

1. When the crack propagates in the vicinity of a circular inclusion of high rigidity, the crack propagation behavior can be divided into three characteristic parts. The 1st is the initial part where the crack propagates nearly straight, the 2nd is the detour part where the crack avoids the inclusion and the effect of reinforcement due to an inclusion is remarkable, and the 3rd is the final part where the reinforcement effect is no more effective.
2. The larger the ratio of the rigidity between the inclusion and the matrix becomes and/or the nearer the distance between the inclusion and the initial crack becomes, the higher reinforcement effect due to the high rigidity inclusion can be anticipated.
3. In double cantilever type specimen, the crack propagation path stability is higher in the order of tensile load, shear load and pure bending.

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## Thermal analysis for cracks near interfaces between piezoelectric materials

Q. Qin, Y. Mai

Department of Mechanics and Mechanical Engineering,  
University of Sydney, NSW 2006, Australia

Email: qin@tiny.me.su.oz.au

## Abstract

A crack of arbitrary size and orientation near bimaterial interfaces between dissimilar thermopiezoelectric materials is considered. A system of singular integral equations for the unknown thermal analog of dislocation density defined on the crack faces is derived by way of the Stroh's formulation and the thermoelectroelastic Green's functions developed recently. The stress and electric displacement intensity factors are then computed by numerically solving these singular integral equations. Numerical results are obtained to elucidate the effects of crack orientation.

## 1 Introduction

The widespread use of piezoelectric materials in structural applications has generated renewed interest in thermoelectroelastic behavior. In particular, information on thermal stress concentrations around material defects in piezoelectric solids will have applications in high-temperature composite materials. In this context, Florence and Goodier<sup>1</sup> studied the thermal stresses for an isotropic elastic medium containing an ovaloid hole by the method of Muskhelishvili<sup>2</sup>. Using the complex variable technique, Chen<sup>3</sup> investigated the orthotropic elastic medium with a circular or elliptic hole, and obtain a complex form solution for the hoop stress around the hole. With respect to anisotropic elastic materials, Atkinson and Clements<sup>4</sup> presented a solution for a two-dimensional

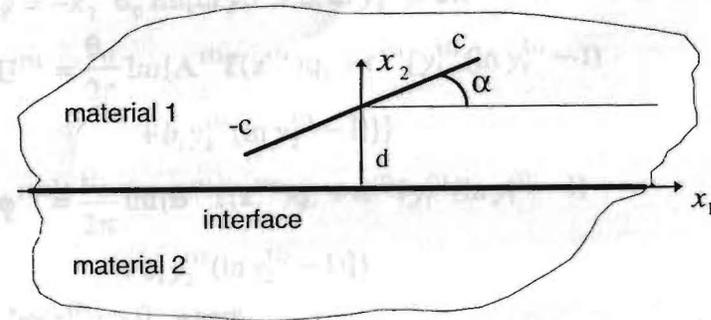


Fig. 1 Geometry of the inclined crack

constructing a uniform heat flux in a general anisotropic elastic medium. Gdoutos and Barber<sup>5</sup> considered the cases of specified temperature on the crack faces and also that of a specified heat flux across the crack by way of Green's function formulation. Based on the Stroh formalism and conformal mapping, Hwu<sup>6</sup> obtained the stress formulation for an anisotropic elastic plate with an elliptic hole subjected to remote uniform flow in  $x_2$ -direction. Recently, Qin, Mai and Yu<sup>7</sup> presented a general formulation for a piezoelectric solid with a hole of various shapes subjected to mixed mechanical, electric and thermal load. However, the problem of a crack at or near interfaces between two thermopiezoelectric media does not seem to have been studied. In the following sections, the extended Stroh formalism and the Green's functions for bimetals are used to study the thermo-electroelastic behavior of a crack near interfaces between piezoelectric media. The geometry of the problem is shown in Fig. 1. The inference on thermo-electroelastic fields due to the inclined crack and the interface is modeled by a system of singular integral equations. These equations can be solved numerically and used to calculate the related stress intensity factors.

## Basic Formulation

Using the notation in [8], the general solution of a 2-D thermopiezoelectric problem can be expressed as

$$\begin{aligned} \mathbf{U} &= \text{Im}[\mathbf{A}\mathbf{f}(z)\mathbf{q} + \mathbf{c}g(z_r)q_r], \quad \phi = \text{Im}[\mathbf{B}\mathbf{f}(z)\mathbf{q} + \mathbf{d}g(z_r)q_r] \\ \Pi_1 &= -\phi_{,2}, \quad \Pi_2 = \phi_{,1}, \quad \theta = \text{Im}[g'(z_r)q_r], \\ \psi &= \text{Im}[-ik_2g'(z_r)q_r], \quad h_1 = -\psi_{,2}, \quad h_2 = \psi_{,1} \end{aligned} \quad (1)$$

where overbars denote complex conjugation, Im stands for the imaginary part,  $\mathbf{q}$  and  $q_r$  are constants to be determined by the boundary conditions,  $\mathbf{U} = \{u_1 \ u_2 \ u_3 \ \phi\}^T$ ,  $\Pi_j = \{\sigma_{1j} \ \sigma_{2j} \ \sigma_{3j} \ D_j\}^T$ ,  $j=1,2$ ;  $k_2 = \sqrt{k_{11}k_{22} - k_{12}^2}$ ,  $i = \sqrt{-1}$ ,  $\mathbf{f}(z) = \{f(z_1) \ f(z_2) \ f(z_3) \ f(z_4)\}^T$ ,  $f_i$  and  $g$  are functions of the generalized complex variables  $z_i$  and  $z_r$ , defined by  $z_i = x_1 + p_i x_2$  and  $z_r = x_1 + p_r x_2$ ,  $p_i$  and  $p_r$  are the heat and electroelastic eigenvalues of the materials whose imaginary parts are positive, respectively,  $\mathbf{A}$ ,  $\mathbf{B}$ ,  $\mathbf{c}$  and  $\mathbf{d}$  are well-defined in the literature [9].

The geometrical configuration of the problem to be solved is depicted in Fig. 1, showing a crack with an orientation angle  $\alpha$  and length  $2c$  near an interface between materials 1 and 2. The corresponding boundary conditions are as follows:

*Along the inclined crack*

$$\mathbf{t}_n = -\Pi_1 \sin \alpha + \Pi_2 \cos \alpha = 0, \quad h_n = -h_1 \sin \alpha + h_2 \cos \alpha = 0 \quad (2)$$

*At infinity*

$$h_2^\infty = h_0, \quad \Pi_1^\infty = \Pi_2^\infty = h_1^\infty = 0 \quad (3)$$

where  $n$  stands for the normal direction to the lower face of the inclined crack,  $\mathbf{t}_n$  is the surface traction and charge vector.

It is convenient to represent the solution as the sum of a uniform heat flux in an unflawed solid (which involves no thermal stress) and a corrective solution in which the boundary conditions are

*Along the inclined crack*

$$\mathbf{t}_n = -\Pi_1 \sin \alpha + \Pi_2 \cos \alpha = 0, \quad h_n = -h_0 \cos \alpha \quad (4)$$

*At infinity*

$$\Pi_1^\infty = \Pi_2^\infty = q_1^\infty = q_2^\infty = 0 \quad (5)$$

In the following sections, we will use (4) and (5) instead of (2) and (3).

## 3 Singular Integral Equations

Consider a bimaterial solid for which the upper half-plane ( $x_2 > 0$ ) is occupied by material 1, and the lower half-plane ( $x_2 < 0$ ) is occupied by material 2. The problem will be formulated in terms of the

thermoelastic Green's functions derived in our previous paper<sup>9</sup> and isothermal Green's functions obtained by Ting<sup>10</sup>. For a bimaterial plate, the basic solutions in material 1 due to a discrete temperature discontinuity of magnitude  $\theta_0$  at  $[(0, d), \text{see Fig. 1}]$ , are given by<sup>9</sup>

$$\psi = -k_2^{(1)}\theta_0 \text{Im}[\ln y_1^{(1)} + b_1 \ln y_2^{(1)}] / 2\pi \quad (6)$$

$$\mathbf{U}^{(1)} = \frac{\theta_0}{2\pi} \text{Im}\{\mathbf{A}^{(1)}\mathbf{f}(\mathbf{z}^{(1)})\mathbf{q}_1 + \mathbf{c}^{(1)}[y_1^{(1)}(\ln y_1^{(1)} - 1) + b_1 y_2^{(1)}(\ln y_2^{(1)} - 1)]\} \quad (7)$$

$$\phi^{(1)} = \frac{\theta_0}{2\pi} \text{Im}\{\mathbf{B}^{(1)}\mathbf{f}(\mathbf{z}^{(1)})\mathbf{q}_1 + \mathbf{d}^{(1)}[y_1^{(1)}(\ln y_1^{(1)} - 1) + b_1 y_2^{(1)}(\ln y_2^{(1)} - 1)]\} \quad (8)$$

for  $\text{Im}(z_i^{(1)}) > 0$ , where

$$\begin{aligned} \mathbf{f}(\mathbf{z}^{(1)}) &= \text{diag}[f(y_1^{*(1)}), f(y_2^{*(1)}), f(y_3^{*(1)}), f(y_4^{*(1)})], \\ \mathbf{q}_1 &= [\mathbf{B}^{(1)} - \mathbf{B}^{(2)}\mathbf{A}^{(2-1)}\mathbf{A}^{(1)}]^{-1} \{ [b_2\mathbf{d}^{(2)} + b_1\bar{\mathbf{d}}^{(2)} - \mathbf{d}^{(1)}] \\ &\quad - \mathbf{B}^{(2)}\mathbf{A}^{(2-1)} [b_2\mathbf{c}^{(2)} + b_1\bar{\mathbf{c}}^{(2)} - \mathbf{c}^{(1)}] \} \end{aligned} \quad (9)$$

together with

$$\begin{aligned} b_1 &= \frac{k_2^{(2)} - k_2^{(1)}}{k_2^{(2)} + k_2^{(1)}}, \quad b_2 = \frac{2k_2^{(1)}}{k_2^{(2)} + k_2^{(1)}}, \quad y_1^{(1)} = z_i^{(1)} - \hat{z}_i^{(1)} \\ y_2^{(1)} &= z_i^{(1)} - \hat{z}_i^{(1)}, \quad f(y) = y(\ln y - 1), \\ y_i^{*(1)} &= z_i^{(1)} - \hat{z}_i^{(1)}, \quad (i = 1, 2, 3, 4) \end{aligned}$$

where the superscripts (1) and (2) label the quantities relating to the material 1 and 2.

The boundary conditions, (4)<sub>2</sub>, can be satisfied by redefining the discrete Green's functions  $\theta_0$  in (6) in terms of distributed Green's functions  $\theta_0(\xi)$  defined along the crack line,  $z_i^{(1)} = p_*^{(1)}d + \eta z^*$ ,  $\hat{z}_i^{(1)} = p_*^{(1)}d + \xi z^*$ ,  $z^* = \cos\alpha + p_*^{(1)}\sin\alpha$ , where  $d$  and  $\alpha$  are shown in Fig. 1. Enforcing the satisfaction of the applied heat flux conditions on the crack faces, a singular integral equation for the Green's function is obtained as

$$\frac{1}{\pi} \text{Re} \left[ \int_{-1}^1 \left[ \frac{1}{t_0 - t} + K_0(t_0, t) \right] \hat{\theta}_0(t) dt \right] = \frac{2h_0 \cos\alpha}{k_2^{(1)}} \quad (10)$$

where  $\hat{\theta}_0(t) = \theta_0(\xi)$ ,  $t = c\xi$ ,  $t_0 = c\eta$ , and

$$K_0(t_0, t) = b_1 \text{Im} \left[ \frac{z^*}{ct_0 z^* - ct\bar{z}^* + (p_*^{(1)} - \bar{p}_*^{(1)})d} \right] \quad (11)$$

is a regular function.

In addition to (10), the single valuedness of the temperature around a closed contour surrounding the whole crack requires that

$$\int_{-1}^1 \hat{\theta}_0(t) dt = 0 \quad (12)$$

The singular integral equation (10) combined with (12) can be solved numerically<sup>11</sup>. To this end, let

$$\hat{\theta}_0(t) = \Theta(t) / \sqrt{1-t^2} \approx \sum_{k=1}^n B_k T_k(t) / \sqrt{1-t^2} \quad (13)$$

where  $\Theta(t)$  is a regular function defined in a closed interval  $|t| \leq 1$ ,  $B_k$  are the real unknown coefficients, and  $T_k(t)$  the Chebyshev polynomial. Thus the discretized form of (10) and (12) may be written as<sup>11</sup>

$$\sum_{k=1}^n \frac{\Theta(t_k)}{n} \left[ \frac{1}{t_{0r} - t_k} + K_0(t_{0r}, t_k) \right] = 2h_0 \cos\alpha / k_2^{(1)} \quad (14)$$

$$\sum_{k=1}^n \Theta(t_k) = 0$$

where

$$\begin{aligned} t_k &= \cos[(2k-1)\pi/2n], \quad (k = 1, 2, \dots, n) \\ t_{0r} &= \cos(r\pi/n), \quad (r = 1, 2, \dots, n-1) \end{aligned}$$

Equation (14) provides a system of  $n$  linear algebraic equations to determine  $\Theta(t_k)$ , and then  $B_k$ . Once the function  $\Theta(t)$  has been found, the corresponding SEP can be given from (1) in the form

$$\begin{aligned} \Pi_1^{(1)} = -\phi_{2,1}^{(1)} &= -\frac{1}{2\pi} \int_{-c}^c \text{Im}[\mathbf{B}^{(1)}\mathbf{P}^{(1)} \langle \ln \mathbf{z}^{(1)} \rangle \mathbf{q}_1 \\ &\quad + \mathbf{d}^{(1)} p_*^{(1)} (\ln y_1^{(1)} + b_1 \ln y_2^{(1)}) \theta_0(\xi) d\xi \end{aligned} \quad (15)$$

$$\begin{aligned} \Pi_2^{(1)} = \phi_{1,1}^{(1)} &= \frac{1}{2\pi} \int_{-c}^c \text{Im}[\mathbf{B}^{(1)} \langle \ln \mathbf{z}^{(1)} \rangle \mathbf{q}_1 + \mathbf{d}^{(1)} (\ln y_1^{(1)} \\ &\quad + b_1 \ln y_2^{(1)}) \theta_0(\xi) d\xi \end{aligned}$$

where

$$\begin{aligned} \langle \ln \mathbf{z}^{(1)} \rangle &= \text{diag}[\ln y_1^{*(1)}, \ln y_2^{*(1)}, \ln y_3^{*(1)}, \ln y_4^{*(1)}], \\ y_i^{*(1)} &= z_i^{(1)} - \hat{z}_i^{(1)} = z_i^{(1)} - \xi z^* - p_*^{(1)}d, \quad (i = 1, 2, 3, 4) \end{aligned} \quad (16)$$

Thus the traction-charge vector on the crack faces is of the form

$$\begin{aligned} \mathbf{t}_n(\eta) &= -\Pi_1^{(1)}(\eta)\sin\alpha + \Pi_2^{(1)}(\eta)\cos\alpha \\ &= \frac{1}{2\pi} \int_{-c}^c \text{Im}[\mathbf{B}^{(1)}(\mathbf{I}\cos\alpha + \mathbf{P}^{(1)}\sin\alpha)\langle \ln \mathbf{z}^{(1)} \rangle \mathbf{q}_1 \\ &\quad + \mathbf{d}^{(1)}(\cos\alpha + p_*^{(1)}\sin\alpha)(\ln y_1^{(1)} + b_1 \ln y_2^{(1)})] \theta_0(\xi) d\xi \end{aligned} \quad (17)$$

It is obvious that  $\mathbf{t}_n(\eta) \neq 0$  on the crack faces  $|\xi| \leq c$ . To satisfy the traction-charge free condition (4)<sub>1</sub>, we must superpose a solution of the corresponding isothermal problem with a traction-charge vector equal and opposite to that of (17) in the range  $|\xi| \leq c$ . The elastic solution for a singular dislocation of strength  $\mathbf{b}_0$  has been obtained by Ting<sup>10</sup>. This solution can be straightforwardly extended to the case of electroelastic problem as

$$\begin{aligned} \Pi_1^{(1)} &= -\frac{1}{\pi} \text{Im}[\mathbf{B}^{(1)}\langle p_i^{(1)}(z_i^{(1)} - p_i^{(1)}d)^{-1} \rangle \mathbf{B}^{(1)T}] \mathbf{b}_0 \\ &\quad - \frac{1}{\pi} \sum_{\beta=1}^4 \text{Im}[\mathbf{B}^{(1)}\langle p_i^{(1)}(z_i^{(1)} - \bar{p}_\beta^{(1)}d)^{-1} \rangle \mathbf{B}^* \mathbf{I}_\beta \bar{\mathbf{B}}^{(1)T}] \mathbf{b}_0 \end{aligned} \quad (18)$$

$$\begin{aligned} \Pi_2^{(1)} &= \frac{1}{\pi} \text{Im}[\mathbf{B}^{(1)}\langle (z_i^{(1)} - p_i^{(1)}d)^{-1} \rangle \mathbf{B}^{(1)T}] \mathbf{b}_0 \\ &\quad + \frac{1}{\pi} \sum_{\beta=1}^4 \text{Im}[\mathbf{B}^{(1)}\langle (z_i^{(1)} - \bar{p}_\beta^{(1)}d)^{-1} \rangle \mathbf{B}^* \mathbf{I}_\beta \bar{\mathbf{B}}^{(1)T}] \mathbf{b}_0 \end{aligned} \quad (19)$$

where  $\mathbf{I}_\beta = \text{diag}[\delta_{1\beta} \delta_{2\beta} \delta_{3\beta} \delta_{4\beta}]$ ,  $\delta_{ij} = 1$  for  $i=j$ ;  $\delta_{ij} = 0$  for  $i \neq j$ , and

$$\begin{aligned} \langle (\cdot)_i \rangle &= \text{diag}[(\cdot)_1 (\cdot)_2 (\cdot)_3 (\cdot)_4] \\ \mathbf{B}^* &= \mathbf{B}^{(1)-1} [\mathbf{I} - 2(\mathbf{M}_1^{-1} + \bar{\mathbf{M}}_2^{-1})^{-1} \mathbf{L}^{-1}] \end{aligned}$$

with

$$\mathbf{L} = -2i\mathbf{B}^{(1)}\mathbf{B}^{(1)T}, \quad \mathbf{M}_j = -i\mathbf{B}^{(j)}\mathbf{A}^{(j)-1}, \quad (j=1,2),$$

Therefore the boundary condition (4)<sub>1</sub> will be satisfied if

$$\frac{\mathbf{L}}{2\pi} \int_{-c}^c \frac{\mathbf{b}_0(\xi) d\xi}{\eta - \xi} + \int_{-c}^c \mathbf{K}_0(\eta, \xi) \mathbf{b}_0(\xi) d\xi = -\mathbf{t}_n(\eta) \quad (20)$$

where

$$\mathbf{K}_0(\eta, \xi) = \frac{1}{\pi} \sum_{\beta=1}^4 \text{Im}[\mathbf{B}^{(1)}\langle z_j^*(z_j^{(1)} - \hat{z}_{\beta c}^{(1)})^{-1} \rangle \mathbf{B}^* \mathbf{I}_\beta \bar{\mathbf{B}}^{(1)T}] \quad (21)$$

with

$$z_j^* = \cos\alpha + p_j^{(1)}\sin\alpha, \quad z_j^{(1)} = \eta z_j^* + p_j^{(1)}d, \quad \hat{z}_{\beta c}^{(1)} = \xi z_\beta^* + p_\beta^{(1)}d$$

For single valued displacements and electric potential around a closed contour surrounding the whole crack, the following conditions have also to be satisfied:

$$\int_{-c}^c \mathbf{b}_0(\xi) d\xi = 0 \quad (22)$$

As was done previously, let  $\eta = ct_0$ ,  $\xi = ct$ , and

$$\mathbf{b}_0(\xi) = \Theta(\xi) / \sqrt{c^2 - \xi^2} \approx \sum_{k=1}^n \mathbf{E}_k T_k(t) / \sqrt{1-t^2} \quad (23)$$

where  $\mathbf{E}_k = \{E_{k1}, E_{k2}, E_{k3}, E_{k4}\}^T$ . Thus, from (20) and (22), we obtain

$$\begin{aligned} \sum_{k=1}^m \frac{1}{n} \left[ \frac{\mathbf{L}}{2c(t_{0r} - t_k)} + \mathbf{K}_0(t_{0r}, t_k) \right] \Theta(ct_k) &= -\mathbf{t}_n(t_{0r}) \\ \sum_{k=1}^m \Theta(ct_k) &= 0 \end{aligned} \quad (24)$$

Equations (24) provide a system of  $4m$  linear algebraic equations to determine  $\Theta(ct_k)$  and then  $\mathbf{E}_k$ . Once the function  $\Theta(ct_k)$  has been found from (24), the stresses and electric displacements,  $\Pi_n(\eta)$ , in a coordinate local to the crack line can be expressed in the form

$$\Pi_n(\eta) = \Omega(\alpha) \left\{ \frac{\mathbf{L}}{2\pi} \int_{-c}^c \frac{\mathbf{b}_0(\xi) d\xi}{\eta - \xi} + \int_{-c}^c \mathbf{K}_0(\eta, \xi) \mathbf{b}_0(\xi) d\xi + \mathbf{t}_n(\eta) \right\} \quad (25)$$

where the  $4 \times 4$  matrix  $\Omega(\alpha)$  whose components are the cosine of the angle between the local coordinates and the global coordinates is well-documented in References 8.

Using (25) we can evaluate the stress intensity factors  $\mathbf{K}^* = (K_{II}, K_I, K_{III}, K_D)^T$  at the tips, e.g., at the right tip ( $\xi=c$ ) of the crack by following definition:

$$\mathbf{K}^* = \lim_{\xi \rightarrow c^+} \sqrt{2\pi(\xi - c)} \Pi_n(\xi) \quad (26)$$

Combined with the results of (25), one then leads to

$$\mathbf{K}^* \approx \sqrt{\frac{\pi}{4c}} \Omega(\alpha) \mathbf{L} \Theta(c) \quad (27)$$

Thus the solution of the singular integral equation enables the direct determination of the stress intensity factors.

## 4 Numerical Results

In this section some sample calculations are presented to illustrate the applications of the proposed formulation. For simplicity, we only consider an inclined crack near interface between two transversely isotropic materials. The upper and lower materials are assumed to be BatiO<sub>3</sub><sup>12</sup> and Cadmium Selenide<sup>13</sup> respectively. The material constants for the two materials are as follows:

(1) material properties for BatiO<sub>3</sub><sup>12</sup>:

$$c_{11} = 150\text{Gpa}, c_{12} = 66\text{Gpa}, c_{13} = 66\text{Gpa}, c_{33} = 146\text{Gpa},$$

$$c_{44} = 44\text{Gpa}, \alpha_{11} = 8.53 \times 10^{-6} / \text{K}, \alpha_{33} = 1.99 \times 10^{-6} / \text{K},$$

$$\lambda_3 = 0.133 \times 10^5 \text{ N / CK}, e_{31} = -4.35 \text{ C / m}^2, e_{33} = 17.5 \text{ C / m}^2,$$

$$e_{15} = 11.4 \text{ C / m}^2, \kappa_{11} = 1115\kappa_0, \kappa_{33} = 1260\kappa_0,$$

$$\kappa_0 = 8.85 \times 10^{-12} \text{ C}^2 / \text{Nm}^2$$

(2) material properties for Cadmium Selenide<sup>13</sup>

$$c_{11} = 74.1\text{Gpa}, c_{12} = 45.2\text{Gpa}, c_{13} = 39.3\text{Gpa}, c_{33} = 83.6\text{Gpa},$$

$$c_{44} = 13.2\text{Gpa}, \gamma_{11} = 0.621 \times 10^6 \text{ NK}^{-1} \text{m}^{-2},$$

$$\gamma_{33} = 0.551 \times 10^6 \text{ NK}^{-1} \text{m}^{-2}, \chi_3 = -0.294 \text{ CK}^{-1} \text{m}^{-2},$$

$$e_{31} = -0.160 \text{ Cm}^{-2}, e_{33} = 0.347 \text{ Cm}^{-2}, e_{15} = 0.138 \text{ Cm}^{-2},$$

$$\kappa_{11} = 82.6 \times 10^{-12} \text{ C}^2 / \text{Nm}^2, \kappa_{33} = 90.3 \times 10^{-12} \text{ C}^2 / \text{Nm}^2$$

Since the values of the coefficient of heat conduction both for BatiO<sub>3</sub> and Cadmium Selenide could not be found in the literature, the value  $k_{33}/k_{11}=1.5$  and  $k_{13}=0$  are assumed.

In our analysis, the plane strain deformation is assumed and the crack line is assumed to be in the  $x_1$ - $x_3$  plane, i.e.,  $D_2=u_2=0$ . Therefore the stress intensity factor vector  $K^*$  now has only three components ( $K_I, K_{II}, K_D$ ). Figures 2 shows the numerical results for the coefficients of stress intensity factors  $\beta_i$  versus the crack orientation  $\alpha$ , where  $\beta_i$  are defined by

$$\begin{aligned} K_I &= h_0 c \sqrt{\pi c} \gamma_{11} \beta_1(\alpha) / k_2^{(1)}, & K_{II} &= h_0 c \sqrt{\pi c} \gamma_{33} \beta_2(\alpha) / k_2^{(1)} \\ K_D &= h_0 c \sqrt{\pi c} \chi_3 \beta_D(\alpha) / k_2^{(1)} \end{aligned} \quad (28)$$

It can be seen from figure 2 that the crack orientation affects strongly the values of  $K^*$ .

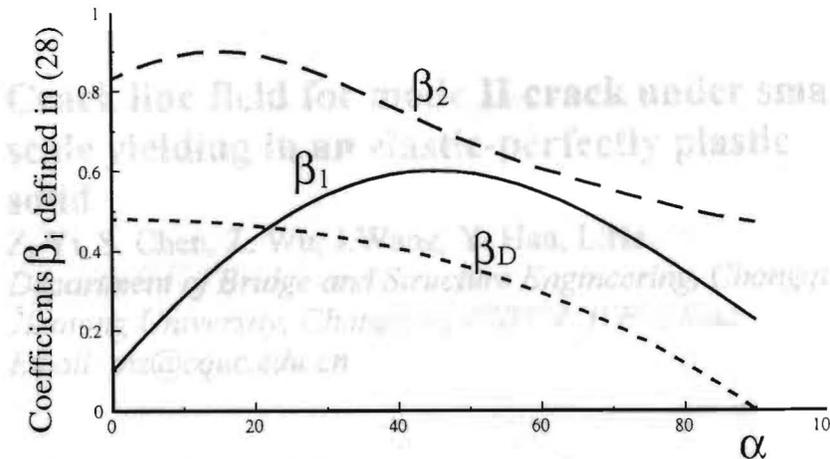


Fig. 2. Normalized coefficients  $\beta_i$  vs angle  $\alpha$

## 5 Conclusion

This paper has presented a formulation and solution for the problem involving arbitrary-oriented cracks near interfaces between dissimilar thermopiezoelectric materials, subjected to remote heat flow. Based on the Stroh's formulation and the thermoelectroelastic Green's function developed recently<sup>9</sup>, a system of singular integral equations is derived to model the crack problem. The stress and electric displacement intensity factors can be computed by numerically solving these singular integral equations. It is found from figure 2 that the crack orientation affects strongly the values of  $K^*$ . The values of  $\beta_1$  and  $\beta_2$  reach their maximum values at about 44° and 15°, respectively.

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## Crack line field for mode II crack under small scale yielding in an elastic-perfectly plastic solid

Z. Yi, S. Chen, Z. Wu, J. Wang, Y. Han, L. He

Department of Bridge and Structure Engineering, Chongqing Jiaotong University, Chongqing 400074, P.R. China

Email: yiz@cquc.edu.cn

### Abstract

A more reasonable crack line solution for a mode II crack under plane stress condition, as well as a mode II crack under plane strain condition in an elastic-perfectly plastic material has been obtained in the present paper. The definition of the small scale yielding in the present paper is different from the usual definition. The assumption of the usual small scale yielding that the plastic zone makes the crack tip elastic field to move a distance along the crack line has been abandoned.

### 1 Introduction

By using the near crack line analysis method, Guo and Li [1][2] obtained the near crack line elastic-plastic field for a mode II crack under plane stress condition and a mode II crack under plane strain condition respectively in an elastic-perfectly plastic material under the small scale yielding. Either under the plane stress condition or under the plane strain condition, the near crack line plastic stress field obtained by them is only a particular solution. Moreover, the matching conditions taken by them are inappropriate.

The usual definition of the small scale yielding states that the plastic zone ahead of the crack tip is so small that the elastic field out of the crack tip plastic zone is the elastic-dominant field for a crack (i.e. the usual K-