Cell based full field displacement calculation for foam material

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Abstract

This paper deals with the calculation of full-field displacement over a surface of metallic foam. Owing to the failure of traditional displacement measurement which calculates the correlation of intensity pattern between two regions, we propose a cell based full field displacement calculation. The proposed approach involves multiple-step image processing including cell region segmentation, cell region matching and nodal matching. Metallic foam is used for assessing the effectiveness and accuracy of the proposed algorithm. The results show that local displacements around cell boundaries on the surfaces of the specimen can be effectively determined with the proposed method, thus it appears that the method is promising for determining displacements of complex cellular solids.

Introduction

Metallic foams are of light-weight and high strength in shear/compression [1]. Researchers have been endeavoured in understanding relationships between the morphology and the properties of metallic foams in order to effectively exploit these properties in engineering design. On the other hand, a number of methods for calculating optical full-field of displacement/strain and then identifying effective material properties has been recently proposed, for analyzing performance of various engineering materials [2-6]. Among these methods, the digital imaging correlation (DIC) has become increasingly popular in the past two decades due to its relatively simple principle and flexibility in its adjustable scales from micro to nanoscale [2,7,8]. With the DIC method, nodal displacements on the surface of a planar specimen are obtained by comparing correlatively a pair of digital images taken before and after the imposed deformation of the specimen [9]. As the main streams in DIC technique, various subset-based DIC methods [10] have been developed. These methods find the displacement of a point (node) cantered at a subset. The so-called extended digital image correlation (X-DIC) had been used by many researchers [11] to handle problems of a specimen with sparse distributed discontinuities. The X-DIC discretizes an image area into some finite elements which are linked by nodes. Unlike subset based DIC methods, all the nodal values in the X-DIC approach are correlated simultaneously by various matching algorithms. The full field displacement can be obtained via interpolation using the nodal values or finite element smoothing [12,13].

Current displacement field calculations with both original DIC and X-DIC are pixel-based, where intensity patterns in images are compared via correlation metrics. The performance of both DIC and X-DIC still relies on the subset size and the texture pattern of pixel’s intensity in the subset. For a cellular solid, its surface is dominant with holes and its cell walls are relatively thin. The subset size in cell wall regions can only be very small that it is inefficient to spatially characterize the intensity pattern in such a subset area, dampening the displacement field calculation performance using original subset-based or X-DIC methods.

In this study, we develop a new cell-based method aiming at retrieving displacement results of metallic foams. The displacement field on the surface of a metallic foam is obtained from the interpolation on the known nodal values. In the process of nodal displacement calculation, a set of nodes from a reference cell and the corresponding node set of the deformed cell are matched by a TPS-RPM method, which was originally developed in computer vision and pattern recognition [14]
for image registration and shape matching. The proposed nodal value calculating involves a multiple-step image processing to locate cell boundaries from raster images and establish mapping between nodes before and after deformation.

**Procedure of Displacement field calculation**

The proposed approach for displacement field calculation involves a series of image processing techniques, including cell region segmentation, cell region matching and cell boundary point matching.

Image segmentation is used to locate cell regions and their boundaries in an image of a surface of metallic foam. In this study, matching a reference cell region to its deformed cell region is a problem of having a region in an image and finding its closet match among a set of regions in another image. A so called maximum-overlapping-area criterion is proposed for processing cell region matching: let $R_s$ be the region of cell $k$ in the reference image and $R'_s$ be the corresponding region of cell $k$ in the deformed image. $R'_s$ is identified by

$$S(R_s, R'_s) = \max \{ S(R_s, R'_s) | i = 1, \ldots, n \}$$

Here $S(R_s, R'_s)$ is the number of pixels in the intersecting area of the two regions $R'_s$ and $R_s$, $n$ is the total number of cell regions in the deformed image.

In the proposed approach, boundary points in a cell region are extracted as nodes. The nodes in a reference cell are matched to their corresponding nodes in the deformed cell by a TPS-RPM described in next section. Nodal displacements are calculated from the pairs of matched nodes. Displacements at any point in the open space bound by cell boundaries are then found by the natural neighbour interpolation. The interpolation is based on the method of Delaunay triangulations [15].

**TPS-RPM for node matching**

Consider two point sets, $X = \{ x_i : i = 1, 2, \ldots, N_1 \}$ along the reference cell boundary and $Y = \{ y_j : j = 1, 2, \ldots, N_2 \}$ along the deformed cell’s boundary, we apply TPS-RPM to determine the correspondence between $X$ and $Y$ and match them accordingly [14]. According to Ref. [14], the following energy function can be minimized in the form:

$$E(M, f) = \sum_{i=1}^{N_1} \sum_{j=1}^{N_2} m_{ij} \| y_j - f(x_i) \|^2 + \lambda_1 \| L f \|^2 + \gamma \sum_{i=1}^{N_1} \sum_{j=1}^{N_2} m_{ij} \log m_{ij} - \zeta \sum_{j=1}^{N_2} \sum_{i=1}^{N_1} m_{ij}$$

which subjects to the following constraints,

$$0 \leq m_{ij} \leq 1 \quad \text{for } i = 1, 2, \ldots, N_1; \quad j = 1, 2, \ldots, N_2 + 1$$

$$\sum_{j=1}^{N_2} m_{ij} = 1 \quad \text{for } i = 1, 2, \ldots, N_1$$

$$\sum_{i=1}^{N_1} m_{ij} = 1 \quad \text{for } j = 1, 2, \ldots, N_2$$

where the matrix $M$ consists of two parts. The $N_1 \times N_2$ inner sub-matrix defines the correspondence between $X$ and $Y$. If $x_i$ corresponds to $y_j$, then $m_{ij} = 1$, otherwise $m_{ij} = 0$. The $(N_2 + 1)_{th}$ column and the $(N_1 + 1)_{th}$ row define the outliers in $X$ and $Y$, respectively. If $x_i$ (or $y_j$) is an outlier, $m_{i,N_1+1} = 1$ (or $m_{N_1+1,j} = 1$).

Details of each term in Eq. 2 can be found in Ref [14]. The algorithm is featured with a two-step up-date process: update the correspondences by differentiating the energy function in Eq. 2 with respect to $M$, and setting the result to zero; update the transformation by the least-squares approach to solve for the TPS parameters. The update process is controlled by the annealing scheme. According to a linear annealing rate $r$ ($r < 1$), the temperature $T$ is reduced with $T_{n+1} = T_n \cdot r$, starting
from \( T_0 = T_{\text{initial}} \). Repeat the two-step updates till the object function convergence at a final temperature \( T_{\text{final}} \).

**Assessment and discussion**

To test the feasibility and efficiency of the proposed algorithm, an example of a metallic foam specimen and its deformation is considered. The in-plane displacements of these cellular structures are calculated using the proposed algorithm.

The accuracy of the proposed displacement calculation depends on the performance of the point matching. In our assessment, the TPS-RPM algorithm rapidly brings two cell boundaries closer in space and more similar in shape from the initial iterations of the deterministic annealing procedure. This assessment shows that the setting of final temperature \( T_{\text{final}} \) is deformation dependent. For the pairs of cell point sets that have small deformation, good matching is achieved at final iteration, and the sequential ordering of points is preserved. For some pairs of cell boundaries that are significantly deformed, when \( T \) is too small, the affine transform flipped, causing some points in a cell boundary map to points far away from their corresponding points in its deformed cell boundary and the sequential ordering of points is broken. Under a setting of \( T_{\text{final}} = 500 \), the algorithm achieves satisfied point matching result.

An image (Fig. 1 (a)) of a designed metallic foam is obtained from [16]. The specimen has at least one complete cell located at the center of effective test zone. It was sectioned by electro-discharge machining to avoid local damage to the cell walls. A simulated synthetic image (Fig. 1 (b)) is generated with a shear transform \((f : (x, y) \rightarrow (x', y'))\) on the image of Fig. 1(a). \( f \) is the shearing parallel to the \( y \) axis, in the transformation form of

\[
x' = x, \quad y' = y + kx
\]

For a given point \( P \), the relative deviation between the mapped point of \( P \) obtained by TPS-RPM and the point of \( P \)'s shear transform is used for the point matching evaluation as

\[
D = \frac{|u_y - u'_y|}{u_y},
\]

in which \( u_y \) is the displacement in \( y \) direction obtained from the shear transform shown in Eq. 4; \( u_y', \) is the displacement in \( y \) direction calculated by the proposed method.

Most nodes in Fig. 1(a) match their correspondences in Fig. 1(b); a number of points mismatch to the points a few pixels away from their correspondences. Fig. 2 depicts the average relative deviations in \( y \) direction for the 4th cell shown in Fig. 1(a), where the shear element \( k = \{0.01,0.03,0.05,0.1,0.2,0.3\} \) respectively. All the average relative deviations are between 2\%~5\% in \( y \) direction. The average relative deviation values for the metal foam increase with the increase of \( k \) as the number of mismatched point pairs increases greatly with the increase of \( k \), causing the increasing of the average deviation value faster than the increasing of average displacement.
Fig. 2 The average relative deviation and theoretical displacement of the ordered points at different for the cell shown in Fig 1 (1 pixel = 0.03mm).

Figs. 3(a) and (b) presents the substrate displacement fields in x and y directions obtained by the proposed algorithm for the metallic foam between the original and the sheared image. The reference point is at (0,0) in both images (Figs. 1(a) and (b)). The ground true of displacement is a shear represented in Eq. 4. The results from the proposed method are the displacement in y direction changing linearly with respect to x; displacement in x direction for most regions is less than 0.03mm. Larger displacement occurs in sparse areas where the point pairs are mismatched.

(a) Displacement in x direction.  
(b) Displacement in y direction.  
Fig. 3 Displacement of the metallic foam in x and y direction (1 pixel = 0.03mm).

Conclusion
The shape information of cells is used in this study for the full field displacement measurement of metallic foam. The accuracy of the proposed method for displacement measurement is mainly determined by the performance of point matching. The assessment shows that for cells with complex geometry shape, the number of mismatched point pairs monotonically increases with the increase of k when k > 0.01. The average relative deviation is between 2%~5%. The results indicate that local displacements around holes’ boundaries of a material’s surface can be effectively determined with the proposed method (under elastic deformation) and it appears that the proposed method is promising for predicting displacements of complex cellular solids.

References