Numerical simulation of thermal properties in 2D biological tissue using the method of fundamental solution

Hui Wang¹, Wencong Zhang², Qing-Hua Qin²

Summary
Non-invasive thermal diagnostics of biological tissues usually require general solutions for transient bio-heat transfer. In this paper, a meshless model is developed to investigate behavior of heat conduction in isotropic skin tissues to provide data for clinical diagnosis. The well-known Pennes bioheat model is used to describe the process of heat conduction in skin tissues. Firstly, a time stepping θ-method is used in handling time variable in the transient bioheat transfer problem. Then, the analog equation method is utilized to convert the modified bioheat equation into an equivalent standard Poisson’s equation. Subsequently, the particular solution with respect to the right-hand side term of the Poisson’s equation is approximated by a linear combination of radial basis functions, while the homogenous solution is approximately determined by the method of fundamental solution. The uniform structure of matrix is constructed by making the complete solution consisting of the particular and homogeneous parts satisfying the governing equation and the specified boundary conditions. The resulting equation system is finally solved by an iteration approach. Three examples including the skin burn injuries and tumor detection are considered to assess the accuracy and efficiency of the proposed method. It appears to be a promising method for detecting and evaluating the thermal damage on skin materials.

Keywords: Bioheat transfer, meshless method, method of fundamental solutions, radial basis function

Nomenclature

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<th>Definition</th>
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<tr>
<td>L</td>
<td>depth of tissue (m)</td>
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<tr>
<td>c</td>
<td>specific heat of tissue (J/kg/°C)</td>
</tr>
<tr>
<td>c_b</td>
<td>specific heat of blood (J/kg/°C)</td>
</tr>
<tr>
<td>ρ</td>
<td>density of tissue (kg/m³)</td>
</tr>
<tr>
<td>ρ_b</td>
<td>density of blood (kg/m³)</td>
</tr>
<tr>
<td>ω_b</td>
<td>blood perfusion (m³/s/m² tissue)</td>
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<td>k</td>
<td>thermal conductivity of tissue (W/m/°C)</td>
</tr>
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<td>t</td>
<td>time (s)</td>
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<tr>
<td>Q_m</td>
<td>metabolic heat of tissue (W/m³)</td>
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<td>Q_r</td>
<td>spatial heating (W/m³)</td>
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<td>normal heat flux (W/m²)</td>
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<td>artery temperature (°C)</td>
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<td>environmental temperature (°C)</td>
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<td>u_0</td>
<td>initial temperature (°C)</td>
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Introduction

Skin has a significantly vital impact on the meditative transactions in our lives. It covers flexibly and continuously the whole body and protect internal organs from the external environment. Generally, the body surface temperature is controlled by the blood circulation underneath the skin, local metabolism, heat exchange between the skin and its environment, and the internal spatial heating like laser and microwave heating. Changes of any of these factors can induce variations of temperature at skin surface, reflecting the physiological state of the human body. For instance, in burn injuries, different environmental fluids generally cause different temperature distribution at skin surface, which can be used to perform burn injury diagnosis; the surface temperature of skin with tumor has been revealed to be higher than that of healthy tissue [1-4]. Apparently, the abnormal temperature distribution at the skin surface might indicate irregular peripheral circulation which can be used in clinical diagnosis. On the other hand, compared to other non-invasive thermometry like MRI, microwave, and ultrasound, thermal methods appear to be more economic and safer [4]. However, the temperature measurement at skin surface requires solutions to transient bioheat transfer problems under various specific internal and boundary conditions. At present, based on the most popular bioheat model, which is Pennes equation, the finite element method (FEM) [5-11], the finite difference method (FDM) [12-14], the Monte Carlo method (MCM) [15, 16], the boundary element method (BEM)/the dual reciprocity boundary element method (DRBEM) [17-19] have been well studied to determine steady-state or transient temperature at skin tissue. In addition, Trefftz FEM has also been successfully used to solve transient heat conduction problem [20]. Among them, the FEM involving time-consuming domain discretization with finite cells is widely employed due to its good adaptability to complex shapes. In addition, FDM depending on finite point difference net cannot osculate the coordinates of the complex biological shape whereas BEM is an advantageous boundary-type method, which is only involving the boundary discretization. However, it is difficult to treat transient or non-homogeneous problems, in which the fundamental solutions required in conventional BEM and domain integrals are usually difficult to deal with and time consuming. Fortunately, the appearance of DRBEM [21] based on the conventional BEM and radial basis functions can partly overcome these shortcomings and extends the application of the BEM to complex problems. The remaining MCM mentioned above is another special numerical method using random process approach, so it is different with the classic numerical methods listed above and is weakly depending on the dimensions of the problem. In general, MCM is viewed as a branch of experimental mathematics.

Unlike the above approaches, in the paper, a meshless model combing the
fundamental solutions and radial basis functions (RBF) approaches [22, 23] is developed to predict the temperature distribution in the skin tissue. After time-marching process, the modified system is replaced by an equivalent Poisson’s equation which is introduced according to the idea of the analog equation method (AEM)[24]. Then, the homogeneous and particular solutions are constructed by the method of fundamental solutions (MFS) and RBF approximation, respectively. Subsequently, the uniform structure of matrix is constructed by making the complete solution consisting of the particular and homogeneous parts satisfy the governing equation and the specified boundary and the resulting equations are finally solved by time iteration.

The paper is arranged as follows. Firstly the Pennes’ bioheat mathematical model is presented for completeness in Section 2. Detailed solution procedure is described in section 3 and three numerical examples are considered in Section 4 for algorithm validation and assessment. Finally, some remarks are concluded in Section 5.

**Pennes bioheat mathematic model**

The well-known Pennes bioheat transfer equation was commonly used to simulate the thermal behavior in the biological tissue. Generally, this equation is written as

\[
\rho c \frac{\partial u(x,t)}{\partial t} = \nabla \cdot [k \nabla u(x,t)] + \omega_b \rho_b c_b [u_a - u(x,t)] + Q_m + Q_r(x,t)
\]  

(1)

in which \( x = (x_1, x_2) \in \Omega \subset \mathbb{R}^2 \) and the term on the left is associated with sensible energy storage in the tissue. The first term on the right represents conduction of heat in the tissue, caused by the temperature gradient, and the second term describes the heat transport between the tissue and microcirculatory blood perfusion. The third term on the right depicts internal heat generation due to metabolism and the last term is spatial heating caused by external heat sources. The Pennes equation accounts for the ability of tissue to remove heat by both passive conduction and perfusion of blood. The blood perfusion is defined as the mass flow rate of blood flow per unit volume in a region that contains sufficient capillaries. Most living tissues, including much of the skin and brain, are highly perfused.

For convenience, a new symbol \( Q(x,t) = Q_r(x,t) + Q_m \) including metabolic heat and special heating is introduced. So, the simplified mathematic model is

\[
\rho c \frac{\partial u}{\partial t} = \nabla \cdot (k \nabla u) + \omega_b \rho_b c_b [u_a - u] + Q
\]

(2)

Besides the above bioheat governing equation, the following boundary conditions and initial condition are applied to the four boundaries to keep the system
complete:

Dirichlet/necessary condition: \( u(x,t) = a(x,t) \quad x \in \Gamma_a \)  

Newman/nature condition: \( q(x,t) = q(x,t) \quad x \in \Gamma_q \)  

Convective condition: \( q(x,t) = h_e [u(x,t) - u_e] \quad x \in \Gamma_c \)  

Initial condition: \( u(x,0) = u_0 \quad x \in \Omega \)  

where \( q \) represents the boundary normal heat flux defined as \( q = -\frac{k \partial T}{\partial n} \) and \( n \) is the unit outward normal to the boundary \( \Gamma \) of the interesting domain \( \Omega \).

Practically, due to the special structure of skin tissue, the rectangular domain usually is studied. For this case, the adiabatic conditions are applied to the upper and bottom edges by assuming that tissue far from the area of interest is not affected by the imposed thermal disturbance on the skin surface, and the temperature on the right edge keeps constant core temperature (see Figure 1).

![Figure 1: Schematic diagram of single-layer tissue](image)

For simplicity, the boundary conditions (3)-(5) are expressed in a general form as

\[
B_1 u(x,t) + B_2 q(x,t) = B_3 q(x,t)  
\]

where \( B_1, B_2, \) and \( B_3 \) are known coefficients and can be written respectively as

\[
\begin{align*}
B_1 &= 1, \quad B_2 = 0, \quad B_3 = 1 & \text{on } \Gamma_a \\
B_1 &= 0, \quad B_2 = 1, \quad B_3 = q & \text{on } \Gamma_q \\
B_1 &= h_e, \quad B_2 = -1, \quad B_3 = h_e u_e & \text{on } \Gamma_c
\end{align*}
\]
Numerical scheme

Time-marching scheme

For a typical time interval \([t^n, t^{n+1}] \subset [0, T]\), \(u(x,t)\), its derivative to time variable \(t\) and \(Q_t(x, t)\) are approximated as

\[
\begin{align*}
\frac{u(x,t)}{t} &= \theta u^{n+1}(x) + (1 - \theta) u^n(x) \\
\frac{Q_t(x, t)}{t} &= \theta Q^{n+1}_t(x) + (1 - \theta) Q^n_t(x) \\
\frac{\partial u(x,t)}{\tau} &= u^{n+1}(x) - u^n(x)
\end{align*}
\]  

(9)

where the superscripts \(n\) and \(n + 1\) refer to subsequent time instances and \(\tau = t^{n+1} - t^n\) is the time step size. \(\theta\) represents dimensionless parameter chosen in \([0, 1]\). In general, this system is only unconditionally stable (i.e., errors will not grow unboundedly) if \(\theta \geq 0.5\). Common choices would be \(\theta = 0.5\) (central or Crank-Nicolson method), \(\theta = 2/3\) (Galerkin implicit method) and \(\theta = 1\) (backward difference method).

Substituting Eqs. (9) into the partial differential equation (PDE) (2) and rearranging it give the following Helmholtz-type equation

\[
\begin{align*}
&u^{n+1}(x) - \frac{\tau k}{\rho c} \theta \nabla^2 u^{n+1}(x) + \frac{\tau s_b}{\rho c} \theta u^{n+1}(x) \\
&u^n(x) + \frac{\tau k}{\rho c} (1 - \theta) \nabla^2 u^n(x) - \frac{\tau s_b}{\rho c} (1 - \theta) u^n(x) + \frac{\tau s_b}{\rho c} u^n + \frac{\tau}{\rho c} Q^n_t(x)
\end{align*}
\]  

(10)

with \(s_b = s_b \rho c \). 

Furthermore, the related boundary condition (7) is changed to

\[
\theta [B_1 u^{n+1}(x) + B_3 q^{n+1}(x)] = \theta B^0_3(x) - (1 - \theta) [B_1 u^n(x) + B_3 q^n(x)]
\]  

(11)

where \(B^0_3 = \theta B_3^{n+1} + (1 - \theta) B_3^n\).

Analog equation method (AEM) [24]

Under the assumption that the sought solution \(u^{n+1}(x)\) is at least two-order continuously differentiable in terms of spatial variable \(x\) in the domain, the standard Laplace operator is applied to this function, we can finally has

\[
\nabla^2 u^{n+1}(x) = h(x) \quad x \in \Omega
\]  

(12)

which indicates that the solution of Eq. (10) can be established by solving this linear Poisson equation with same boundary conditions, if the fictitious source term
introduced by means of the AEM is known. However, we can deal with this obstacle by the following procedure using the method of fundamental solution with radial basis functions [25].

Due to the linear property of Eq. (12), its solution can be expressed as a summation of homogeneous solution $u_h^{n+1}$ and particular solution $u_p^{n+1}$, that is

$$u^{n+1} = u_h^{n+1} + u_p^{n+1}$$  \hspace{1cm} (13)

where $u_h^{n+1}$ and $u_p^{n+1}$, respectively, satisfy

$$\nabla^2 u_p^{n+1}(x) = b(x) \quad x \in \mathbb{R}^2$$  \hspace{1cm} (14)

and

$$\nabla^2 u_h^{n+1}(x) = 0 \quad x \in \Omega$$  \hspace{1cm} (15)

**Particular solution with radial basis function interpolation**

The next step of the proposed approach is to evaluate the part of particular solution by RBF approximation [26]. The right-hand term of equation (14) can be approximated by

$$b(x) = \sum_{i=1}^{N_i} \alpha_i^{n+1} \phi_i(x) \quad x \in \Omega$$  \hspace{1cm} (16)

where $N_i$ is the number of interpolation points in the domain under consideration. $\phi_i(x) = \phi(r) = \phi(|x - x_i|)$ denotes radial basis functions with different reference points $x_i$ and $\alpha_i^{n+1}$ are interpolating coefficients to be determined.

Simultaneously, the particular solution $u_p^{n+1}$ is similarly expressed as

$$u_p^{n+1}(x) = \sum_{i=1}^{N_i} \gamma_i^{n+1} \hat{u}_i(x)$$  \hspace{1cm} (17)

where $\hat{u}_i$ represent corresponding approximated particular solutions which satisfy analytically

$$\nabla^2 \hat{u}_i = \phi_i$$  \hspace{1cm} (18)

according to the relation of the particular solution $u_p^{n+1}$ and right-handed term $b$ in Eq. (14).

Specially, for the case of thin plate spline (TPS), that is, $\phi_i = r^2 \ln r$, the corresponding $\hat{u}_i$ is given by

$$\hat{u}_i = \frac{r^4}{16} \ln r - \frac{r^4}{32}$$  \hspace{1cm} (19)

Since the inhomogeneous term $b$ is a unknown function depending on the sought temperature field $u^{n+1}$, the coefficients $\alpha_i^{n+1}$ can not be determined directly through solving Eq. (16).
Method of fundamental solution for the homogeneous solution

To obtain a weak solution of the Laplace equation (15), \( N_S \) fictitious source points \( y_j \) \((j = 1, 2, \ldots, N_S)\) are chosen outside the domain. According to superposition theory and the physical property of the fundamental solution, the potential \( u_h^{n+1} \) at arbitrary field point \( x \) in the domain can be expressed by a linear combination of fundamental solutions in terms of fictitious sources located outside the domain, that is [27, 28]

\[
u_h^{n+1}(x) = \sum_{j=1}^{N_S} \beta_j^{n+1} u_j^*(x) \quad \forall x \in \Omega, \quad y_j \notin \Omega
\] (20)

where \( u_j^*(x) = u^*(x, y_j) \) are the fundamental solutions related to the Laplace operator, and \( \beta_j^{n+1} \) are corresponding values of source (see Figure 2).

![Diagram](image)

**Figure 2:** Illustration of source points properly, points and boundary field points

It is clear that Eq. (20) naturally satisfy the homogeneous equation (15) according to the definition of the fundamental solutions

\[
\nabla^2 u^*(x, y) = \delta(x, y) \quad \forall x, y \in \mathbb{R}^2
\] (21)

where \( \delta \) is a standard Delta function satisfying

\[
\delta(x, y) = \begin{cases} 
0 & x \neq y \\
\infty & x = y
\end{cases}
\] (22)
Typically, for the two-dimensional cases, there is

\[ u^*(x, y) = \frac{1}{2\pi} \ln \frac{1}{r(x, y)} \]  \hspace{1cm} (23)

Additionally, the generation of source points outside the domain is a curious problem, which affects the accuracy and stability. At present, there is not a uniform approach to generate these source points properly.

In our work, a strategy shown in Figure 2 is employed [29]

\[ y_j = x_j + \gamma (x_j - x_c) \]  \hspace{1cm} (24)

where \( x_j \) are boundary nodes, \( x_c \) is the geometric center of the domain and \( \gamma \) is a dimensionless parameter, which can be chosen as 2.0 for our thermal analysis in biological tissue.

**Construction of solving equations**

Based on above operations, the complete solution \( u^{n+1}(x) \) can be written as

\[ u^{n+1}(x) = \sum_{i=1}^{N_\Omega} \alpha_i^{n+1} \tilde{u}_i(x) + \sum_{j=1}^{N_\Omega} \beta_j^{n+1} u_j^*(x) \quad x \in \Omega \]  \hspace{1cm} (25)

which also should be the solution we are seeking to the system consisting of Eqs. (10) and (11).

For the sake of convenience, we express the equation (25) above in a vector form

\[ u^{n+1}(x) = \{U(x)\} \{A^{n+1}\} \quad x \in \Omega \]  \hspace{1cm} (26)

with

\[ \{A^{n+1}\} = \begin{pmatrix} \alpha_{1}^{n+1} & \alpha_{2}^{n+1} & \cdots & \alpha_{N_\Omega}^{n+1} & \beta_{1}^{n+1} & \beta_{2}^{n+1} & \cdots & \beta_{N_\Omega}^{n+1} \end{pmatrix}^T \]

\[ \{U(x)\} = \{\tilde{u}_1(x), \tilde{u}_2(x), \cdots, \tilde{u}_{N_\Omega}(x), u_1^*(x), u_2^*(x), \cdots, u_{N_\Omega}^*(x)\} \]

Furthermore, the normal heat flux can be derived as

\[ q^{n+1}(x) = \{Q(x)\} \{a^{n+1}\} \]  \hspace{1cm} (27)

where \( \{Q(x)\} = -k \frac{\partial u^*(x)}{\partial n} \).

In order to determine all unknowns, making Eq. (26) satisfies the governing equation (10) at \( N_I \) interpolation points \( s_i^* \) \( (i = 1, 2, \ldots, N_I) \) within the domain produces

\[ [K_I] \{A^{n+1}\} = [M_I] \{A^n\} + \{F_I\} \]  \hspace{1cm} (28)
Additionally, the substitution of Eq. (26) into the boundary condition at \( N_S \) boundary nodes \( x_j^B \) \((j = 1, 2, \ldots, N_S)\) yields

\[
[K \{A^{*+1}\}] = [M_2] [A^*] + \{F_2\}
\]

(29)

Evidently, the final solution system consisting of Eqs. (28) and (29) is given by

\[
\begin{bmatrix}
[K_1] \\
[K_2]
\end{bmatrix} [A^{*+1}] = \begin{bmatrix}
[M_1] \\
[M_2]
\end{bmatrix} [A^*] + \begin{bmatrix}
\{F_1\} \\
\{F_2\}
\end{bmatrix}
\]

(30)

from which all unknowns at different time instances can be determined. Further, the temperature field at arbitrary point within the domain can be evaluated by means of Eq. (25).

**Numerical validation**

**Example 1** Constant surface temperature heating

In this case, the surface temperature at \( x_1 = 0 \) was instantaneously changed to \( u_s = 45^\circ\text{C} \). For validation of the algorithm, the following steady-state analytical solution without metabolic heat is given [30]

\[
u(x_1) = u_a + \frac{(u_a - u_s) \sinh(\mu (L - x_1)) + (u_c - u_s) \sinh(\mu x_1)}{\sinh(\mu L)}
\]

(31)

with \( \mu = \sqrt{\frac{\rho_b c_b}{k}} \).

![Figure 3: Steady-state temperature distribution along the depth of skin for exposure to a constant surface temperature](image1)

![Figure 4: Variation of temperature in skin at different time instances with m3/s/m3 for exposure to a constant surface temperature](image2)
Figure 3 shows the final steady-state temperature along the depth of skin tissue under the constant applied surface temperature and we can see clearly that the numerical results by MFS agree well with the exact ones. The required iteration time to achieve the final steady state is 3830s, 3025s, and 2495s, respectively. Simultaneously, Figure 3 also depicts the influence of blood perfusion rates. Larger blood perfusion rates are, steeper thermal gradients in the region near the skin surface are. This phenomenon also can be explained that the large blood perfusion can effectively remove the heat and decrease the damage to deeper tissue.

Additionally, the heating history in the skin is presented in Figure 4 with \( \omega_b = 0.0005 m^3/s/m^2 \), from which we can see that the propagation rate of the thermal wave into the tissue is slow, due to the relatively mild surface temperature (45°C).

**Example 2** Tumor hyperthermia

If there is a tumor at the skin surface, the tumor will affect the distribution of blood perfusion due to different metabolic heat generation. The different thermal state expressions at the skin surface will indicate the abnormal blood perfusion or metabolic heat generation. It is a benefit that this difference can be used for noninvasive diagnostics for the physiological status of the biological body such as tumor[31]. To demonstrate this, a classical example will be considered. Figure 5 shows the location of the tumor beneath the skin surface. There are entirely 56 boundary nodes on the real boundary and 231 interpolation nodes within the domain to be chosen to perform the computation. The boundary conditions (BCs) are shown below:

\[
q(x,y,0) = 0, \quad x,y \in I, \quad II, \ IV \quad \text{and} \quad T(x,y,0) = 37 °C, \quad x,y \in III
\]

For simplicity, the environmental parameters are neglected in this example. The temperature at core III boundary is considered as constant as well as arterial blood temperature (\( a_s = 37°C \)). The following assumptions on blood perfusion and metabolic heat generation are made for a highly vascularized tumor situated underneath the skin. [31]. Thus, for healthy tissue,

\[
\omega_b = 0.0005 ml/s/ml, \quad x,y \in \Omega \quad \text{and} \quad Q_m = 420 J/m^3 s, \quad x,y \in \Omega
\]

For tissue with a tumor,

\[
\omega = \begin{cases} 
0.0005 \text{ ml/s/ml}, & x,y \not\in L \\
0.002 \text{ ml/s/ml}, & x,y \in L
\end{cases}
\]

where \( L \subseteq |y| \leq 0.01m, \ 0.005m \leq x \leq 0.015m \) is prearranged as tumor domain and \( \Omega \) is the entire domain.
Figure 6 clearly shows the differences of the temperature distribution between tumor site and healthy tissues due to different blood perfusion and metabolic heat generation. For healthy tissue, the temperature distribution is perfectly obtained while there is a little distinction for tissues with tumor, compared to those in reference [31]. However, with the advantage of time saving, this tiny difference is accepted by the accuracy of $10^{-4}$. Moreover, Figure 7 and 8 demonstrates how the temperature varies from 0 s to 10000 s at a point of $(0, \theta, 0.014)$ when applied different values of theta and stepping time for healthy tissue and tissues with tumor respectively.

**Figure 5: Illustration of tissue with tumor**

**Figure 6: Steady state temperature distributions at $\theta = 0.6$, $\Delta t = 10$**

**Example 3 Cryosurgery**
Cryosurgery is a freezing technique used in the clinic medical surgery for the cancer or tumors. It utilizes tremendously low temperature into frozen range in situ to kill or remove cancer tissues, tumors or tissues in target. Cryosurgery can select undesirable area to destroy. It has been used in various medical surgeries for different parts of bodies by a large number of surgeons, doctors and experts, especially when the targeted tissues can be easily accessed. The cryosurgery can be found in many areas, such dermatology, genealogy etc. However, the extent of the irregular shape of the freezing section and the growth of ice balls is difficult to measure and understand. It is necessary to simulate the process of cryosurgery in clinics. The algorithm proposed in this paper can handle with these problems with moving boundary. We consider one probe touching in the middle of the surface of the skin as shown in Figure 9. The entire skin tissue has a length of 0.1 m in the Y direction and a width of 0.05 m along X direction.
The boundary conditions are given by

\[
\begin{align*}
    \left. k \frac{\partial u}{\partial x} \right|_{x=0} &= h_{\text{in}}(u - u_0), & \text{do not contact with cryoprobe} \\
    \left. u \right|_{x=0} &= u_0, & \text{contact with cryoprobe}
\end{align*}
\]

\[u|_{x=0.05} = u_0, \quad q = -k \frac{\partial u}{\partial y} \bigg|_{y=0} = 0 \quad \text{and} \quad q = -k \frac{\partial u}{\partial y} \bigg|_{y=0.01} = 0.\]

The initial condition is simplified as \( u(x, y, t)|_{t=0} = u_0. \) Other properties used for cryosurgery problem are shown in Table 1.

To solve this problem, we first utilize the effective capacity method to construct a uniform and linear equation proposed in [32]. Subsequently, solving the linear equation can be applied by using our algorithm. The temperature distribution in the tissue is shown in Figure 10a. A more detailed and lucid isotherm map for cryosurgery problem is given by Figure 10b where the dash line and bold text indicate the lower and upper boundaries of phase change. Figure 11 represents that the upper and lower boundaries extending from the skin surface to the internal core with time changing for the case of \( \theta = 0.6, \Delta t = 100. \) Figure 12 indicates the temperature distribution versus time at point of \((0, 0.0486).\)

**Conclusions**

In the paper, a meshless approach combining the method of fundamental solutions and radial basis functions is developed to perform temperature prediction in healthy or abnormal tissues under various surface boundary conditions. Three numerical examples involving constant surface temperature heating, convective heating in different fluids and tumor detection are analyzed for the validation of the proposed method. The solution procedure and final computational results show that
Table 1: Values of properties used in freezing problem

<table>
<thead>
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<th>Symbols</th>
<th>Value</th>
<th>Symbols</th>
<th>Value</th>
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<tr>
<td>( C_p, C_w )</td>
<td>3.6 MJ/m(^3) K(^{-1})</td>
<td>( u_{a}, u_{c} )</td>
<td>37 °</td>
</tr>
<tr>
<td>( C_f )</td>
<td>1.86 MJ/m(^3) K(^{-1})</td>
<td>( u_{ml} )</td>
<td>-8 °</td>
</tr>
<tr>
<td>( k_f )</td>
<td>2 W/m K</td>
<td>( u_{ma} )</td>
<td>-1 °</td>
</tr>
<tr>
<td>( k_w )</td>
<td>0.5 W/m K</td>
<td>( h_{nn} )</td>
<td>10 W/m(^2) K(^{-1})</td>
</tr>
<tr>
<td>( \delta_b )</td>
<td>0.0005 m/s</td>
<td>( u_{nb} )</td>
<td>10 °</td>
</tr>
<tr>
<td>( Q_i )</td>
<td>250 MJ/m(^3)</td>
<td>( u_{0} )</td>
<td>37 °</td>
</tr>
<tr>
<td>( Q_m )</td>
<td>4200 W/m(^3)</td>
<td>( u_{w} )</td>
<td>-120 °</td>
</tr>
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</table>

(a) Three dimensional surface graph (b) Isotherm map

Figure 10: Profile of temperature distribution at \( t = 1000s \) when \( \theta = 0.6, \Delta t = 10 \)

(a) Upper boundary of phase change (b) Lower boundary of phase change

Figure 11: The schematic of moving boundary of phase change versus time
the presented method is an efficient meshless method, which has simple solution procedure, high accuracy and stability, and can be used for fast analysis to identify thermal states of the biological bodies for different purposes.

References


18. Lu WQ, Liu J and Zeng Y. Simulation of the thermal wave propagation in biological tissues by the dual reciprocity boundary element method. Eng

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