fig. 21, however, that the values of the Poisson’s ratio again behave in a more complex fashion with the increase of the aspect ratio, \( c/a \).

4.5. Stress distributions of a cube with multiple particles

The stress distribution for a RVE with a single particle was studied in Section 4.2. This section presents an investigation of the stress distributions of a cube with multiple particles. The stress distribution across the interface of the particle at the center of the RVE with a total particle number \( N_p = 125 \) (Fig. 2a) under single tension of unit load \( \sigma_{33} = 1 \) is presented in Figs. 22 and 23, respectively, for soft and hard spheroidal particles with various
equivalent particle sizes, \( r_0/s \). The stresses are computed across the interfaces of the particle along the \( z_1 \) axis as shown in Fig. 2c where \( z_1(\mathbf{x}_1) = r_0/s \). The stresses very close to the interfaces are evaluated at \( z_1(\mathbf{x}_1) = (1 ± 0.002) \times r_0/s \) and all stresses take peak values at the sides of the matrix. It can be seen from Figs. 22 and 23 that the distributions inside the particles are consistent with the inference of constant stresses or strains inside a particle with a simple shape by Eshelby [1,2] when the equivalent particle sizes are relatively small. However, the agreement becomes less consistent with that inference when the equivalent particle sizes are relatively large, for example, \( r_0/s = 0.6 \) for soft particle in Fig. 22 and \( r_0/s = 0.7 \) for hard particle in Fig. 23, reflecting the stronger interference among particles.

The stress distributions for soft and hard ellipsoidal particles are presented in Figs. 24 and 25, respectively, as a function of \( z_1/s \) with various aspect ratios, \( c/a \), while the equivalent particle sizes, \( r_0/s \), is kept constant. The results of the computed stress distributions show further the validity and the effectiveness of the present algorithm.

4.6. Convergence behavior

The convergence behavior of the proposed algorithm is investigated by examining the required number of iterations. Figs. 26–30 show that the number of iterations varies with a variety of factors including the modulus ratio, the loading mode, the equivalent particle size or volume fraction and the shape of particles, which is considered to reflect the effects on or the disturbances to the stress states at the locations among particles. The required number of iterations of the algorithm with respect to the modulus ratio, \( E_\text{f} / E_\text{m} \), is presented in Figs. 26 and 27, respectively, for the RVE being in tension and pure shear. It is considered that the value of the modulus ratio reflects the degree of mismatch between the materials of particles and matrix.

To study the effect of particle stiffness on the convergence behavior, Figs. 28 and 29 present the required number of iterations of the algorithm as a function of the equivalent particle size, \( r_0/s \), for the RVE with hard and soft particles, respectively. It can be seen that the number of iterations increases monotonically with the increase of equivalent particle size, reflecting an increased disturbance of the particles. The required number of iterations of the algorithm with respect to the aspect ratio, \( c/a \), is presented in Fig. 30, showing the effect of particle shape on the disturbances among particles while those of spheroidal particles are in general the smallest. Fig. 31 shows the CPU time of the algorithm as a function of total particle number, \( N_p \). It is approximately a linear function of \( N_p \). In summary, two principal factors need to be considered that influence the convergence behavior of the present algorithm. The first factor is the mutual disturbances of the stress states among the particles, and the second factor is the mismatches between the particles and the matrix.

5. Conclusions

An efficient computational model and solution procedure are presented for simulating ellipsoidal particle-reinforced composites using the proposed three-dimensional eigenstrain formulation of the BIE in the present study. As the unknowns appear only on the boundary of the solution domain, the solution scale of the problem with the present model remains fairly small in comparison with the conventional algorithm using the FEM or the BEM. To further improve the efficiency of the algorithm, one point computing is examined and employed in the algorithm. The results for the overall properties and stress distributions are presented for a RVE with single or multiple particles. The effects of various factors, including the overall properties of particle-reinforced composites, are examined. It is found that the tangential stresses on the interface from the eigenstrain formulation simply take the average values of the two sides of the particle and the matrix. The effectiveness and efficiency of the proposed algorithm as well as the convergent performance of the solution procedure are assessed numerically.

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References