Theoretical Prediction of Surface Bone Remodeling under Axial and Transverse Loads

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Abstract

Theoretical prediction of surface bone remodeling in the diaphysis of the long bone under various transverse loads are made within the framework of adaptive elastic theory. These loads include external lateral pressure, electric, magnetic and thermal loads. An explicit solution is obtained for analyzing problems of surface bone remodeling. Numerical results are presented to verify the proposed formulation and to show the effects of mechanical, thermal and magnetoelectric loads on bone remodeling process. The theory can also be applied to the problem of predicting surface remodeling in the bone subjected to coupled axial force, external lateral pressure, electric, magnetic and thermal loads as a result of a force-fitted medullary pin.

Keywords: surface bone remodeling, biomaterial, biomechanics, magnetoelectroelastic

1. Introduction

Surface bone remodeling is the deposition and resorption of the surface layers of bone tissues. It is well-known that outer environment can significantly affect bone surface remodeling. For example, the bone of a person who often exercises himself physically is much stronger and thicker than that of a less exercised one. The mechanism of this process attracted wide attention from biological scientists and mechanical engineers. Many experimental investigations related to this subject have been carried out and a lot of hypothesis have been employed to explain the bone remodeling [1-6]. Among these theories, “adaptive elasticity” proposed by Cowin and Hegedus[2] is the most popular one.


In this work, an explicit solution for thermomagnetoelectroelastic problems of surface bone
remodelling, based on the theory of adaptive elasticity [2], is presented to study the effects of mechanical, thermal and magnetoelectric loads on surface bone remodelling process. The analytical solution is used for investigating surface bone remodelling process on the basis of assuming a homogeneous bone material [4]. Numerical results are presented to show applicability of the proposed solution and the effect of thermal and magnetoelectric loads on the bone surface remodelling process.

2. Solution of surface modeling for a homogeneous hollow circular cylindrical bone

2.1 Linear theory of theromagnetoelectroelastic solid

Consider a hollow circular cylinder composed of linearly a theromagnetoelectroelastic bone material subjected to axisymmetric loading. The axial, circumferential and normal to the middle-surface co-ordinate length parameters are denoted with \( z \), \( \theta \) and \( r \), respectively. The constitutive equations for the theromagnetoelectroelastic field in the cylindrical co-ordinate system are [18]:

\[
\begin{align*}
\sigma_{rr} &= c_{11} s_{rr} + c_{12} s_{\theta\theta} + c_{13} s_{zz} - e_{31} E_z - \alpha_{31} H_z - \beta_1 T \\
\sigma_{\theta\theta} &= c_{12} s_{rr} + c_{11} s_{\theta\theta} + c_{13} s_{zz} - e_{31} E_z - \alpha_{31} H_z - \beta_1 T \\
\sigma_{zz} &= c_{13} s_{rr} + c_{11} s_{\theta\theta} + c_{33} s_{zz} - e_{31} E_z - \alpha_{33} H_z - \beta_3 T \\
\sigma_{rr} &= c_{44} s_{rr} - e_{45} E_r - \alpha_{45} H_r, \\
D_r &= e_{15} s_{rr} + \kappa_1 E_r + d_1 H_r, \\
D_z &= e_{31} (s_{rr} + s_{\theta\theta}) + e_{33} s_{zz} + \kappa_3 E_z + d_3 H_z - p_3 T \\
B_r &= \alpha_{15} s_{rr} + d_1 E_r + \mu_1 H_r, \\
B_z &= \alpha_{31} (s_{rr} + s_{\theta\theta}) + \alpha_{33} s_{zz} + d_3 E_z + \mu_3 H_z - m_3 T \\
h_r &= k_r q_r, \quad h_z = k_z q_z
\end{align*}
\]

where \( \sigma_{ij} \), \( s_{ij} \), \( D_i \), \( E_i \), \( B_i \), \( H_i \) and \( h_i \) are components of stress, strain, electrical displacement, electric field, magnetic induction, magnetic field and heat flow, respectively; \( c_{ij} \) are elastic stiffness; \( e_{ij} \) are piezoelectric constants; \( \alpha_{ij} \) are piezomagnetic constants; \( \kappa_i \) are dielectric permitivities; \( d_i \) are magnetoelectric constants; \( \mu_i \) are magnetic permeabilities; \( T \) denotes temperature change; \( p_3 \) is pyroelectric constant; \( m_3 \) is pyromagnetic constant; \( \beta_i \) are stress-temperature coefficients; \( q_i \) are heat intensity; and \( k_i \) are heat conduction coefficients. The associated strains, electric fields, and heat intensities are respectively related to displacements \( u_i \), electric potential \( \varphi \), magnetic potential \( \psi \), and temperature change \( T \) as

\[
\begin{align*}
s_{rr} &= u_{rr}, \quad s_{\theta\theta} = u_{\theta\theta}, \quad s_{zz} = u_{zz} \quad \sigma_{rr} &= u_{rr,rr} + u_{rr,\theta \theta} + E_r = -\varphi, \\
E_z &= -\varphi, \quad H_r = -\psi, \quad H_z = -\psi, \quad q_r = -T, \quad q_z = -T
\end{align*}
\]

For quasi-stationary behaviour, in the absence of heat source, free electric charge, electric current, and body forces, the set of equations for thermopiezoelectricmagnetic theory of bones is completed by adding the following equations of equilibrium for heat flow, stress, electric displacements and magnetic induction to eqs. (1) and (2).

\[
\begin{align*}
\frac{\partial \sigma_{rr}}{\partial r} + \frac{\partial \sigma_{rr}}{\partial z} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} &= 0, & \frac{\partial \sigma_{\theta\theta}}{\partial r} + \frac{\partial \sigma_{zz}}{\partial z} + \frac{\sigma_{zz}}{r} &= 0, \\
\frac{\partial D_r}{\partial r} + \frac{\partial D_z}{\partial z} + \frac{D_r}{r} &= 0, & \frac{\partial B_r}{\partial r} + \frac{\partial B_z}{\partial z} + \frac{B_r}{r} &= 0 \\
\frac{\partial h_r}{\partial r} + \frac{\partial h_z}{\partial z} + \frac{h_r}{r} &= 0
\end{align*}
\]

2.2 Equation for surface bone remodeling

The equations of the adaptive elastic theory presented in [3] are used and extended to include piezoelectric and piezomagnetic effect in this study. The remodelling rate equation in cylindrical coordinates is
\[ U = C_0 \left( n \cdot Q \right) \left[ s_y \left( Q \right) - s_y^0 \left( Q \right) \right] + C_1 \left[ E_i \left( Q \right) - E_i^0 \left( Q \right) \right] + G_i \left[ H_i \left( Q \right) - H_i^0 \left( Q \right) \right] \]

\[ = C_{rr} s_{rr} + C_{r\theta} s_{r\theta} + C_{rz} s_{rz} + C_{E_r} E_r + C_{E_z} E_z + G_r H_r + G_z H_z - C_0 \]

where \( C_0 = C_{rr} s_{rr} + C_{r\theta} s_{r\theta} + C_{rz} s_{rz} + C_{E_r} E_r^0 + C_{E_z} E_z^0 + G_r H_r^0 + G_z H_z^0 \); \( U \) denotes the velocity of the remodeling normal to the surface; \( C_i \) and \( G_i \) are surface remodeling coefficients.

### 3. Analytical solution

We now consider a hollow circular cylinder of bone, subjected to an external temperature change \( T_0 \), a quasi-static axial load \( P \), an external pressure \( p \), an electric potential load \( \phi_a \) (or/and \( \phi_b \)) and an magnetic potential load \( \psi_a \) (and/or \( \psi_b \)). The boundary conditions are

\[ T = 0, \quad \sigma_r = \sigma_{r\theta} = \sigma_{rz} = 0, \quad \phi = \phi_a, \psi = \psi_a \quad \text{at} \quad r = a \]

\[ T = T_0, \quad \sigma_r = -p, \quad \sigma_{r\theta} = \sigma_{rz} = 0, \quad \phi = \phi_b, \psi = \psi_b \quad \text{at} \quad r = b \]

and

\[ \int_S \sigma_{zz} dS = -P \]

where \( a \) and \( b \) denote, respectively, the inner and outer radii of the bone, and \( S \) is the cross-sectional area.

For a long bone, it is assumed that except the axial displacement \( u_z \), all displacements, temperature and electrical potential is independent of the \( z \) coordinate and that \( u_z \) may have linear dependence on \( z \).

Using (1) and (2), differential equations (3) can be written as

\[ \left( \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} \right) T = 0, \quad c_{11} \left( \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} - \frac{1}{r^2} \right) u_r = \beta_1 \frac{\partial T}{\partial r} \]

\[ c_{14} \left( \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} \right) u_z + e_1 \left( \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} \right) \phi + \alpha_1 \left( \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} \right) \psi = 0 \]

\[ e_1 \left( \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} \right) u_z - \kappa_1 \left( \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} \right) \phi - \mu_1 \left( \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} \right) \psi = 0 \]

The solution to the above equations satisfying boundary conditions (5)-(6) is given by

\[ u_r = \frac{r}{F_3^*} \left( c_{13} \beta_1^* \left[ \beta_2^* T_0 + p(t) \right] + \sigma \left( c_{13} c_{12} - \frac{F_2^* T_0 + P(t)}{\pi (b^2 - a^2)} \right) c_{13} - F_1^* T_0 c_{13} \right) \]

\[ + \frac{a^2 \beta_1^* \beta_2^* T_0 + p(t)}{r(e_{11} - c_{12})} + \frac{\sigma [\ln(r/a) - 1]}{c_{11}} \]

\[ u_z = \frac{z}{F_3^*} \left[ F_1^* T_0 - \frac{F_2^* T_0 + P(t)}{\pi (b^2 - a^2)} \right] (c_{11} + c_{12}) - 2 \frac{c_{13} c_{12} \sigma}{c_{11}} - \frac{e_1 (\phi_b - \phi_a) \ln(r/a)}{c_{44} \ln(b/a)} - \frac{\alpha_1 (\psi_b - \psi_a) \ln(r/a)}{c_{44} \ln(b/a)} \]

\[ \psi = \frac{\ln(r/a)}{\ln(b/a)} (\phi_b - \phi_a) \phi = \frac{\ln(r/a)}{\ln(b/a)} (\psi_b - \psi_a) + \psi_a \]

\[ T = \frac{\ln(r/a)}{\ln(b/a)} \frac{\ln(r/a)}{\ln(b/a)} T_0 \]

where

\[ \sigma = \frac{\beta T_0}{2 \ln(b/a)} F_1^* \quad F_1^* = \frac{1}{\ln(b/a)} \left( \frac{c_{13} \beta_1^*}{c_{11}} - \frac{\beta_3}{2} \right) \]

\[ F_2^* = \pi b^2 \left( \frac{c_{13}}{c_{11}} \beta_1 - \beta_3 \right) \]
The strains and electric field intensity can be found by introducing Eqs. (10)—(13) into (1). They are, respectively,

\[ s_{rr} = \frac{1}{F_3} \left( c_{33} \beta_1^2 [\beta_1^2 T_0 + p(t)] + \sigma \frac{c_{33} c_{12}}{c_{11}} + \frac{F_2^* T_0 + P(t)}{\pi (b^2 - a^2)} c_{13} - F_4^* T_0 c_{13} \right) \]

\[ - \frac{a^2 \beta_1^2 [\beta_1^2 T_0 + p(t)]}{r^2 (c_{11} - c_{12})} + \frac{\sigma \ln(r/a)}{c_{11}} \]  

\[ s_{\theta\theta} = \frac{1}{F_3} \left( c_{33} \beta_1^2 [\beta_1^2 T_0 + p(t)] + \sigma \frac{c_{33} c_{12}}{c_{11}} + \frac{F_2^* T_0 + P(t)}{\pi (b^2 - a^2)} c_{13} - F_4^* T_0 c_{13} \right) \]

\[ + \frac{a^2 \beta_1^2 [\beta_1^2 T_0 + p(t)]}{r^2 (c_{11} - c_{12})} + \frac{\sigma \ln(r/a) - 1}{c_{11}} \]  

\[ s_{\varphi\varphi} = \frac{1}{F_3} \left( \left[ F_4^* T_0 - \frac{F_2^* T_0 + P(t)}{\pi (b^2 - a^2)} \right] (c_{11} + c_{12}) - 2c_{13} \beta_1^2 [\beta_1^2 T_0 + p(t)] \right) \]

\[ \frac{2c_{13} c_{12} \sigma}{c_{11}} \]  

\[ s_{r\varphi} = -\frac{e_{15} (\varphi_b - \varphi_a)}{rc_{44} \ln(b/a)} - \frac{\alpha_{15} (\Psi_b - \Psi_a)}{rc_{44} \ln(b/a)} \]

\[ E_r = -\frac{(\varphi_b - \varphi_a)}{r \ln(b/a)} \quad H_r = -\frac{(\psi_b - \psi_a)}{r \ln(b/a)} \]

Substituting (16)-(20) into (4) yields

\[ U_e = N_{1e}^e \frac{b^2}{b^2 - a^2} + N_{2e}^e \frac{1}{\ln(b/a)} + N_{3e}^e \frac{1}{b^2 - a^2} + N_{4e}^e \frac{1}{a \ln(b/a)} - C_0^e \]

\[ U_p = N_{1p}^p \frac{b^2}{b^2 - a^2} + N_{2p}^p \frac{\alpha^2}{b^2 - a^2} + N_{3p}^p \frac{1}{\ln(b/a)} + N_{4p}^p \frac{1}{b^2 - a^2} \]

\[ + N_{4p}^p \frac{1}{b \ln(b/a)} + N_3^p - C_0^p \]

where

\[ N_{1e}^e = \frac{1}{F_3} \left[ c_{13} \left( \frac{c_{13}}{c_{11}} \beta_1 - \beta_3 \right) T_0 - c_{33} \left( \frac{\beta_1 (c_{12} - 1) T_0 + p(t)}{c_{11}} \right) \right] (C_{rr}^e + C_{\theta\theta}^e) \]

\[ + \frac{1}{F_3} \left[ 2c_{13} C_{\varphi\varphi}^e \left( \frac{\beta_1 (c_{12} - 1) T_0 + p(t)}{c_{11}} \right) - (c_{11} + c_{12}) C_{\varphi\varphi}^e \left( \frac{c_{13}}{c_{11}} \beta_1 - \beta_3 \right) T_0 \right] \]

\[ + \left( C_{rr}^e - C_{\theta\theta}^e \right) \left( \frac{\beta_1 (c_{12} - 1) T_0 + p(t)}{c_{11}} \right) \]

\[ + \frac{c_{11} - c_{12}}{c_{11}} \]  

\[ \frac{1}{F_3} \left[ c_{13} c_{13} \beta_1 T_0 - \left( \frac{c_{13}}{c_{11}} \beta_1 - \beta_3 \right) c_{13} T_0 \right] (C_{rr}^e + C_{\theta\theta}^e) \]

\[ + \frac{C_{\varphi\varphi}^e}{F_3} \left[ (c_{11} + c_{12}) \left( \frac{c_{13}}{c_{11}} \beta_1 - \beta_3 \right) T_0 - \frac{c_{12} c_{13}}{c_{11}} \beta_1 T_0 \right] - \frac{C_{\theta\theta}^e \beta_1 T_0}{2 c_{11}} \]
\[ N_3^e = \frac{1}{F_3} \left[ c_{13} \left( C_{rr}^e + C_{\theta\theta}^e \right) - (c_{11} + c_{12}) C_{zz}^e \right] \frac{P(t)}{\pi} \]  

(24)

\[ N_4^e = \left( \frac{e^{15}}{c_{44}} C_{\theta\theta}^e + C_r \right) \left( \varphi_b - \varphi_a \right) - \left( \frac{\alpha_{15}}{c_{44}} C_{\theta\theta}^e + G_r \right) \left( \psi_b - \psi_a \right) \]  

(25)

\[ N_1^p = \frac{1}{F_3^s} \left[ c_{13} \left( c_{11} \beta_1 - \beta_3 \right) T_0 - c_{13} \left( \frac{\beta_1}{2} \left( c_{12} \frac{1}{c_{11}} - 1 \right) T_0 + P(t) \right) \right] \left( C_{rr}^p + C_{\theta\theta}^p \right) \]  

(26)

\[ N_3^p = \frac{\left( C_{rr}^p - C_{\theta\theta}^p \right) \left( \frac{\beta_1}{2} \left( c_{12} \frac{1}{c_{11}} - 1 \right) T_0 + P(t) \right) + \frac{C_p}{c_{11}} \left( c_{11} + c_{12} \right) C_{\theta\theta}^p \left( \frac{c_{13}}{c_{11}} \beta_1 - \beta_3 \right) T_0 \right]}{2 c_{11}} \]  

(27)

\[ N_2^p = \frac{1}{F_3} \left[ c_{13} c_{11} \beta_1 T_0 - \left( \frac{c_{11}}{c_{11}} \beta_1 - \frac{\beta_3}{2} \right) c_{13} T_0 \right] \left( C_{rr}^p + C_{\theta\theta}^p \right) \]  

(28)

\[ N_3^p = \frac{1}{F_3^s} \left[ c_{13} \left( C_{rr}^p + C_{\theta\theta}^p \right) - (c_{11} + c_{12}) C_{zz}^e \right] \frac{P(t)}{\pi} \]  

(29)

\[ N_4^p = \left( \frac{e^{15}}{c_{44}} C_{\theta\theta}^e + C_r \right) \left( \varphi_b - \varphi_a \right) - \left( \frac{\alpha_{15}}{c_{44}} C_{\theta\theta}^e + G_r \right) \left( \psi_b - \psi_a \right) \]  

(30)

and the subscripts, \( p \) and \( e \), refer to periosteal and endosteal, respectively. Since \( U_e \) and \( U_p \) are the velocities normal to the inner and outer surfaces of the cylinders, respectively, they are calculated as

\[ U_e = -\frac{da}{dt}, \quad U_p = \frac{db}{dt} \]  

(31)

where the minus sign appearing in the expression for \( U_e \) denotes that the outward normal of the endosteal surface is in the negative coordinate direction. Thus, equations (21) can be written as

\[
\begin{align*}
\frac{da}{dt} &= N_1^e \frac{b^2}{b^2 - a^2} + N_2^e \frac{1}{\ln \left( \sqrt{b/a} \right)} + N_3^e \frac{1}{b^2 - a^2} + N_4^e \frac{1}{a \ln \left( \sqrt{b/a} \right)} - C_0^e \\
\frac{db}{dt} &= N_1^p \frac{b^2}{b^2 - a^2} + N_2^p \frac{a^2}{b^2 - a^2} + N_3^p \frac{1}{b^2 - a^2} + N_4^p \frac{1}{b \ln \left( \sqrt{b/a} \right)} + N_3^p \frac{1}{b^2 - a^2} - C_0^p \end{align*}
\]  

(32)

where \( C_0^p = C_0^p - N_3^p \).

These equations are quite similar to those presented in [15] except excluding the terms related to magnetic field. Their solution procedure is similar to that in [15] and we will omit it here.

4. Numerical examples

As numerical illustration of the proposed analytical solutions, we consider a femur with \( a_0 = 25 \) mm and \( b_0 = 35 \) mm to illustrate the proposed analytical solutions. The material properties assumed for the bone are

\[ c_{11} = 15 \text{ GPa}, \quad c_{12} = c_{13} = 6.6 \text{ GPa}, \quad c_{33} = 12 \text{ GPa}, \quad c_{44} = 4.4 \text{ GPa}, \]

\[ \beta_1 = 0.621 \times 10^5 \text{ N K}^{-1} \text{ m}^{-2}, \quad \beta_3 = 0.551 \times 10^5 \text{ N K}^{-1} \text{ m}^{-2}, \quad p_3 = 0.0133 \text{ CK}^{-1} \text{ m}^{-2}, \]
\[ e_{15} = 1.14 \text{C/m}^2, \quad \alpha_{15} = 550 \text{N/Am}, \quad \kappa_1 = 111.5 \kappa_0, \]
\[ \kappa_2 = 126 \kappa_0, \kappa_0 = 8.85 \times 10^{-12} \text{C}^2/\text{Nm}^2 \text{ = permittivity of free space} \]

The surface remodeling rate coefficients are assumed to be
\[ C_{rr}^e = -9.6 \text{m/day}, \quad C_{\theta \theta}^e = -7.2 \text{m/day}, \quad C_{zz}^e = -5.4 \text{m/day}, \]
\[ C_{rr}^p = -8.4 \text{m/day}, \quad C_{\theta \theta}^p = -12.6 \text{m/day}, \quad C_{zz}^p = -10.8 \text{m/day}, \]
\[ C_{rr}^s = -9.6 \text{m/day}, \quad C_{zz}^s = -12 \text{m/day}, \quad C_r = 10^{-9} \text{m/day}, \]
\[ G_r = 10^{-10} \text{m/day}. \]

and
\[ C_0^e = 0.0008373 \text{m/day}, \quad C_0^p = 0.00015843 \text{m/day} \]

We distinguish following five loading cases:

1. \[ p(t) = n \times 2 \text{MPa} \text{ (n=0.8, 0.9, 1.1 and 1.2)}, \ P(t) = 1500 \text{N}, \text{ and no other types of loads are applied.} \]

The extended results for this loading case are shown in Fig. 1 to study the effect of external pressure on the bone remodeling process. It displays that both the transverse and axial loads have the same effect on the bone. The inner radius of the bone decreases, while the outer one increases, as the external pressure increases. This results in an increase in the bone’s cross sectional area and, consequently, a thicker and stronger bone. And when the external pressure decreases, the inner radius increases and the outer one decreases, which means that the bone becomes thinner and weaker. On the other hand, the larger pressure can increase the velocity of bone surface remodeling, which can accelerate the recovery of the injured bone.

Fig. 1 presents an interesting change of radius against time. The outer radius of the bone increases at first as the transverse pressure increases. It begins to decrease after a few days and finally converges to a stable value that is greater than its initial one. Similar result was presented by Cowin and Van Buskirk [3]. The earlier analysis was based on a model of surface remodeling and was due to the surrounding bone material becoming more or less stiff rather than being due to surface movement.

2. \[ T_0(t) = -0.5^\circ C, -0.2^\circ C, 0^\circ C, 0.2^\circ C, 0.5^\circ C, \varphi_b - \varphi_a = 30^\circ V, \psi_b - \psi_a = 1 \text{A}, \ p(t) = 1 \text{MPa} \]
\[ P(t) = 1500 \text{N} \]
Figure 2 shows the effects of temperature change on bone surface remodeling. In general, the radii of the bone decrease when the temperature increases and they increase when the temperature decreases. It can also be seen from Fig.2 that $\varepsilon$ and $\eta$ are almost the same. Since $a_0 < b_0$, the change of the outer surface radius is normally greater than that of the inner one. The area of the bone cross section decreases as the temperature increases. This also suggests that a lower temperature is likely to induce thicker bone structures, while a warmer environment may improve the remodeling process with a less thick bone structure. This result seems to coincide with the actual fact. Thicker and stronger bones maybe make a person living in Russia looks stronger than that living in Vietnam. It should be mentioned here that how this change may affect bone remodeling process is still an open question. As an initial investigation, the purpose of this study is to show how a bone may response to thermal loads and to provide information for possible use of imposed external temperature fields in medical treatment and controlling healing process of injured bones.

(3) $\varphi_b - \varphi_a = -60V, -30V, 30V, and 60V, \ p(t) = 1MPa, \ P(t) = 1500N, \ \psi_b - \psi_a = 1A$ and $T_0 = 0$

Figure 3 shows the variation of $\varepsilon$ and $\eta$ with time t for various values of electric potential difference. It can be seen that the effect of the electric potential is just opposite to that of temperature. A decrease of the intensity of electric field results in a decrease of the inner and outer surface radii of the bone by almost the same magnitude. Theoretically, the results suggest that the remodeling process may be improved by exposing a bone to an electric field. Apparently, further theoretical and experimental studies are needed to investigate the implication of this in medical practice.

(4) $\psi_b - \psi_a = -2A, -1A, 1A, and 2A, \ p(t) = 1MPa, \ P(t) = 1500N, \ \varphi_b - \varphi_a = 30V$ and $T_0 = 0$

Figure 4 shows the variation of $\varepsilon$ and $\eta$ with time t for various values of magnetic potential difference. The change of the outer and inner surfaces of the bone due to the magnetic influence is similar to that in [15].
5. Conclusion

The problem of thermopiezomagnetoelectric bone surface remodeling has been addressed within the framework of adaptive elastic theory. The control equations for bone materials have been derived through use of the adaptive elastic theory. By assuming a homogeneous bone material, the analytical solution can provide explicit solutions for analyzing circular cylindrical bones. Then, in the numerical analysis various load conditions were considered, including transverse pressure, electric load, magnetic load and thermal load.

The numerical results show that electric field, magnetic field, and thermal load can also affect bone remodeling process in addition to mechanical loads. This feature may be considered and utilized in controlling healing process of injured bones. It should be mentioned here that all the results are obtained on the basis of the numerical model which may have difference from those of individual bone materials. Therefore further experimental validation is required before it can apply to the clinical practice.

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