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PREFACE

The Conferences on Boundary Element and Meshless Techniques are devoted to fostering the continued involvement of the research community in identifying new problem areas, mathematical procedures, innovative applications, and novel solution techniques in both boundary element methods (BEM) and boundary integral equation methods (BIEM). Previous successful conferences devoted to Boundary Element Techniques were held in London, UK (1999), New Jersey, USA (2001), Beijing, China (2002), Granada, Spain (2003), Lisbon, Portugal (2004), Montreal, Canada (2005), Paris, France (2006), Naples, Italy (2007), Seville, Spain (2008), Athens, Greece (2009) and Berlin, Germany (2010).

The present volume is a collection of edited papers that were accepted for presentation at the Boundary Element Techniques Conference held at the FINATEC, University of Brasilia during 13th-15th July 2011. Research papers received from 18 counties formed the basis for the Technical Program. The themes considered for the technical program included solid mechanics, fluid mechanics, potential theory, composite materials, fracture mechanics, damage mechanics, contact and wear, optimization, heat transfer, dynamics and vibrations, acoustics and geomechanics.

The conference organizers would also like to express their appreciation to the International Scientific Advisory Board for their assistance in supporting and promoting the objectives of the meeting and for their assistance in the form of reviews of the submitted papers.

We would like to dedicate the conference to the memory of our friend and colleague Prof Wilson Venturini.

Editors
July 2011
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Fundamental solution based FEM for nonlinear thermal radiation problem

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Abstract. A new hybrid finite element model (FEM) with elementary-boundary integrals only is developed for the solution of multimode heat transfer problems involving heat conduction, convection and nonlinear radiation in the paper. Based on the fundamental solution (FS) for the heat transfer problems, the internal and boundary temperature fields are independently assumed and the boundary integral-based variational functional is formulated to produce the final stiffness equation and connection between the two independent fields, and then, the temperature is determined by a special iteration procedure. Two examples involving the convection and radiation boundary conditions are performed using the proposed approach and the results show a good agreement between the proposed method and ABAQUS.

1. Introduction

The determination of temperature distribution in a medium (solid, liquid, gas or combination of these phases) is the main objective of the thermal conduction analysis. As an alternative to the theoretical analysis and experiment measurements, numerical simulations are playing more and more important role for the solution of thermal related problems.

Among all numerical methods, the most popular ones including FEM [1], boundary element method (BEM) [2-4], meshless methods [5-10], and hybrid Trefftz (HT) FEM [11-19] have well-developed to solve heat conduction problems during the past decades. However, they seldom involve multimodal analysis related to surface convection and surface radiation. The difficulties in solving conjugate thermal problems are associated with their nonlinearity and high computing cost.

This paper is devoted to the development and application of a novel hybrid finite element formulation (HFS-FEM) to analyze the conjugate heat transfer problems by constructing a new hybrid variational functional and designing a simple iteration procedure. The developed method is based on the fundamental solution of the problem of interest, rather than the T-complete functions in HT-FEM, and inherits the advantages of HT-FEM against other techniques like FEM and BEM. At the same time, some drawbacks of the HT-FEM are removed in solving many engineering problems [20,21].

2. Basic equations

Let us consider the plane heat conduction equation in Cartesian coordinates $(X_1, X_2)$:

$$
k \left( \frac{\partial^2 u(P)}{\partial X_1^2} + \frac{\partial^2 u(P)}{\partial X_2^2} \right) = 0 \quad P \in \Omega$$

which is complete for any problem only if appropriate boundary conditions stated below are given

\begin{equation}
\begin{aligned}
& u = u_e & \text{on } \Gamma_u \\
& q = q_e & \text{on } \Gamma_q \\
& q = h_t (u - u_e) + \kappa \sigma (u^4 - u_e^4) & \text{on } \Gamma_c
\end{aligned}
\end{equation}
where \( k \) represents the thermal conductivity of materials, \( u \) sought temperature field, \( q = -k \frac{\partial u}{\partial n} \) surface heat flux, \( n \) the unit normal to \( \partial \Omega \), \( u_e \) environment temperature, \( \varepsilon \) the emissivity of the surface, and \( \sigma \) the Stefan Boltzmann constant, which has the value \( 5.669 \times 10^{-8} \text{ W/m}^2\text{K}^4 \). \( \Gamma = \partial \Omega = \Gamma_e + \Gamma_l + \Gamma_r \) is the boundary surface of the domain \( \Omega \).

Evidently, the radiation condition described by the third equation in eq (2) is highly nonlinear, an iteration strategy is, therefore, needed. To this end, we first rewrite the convection and radiation condition as

\[
q = h_e (u - u_e) + \varepsilon \sigma (u^2 - u_e^2) + h (u - u_e)
\]

in which

\[
h(u) = h_e + \varepsilon \sigma (u + u_e) (u^2 + u_e^2)
\]

The formulation (3) is similar to the convective condition stated by the first part of the third expression in eq (2). Then, we can construct the following iteration algorithm to remove the nonlinearity term appeared in eq (4)

\[
q = h(u^{(n-1)}) n^{(n)} - u_e
\]

where \( u^{(n-1)} \) and \( u^{(n)} \) are the temperatures obtained in the iteration step \((n-1)\) and \((n)\), respectively.

When the residue is less than the specified tolerance \( \delta \), that is

\[
\sqrt{\sum_{i=1}^{n} (u^{(n-1)} - u^{(n)})^2} \leq \delta
\]

the iteration stops and yields final convergent results.

For the sake of convenience, the fundamental solution used in our algorithm is given here. If \( P \) and \( Q \) are source and field points, the fundamental solution of a plane heat conduction problem is

\[
u^*(P, Q) = \frac{1}{2k\pi} \ln r
\]

where \( r \) is the distance between \( P \) and \( Q \). The fundamental solution satisfies the governing equation (1) at any field point \( Q \).

3. Fundamental solution based finite element formulation

To deal with the convective condition, the elementary variational integral \( \Pi_{m_e} \) presented in [20] is modified as

\[
\Pi_{m_e} = -\frac{1}{2} \int_{\partial \Omega} \kappa u \frac{\partial u}{\partial n} d\Gamma - \int_{\Omega} q(u - u_e) d\Omega - \frac{1}{2} \int \kappa h (u - u_e) d\Gamma - \frac{1}{2} \int_{\partial \Omega} h (u - u_e) d\Gamma
\]

where \( u \) and \( \tilde{u} \) are independent fields defined in the element and on the element boundary, respectively.

In the hybrid FS FE approach, temperature field within an element, element \( e \), can be approximated by

\[
u_e(P) = \sum_{i=1}^{n} \tilde{u}^* (P, Q) c_i = N_e(P) c_e
\]

where \( c_i \) is unknown coefficients and \( n_e \) is the number of lines source points \( Q_j \) outside the element \( e \).

In order to enforce the conformity of temperature field between adjacent elements, temperature field over the element boundary is independently assumed as

\[
u_b(P) = \sum_{i=1}^{n} \tilde{N}_i d_i = \tilde{N}_b d_b
\]

where \( \tilde{N}_i \) is the shape function defined on the element boundary, and \( d_i \) the nodal temperature. \( n_b \) is the number of nodes on the element edge.

Applying the Gauss theorem, the hybrid variational functional (8) can be converted to the following one involving boundary integrals only:
\[ \Pi_{\text{nc}} = -\frac{1}{2} \int_{\mathcal{G}} q u d\Gamma - \int_{\mathcal{G}} q u \tilde{v} d\Gamma + \int_{\mathcal{G}} q u \tilde{w} d\Gamma - \int_{\mathcal{G}} \frac{h}{2} (\tilde{u} - u_{\text{ref}})^2 d\Gamma \]  

(11)

Substitution of eqs (9)–(10) into eq (11) gives

\[ \Pi_{\text{nc}} = -\frac{1}{2} c^T H_\varepsilon c_\varepsilon - d^T G_\varepsilon c_\varepsilon - c^T G_\varepsilon d_\varepsilon - \frac{1}{2} d^T F_\varepsilon d_\varepsilon + q^T f_\varepsilon - a_\varepsilon \]  

(12)

where

\[ H_\varepsilon = \int_{\mathcal{G}} Q^T N_\varepsilon d\Gamma \quad G_\varepsilon = \int_{\mathcal{G}} Q^T \tilde{N}_\varepsilon d\Gamma \quad d_\varepsilon = \int_{\mathcal{G}} \tilde{N}_\varepsilon q\varepsilon d\Gamma \]

\[ F_\varepsilon = \int_{\mathcal{G}} h\tilde{N}_\varepsilon^T \tilde{N}_\varepsilon d\Gamma \quad f_\varepsilon = \int_{\mathcal{G}} h u_{\text{ref}} \tilde{N}_\varepsilon^T d\Gamma \quad a_\varepsilon = \int_{\mathcal{G}} \frac{h u_{\text{ref}}^2}{2} d\Gamma \]

(13)

The stationary value of \( \Pi_{\text{nc}} \) in eq (12) with respect to \( c_\varepsilon \) and \( d_\varepsilon \) yields the following relations

\[ K_\varepsilon c_\varepsilon = g_\varepsilon - f_\varepsilon \]  

(14)

and

\[ c_\varepsilon = H_\varepsilon^{-1} G_\varepsilon d_\varepsilon \]  

(15)

4. Numerical results

In order to verify the proposed algorithm, two examples including convective and radiation conditions are considered in this paper. The first example described in Fig. 1 involves a rectangular plate with convection conditions. The convection coefficient \( h \) has different values in the computation. The conventional FE solutions obtained from ABAQUS with same mesh as that in the proposed model are provided for the purpose of comparison. As was shown in Fig. 1, due to large temperature change between the upper corner point temperature (380°C) and ambient temperature (25°C), the temperature there might change dramatically, hence, more elements are placed there (see Fig. 2). Results in Fig. 3 show the temperature variation on the convection edge. It can be seen from Fig. 3 that a good agreement between results of HFS-FEM and those of ABAQUS is achieved.

Fig. 1 Rectangular plate with convection edge

Fig. 2 Mesh used in HFS-FEM and ABAQUS with 25 eight-node elements
The second example involves the same plate as the one used in the first example, while a radiation condition is added to the existing convective edge. The material emissivity $\varepsilon$ is 0.725. In the computation, the same mesh as shown in Fig. 2 is employed. If the initial value of temperature is taken to be 25°C and the iteration tolerance is chosen as $10^{-6}$, after 4 iterations, the convergent results can be obtained and the temperature distribution for two convective coefficients ($h_c=0.5$ and 50) is plotted in Fig. 5, from which a good agreement between results of HFS-FEM and ABAQUS can be found. Additionally, the temperature at the corner point (0.4, 0) are 164.4°C ($h_c = 0.5$W/m²K) and 39.5°C ($h_c = 50$W/m²K), respectively, in the presence of thermal radiation, while the temperature values are 168.8°C ($h_c = 0.5$W/m²K) and 39.5°C ($h_c = 50$W/m²K), respectively, in the absence of radiation effect. It is clearly evident that the convection effect is greater than the radiation effect, especially for the larger convective coefficient.
Fig. 5 Temperature variation on the convection and radiation edge

5. Conclusions

A fundamental solution based finite element model for the solution of plane heat conduction with nonlinear radiation condition has been formulated in the paper. Based on the fundamental solution for the potential problem, two independent internal and boundary temperature fields are assumed independently and a modified variational functional is developed. The use of fundamental solution makes all integrals being over the element boundary. Two examples involving the convection boundary condition and radiation boundary condition are performed using the proposed approach and results show a good agreement between the proposed method and ABAQUS.

References


