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The Receding Horizon Approach to H^∞ Control of Nonlinear Systems *

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Abstract

In this paper we construct state feedback H^∞ controllers for nonlinear systems using the receding horizon technique. A method is described for the computation online of the control based on the characteristics of the underlying PDE. The viability of the approach is illustrated with an example.

Keywords. Robust control, H^∞ control, receding horizon control, nonlinear systems.

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1 Introduction

H^∞ control of nonlinear system has received much attention during the 1990's. In the case where the system state is available for control, called the state feedback case, the solution of the problem reduces to the solution of a certain Hamilton Jacobi PDE [15]. In general, however, solving this PDE is computationally a hard problem, with complexity increasing rapidly (exponentially) as the state dimension grows.

The receding horizon technique has been developed for linear time varying and nonlinear systems as a feasible way of calculating optimal controls online without having to solve a PDE at every state, see [9], [10] and [11], and the survey paper on model predictive control [12]. The optimal control problem considered in [9] is an H^2 type problem, and was shown to be stabilizing. The receding horizon approach has been in used for many years and is of considerable practical importance.

The aim of this note is to describe an application of the receding horizon control approach to the state feedback H^∞ control of nonlinear systems. The goal is to combine the virtue of the robust stability guaranteed by the H^∞ objective, and the computational feasibility of the receding horizon technique.

The paper is organized as follows. In Section 2 we state the H^∞ problem to be solved and review some basic facts. The receding horizon controller is defined in Section 3, and calculation of it is discussed in Section 4. Properties of the H^∞ receding horizon controller are given in Section 5. An example is studied in Section 6, and several points concerning the approach are discussed in Section 7.

2 State Feedback H_∞ Control

The state feedback nonlinear H^∞ control problem has been well-studied in the literature. In this section we review the problem and its solution in terms of a HJB equation. For further details, see [15], [1], [3], and the references therein.

Consider a plant model

$$G \ : \ \begin{cases} \dot{\xi} = A(\xi) + B_1(\xi)w + B_2(\xi)u \\ z = C_1(\xi) + D_{12}(\xi)u \end{cases} \tag{2.1}$$

Assumption 2.1 *We assume that all the problem data are smooth functions with bounded first derivatives, that B_1, B_2 and D_{12} are bounded, and that the origin is an equilibrium state: $A(0) = 0$ and $C_1(0) = 0$.*

A state feedback controller $u = K(x)$ is said to solve the H^∞ control problem for G provided the closed-loop system (G, K) is dissipative and internally stable. The closed-loop system is *dissipative* (with gain $\gamma > 0$) if

$$\tfrac{1}{2}\int_0^T |z(s)|^2 ds \le \tfrac{1}{2}\gamma^2 \int_0^T |w(s)|^2 ds + \beta(x_0) \tag{2.2}$$

where $\xi(0) = x_0$, for some non-negative finite function β with $\beta(0) = 0$ and every $w \in L_2[0,T]$, for all $T \geq 0$. *Internal stability* means that if $w \in L_2[0,\infty)$ then $u(\cdot), z(\cdot), \xi(\cdot) \in L_2[0,\infty)$, and consequently $\xi(t) \to 0$ as $t \to \infty$.

If there exists a static state feedback controller $u = K(x)$ such that the closed-loop system is dissipative (with gain $\gamma > 0$) there exists a storage function $V(x) \geq 0$ which satisfies $V(0) = 0$ and the Hamilton-Jacobi-Bellman-Isaacs (HJBI) equation

$$\nabla V(A - B_2 D_{12}' C_1) + \tfrac{1}{2} \nabla V(\gamma^{-2} B_1 B_1' - B_2 E_1^{-1} B_2') \nabla V' + \tfrac{1}{2} C_1'(I - D_{12} E_1^{-1} D_{12}') C_1 = 0. \quad (2.3)$$

The function $V(x)$ need not necessarily be smooth, in which case the PDE (2.3) can be interpreted in the viscosity sense, [14].

For notational convenience we write

$$\begin{aligned} H(x,\lambda) = {} & \lambda(A(x) - B_2(x) D_{12}(x)' C_1(x)) + \tfrac{1}{2}\lambda(\gamma^{-2} B_1(x) B_1(x)' - B_2(x) E_1(x)^{-1} B_2(x)')\lambda' \\ & + \tfrac{1}{2} C_1(x)'(I - D_{12}(x) E_1(x)^{-1} D_{12}(x)') C_1(x) \end{aligned} \quad (2.4)$$

for any $x \in \mathbf{R}^n$ and (row) vector $\lambda \in \mathbf{R}^n$ where $E_1 = D_{12}' D_{12} > 0$. Then (2.3) becomes

$$H(x, \nabla V(x)) = 0. \quad (2.3)'$$

Note that

$$H(x,\lambda) = \min_{u \in \mathbf{R}^m} \max_{w \in \mathbf{R}^s} \{ \lambda(A(x) + B_1(x) w + B_2(x) u) + \tfrac{1}{2} C_1(x) + D_{12}(x) u|^2 - \gamma^2 \tfrac{1}{2} |w|^2 \} \quad (2.5)$$

with optimal u given by

$$u^*(x,\lambda) \stackrel{\triangle}{=} E_1(x)^{-1}(D_{12}(x)' C_1(x) + B_2(x)' \lambda'). \quad (2.6)$$

On the other hand, if (2.3) has a *smooth* solution $V(\cdot) \geq 0$, $V(0) = 0$, then the state feedback controller

$$K_{state}^*(x) \stackrel{\triangle}{=} u^*(x, \nabla V(x)) \quad (2.7)$$

renders the closed loop (G, K_{state}^*) dissipative (with gain $\gamma > 0$), since integration yields the dissipation inequality

$$V(\xi(t)) + \tfrac{1}{2} \int_0^t |z(s)|^2 ds \leq \tfrac{1}{2} \gamma^2 \int_0^t |w(s)|^2 ds + V(\xi(0)). \quad (2.8)$$

The function β in the definition (2.2) can be taken to be V. Since the control law depends on ∇V, we assume that $V(\cdot)$ is differentiable in some sense (C^1 with globally Lipschitz derivative is enough). The stability of the closed loop follows from the dissipation inequality and detectability. By *detectability* we mean that $w, z \in L_2[0,\infty)$ implies $\xi \in L_2[0,\infty)$.

Thus to solve the nonlinear state feedback H^∞ control problem, one needs to solve the PDE (2.3). Actually, it is enough to solve the PDI

$$H(x, \nabla V(x)) \leq 0 \quad (2.9)$$

for a function $V \geq 0$, with $V(0) = 0$, since integration of (2.9) also gives the dissipation inequality (2.8).

The minimal solution V_a of (2.3) or (2.9) is called the *available storage*, and is the solution usually used, since it corresponds to the optimal minimax controller (obtained by using $V = V_a$ in (2.7)). Indeed:

$$V_a(x) = \inf_K \sup_{T \geq 0,\, w \in L_2[0,T]} \{ \tfrac{1}{2} \int_0^T [|z(s)|^2 - \gamma^2 |w(s)|^2] ds \ : \ \xi(0) = x \} \tag{2.10}$$

where $u(s) = K(\xi(s))$, and ξ, z are solutions of (2.1).

3 Receding Horizon H_∞ Control

Receding horizon control is an important and applicable technique for finding optimal state feedback controllers *online* without having to solve dynamic programming PDEs for all states. The technique is shown to give a stable closed loop in [9]; see also [10], [11]. In this section we adapt this technique to the nonlinear state feedback H^∞ control problem. For similar results for linear systems, see [7].

The receding horizon controller is constructed from a finite time horizon optimal control problem which we now describe. Let Φ be a smooth solution of the PDI (2.9) satisfying $\Phi(x) \geq 0$, $\Phi(0) = 0$. Let $T > 0$ be a fixed time *horizon*. Define a finite horizon optimal control problem on $[0, T]$:

$$V^{\Phi,T}(x, 0) = \inf_{u \in PC[0,T]} \sup_{w \in L_2[0,T]} \{ \tfrac{1}{2} \int_0^T [|z(s)|^2 - \gamma^2 |w(s)|^2] ds + \Phi(\xi(T)) \ : \ \xi(0) = x \} \tag{3.1}$$

where the control $u(\cdot)$ is piecewise continuous and square integrable, and ξ, z are solutions of (2.1) on $[0, T]$ with initial condition $\xi(0) = x$. Let

$$s \mapsto u^{*\Phi,T}_{x,0}(s), \quad 0 \leq s \leq T, \tag{3.2}$$

denote the optimal control. The optimal control value at the initial time $s = 0$ is then $u^{*\Phi,T}_{x,0}(0)$.

The receding horizon controller is defined as follows: when in state x, apply the control value $u^{*\Phi,T}_{x,0}(0)$. To elaborate, suppose now that $\xi(\cdot)$ is the trajectory of (2.1) on $[0, \infty)$ with initial condition $\xi(0) = x_0$ corresponding to the receding horizon controller. At time $t \geq 0$ the current state is $x = \xi(t)$ and consider the time horizon $[t, t + T]$ and the above finite time horizon optimal control problem shifted to this interval. By time invariance, the optimal control for this time-shifted problem on $[t, t + T]$ is

$$s \mapsto u^{*\Phi,T}_{x,0}(s - t), \quad t \leq s \leq t + T. \tag{3.3}$$

This control is not applied, but rather the receding horizon control policy is to apply at time t the "initial" control value corresponding to $s = t$:

$$K_{rh}(x) \stackrel{\triangle}{=} u^{*\Phi,T}_{x,0}(0) \tag{3.4}$$

where $x = \xi(t)$ is the current state. To be specific, the control at time $t \geq 0$ is

$$u(t) = K_{rh}(\xi(t)) \stackrel{\triangle}{=} u^{*\Phi,T}_{\xi(t),0}(0) \tag{3.5}$$

This process is repeated at each time t, so we are repeatedly applying the initial optimal control value for a sequence of finite horizon control problems, indexed by the current state $x = \xi(t)$. Associated with the controller $K_{rh}(x)$ is a function

$$V_{rh}(x) = V^{\Phi,T}(x, 0). \tag{3.6}$$

4 Calculating the Receding Horizon Controller

We turn now to the issue of determining the control values $u^{*\Phi,T}_{x,0}(0)$. By the invariance principle of optimal control, define the value function for the finite time horizon problem (for $t \in [0, T]$) by

$$V^{\Phi,T}(x,t) = \inf_{u \in PC[t,T]} \sup_{w \in L_2[t,T]} \{ \tfrac{1}{2} \int_t^T [|z(s)|^2 - \gamma^2 |w(s)|^2] ds + \Phi(\xi(T)) \ : \ \xi(t) = x \} \tag{4.1}$$

where the control $u(\cdot)$ is piecewise continuous and square integrable, and ξ, z are solutions of (2.1) on $[t, T]$ with initial condition $\xi(t) = x$. By dynamic programming, $V^{\Phi,T}$ is the unique solution of the PDE problem:

$$\begin{aligned} \tfrac{\partial V}{\partial t}(x,t) + H(x, \nabla V(x,t)) &= 0 \\ V(x,T) &= \Phi(x) \end{aligned} \tag{4.2}$$

If $V^{\Phi,T}$ is smooth, the optimal control for the starting point $(x,t) \in \mathbf{R}^n \times [0,T]$ is given by

$$u^{*\Phi,T}_{x,t}(s) = u^*(\xi(s), \nabla V^{\Phi,T}(\xi(s), s)) \tag{4.3}$$

for $s \in [t, T]$, where $u^*(x, \lambda)$ is defined by (2.6), and ξ is the corresponding solution of (2.1) with $\xi(t) = x$. Then we have

$$\begin{aligned} K_{rh}(x) &\stackrel{\triangle}{=} u^{*\Phi,T}_{x,0}(0) \\ &= u^*(x, \nabla V^{\Phi,T}(x,0)) \\ &= u^*(x, \nabla V_{rh}(x)) \end{aligned} \tag{4.4}$$

Thus the receding horizon controller can be found from the solution to the PDE (4.2). The receding horizon methodology seeks to avoid this, and only computes the needed values as the system evolves online. This can be done using the characteristic equations for (4.2):

$$\begin{aligned} \dot{\xi}(s) &= \nabla_\lambda H(\xi(s), \lambda(s)), \quad \xi(0) = x, \\ \dot{\lambda}(s) &= -\nabla_x H(\xi(s), \lambda(s)), \ \lambda(T) = \nabla_x \Phi(\xi(T)). \end{aligned} \tag{4.5}$$

Thus

$$K_{rh}(x) = u^*(x, \lambda(0)), \tag{4.6}$$

so the online implementation requires repeated solution of the two point boundary value problem (4.5) as the system evolves.

5 Properties of the Receding Horizon Controller

Before establishing properties of the receding horizon controller, we collect together some results concerning the finite horizon problem.

For $t_1 \le t_2$, define the operator S_{t_1,t_2} by

$$(S_{t_1,t_2}\Phi)(x) = \inf_{u \in PC[t_1,t_2]} \sup_{w \in L_2[t_1,t_2]} \{ \tfrac{1}{2} \int_{t_1}^{t_2} [|z(s)|^2 - \gamma^2 |w(s)|^2] ds + \Phi(\xi(t_2)) \ : \ \xi(t_1) = x \} \tag{5.1}$$

where the control $u(\cdot)$ is piecewise continuous and square integrable, and ξ, z are solutions of (2.1) on $[t_1, t_2]$ with initial condition $\xi(t_1) = x$.

Assumption 5.1 *Assume $\Phi \ge 0$ is finite, smooth, satisfies $\Phi(0) = 0$, and solves*

$$H(x, \nabla \Phi(x)) \le 0. \tag{5.2}$$

Lemma 5.2 *Make Assumption 2.1. Then:*

(i) If $\Phi_1 \le \Phi_2$, then

$$S_{t_1,t_2}\Phi_1 \le S_{t_1,t_2}\Phi_2 \tag{5.3}$$

(ii) For any τ,

$$S_{t_1,t_2}\Phi = S_{t_1+\tau,t_2+\tau}\Phi \tag{5.4}$$

If in addition Assumption 5.1 holds, we have

(iii) *For all $0 \le t \le T$,*

$$V^{\Phi,T}(x,t) \le \Phi(x) \tag{5.5}$$

(iv) *For* $0 \le t_1 \le t_2 \le T$,

$$V^{\Phi,T}(x,t_1) \le V^{\Phi,T}(x,t_2) \tag{5.6}$$

v) *For all* $0 \le t \le T$,

$$V^{\Phi,T}(x,t) \ge \inf_{u\in PC[t,T]} \sup_{w\in L_2[t,T]} \{\tfrac{1}{2}\int_t^T [|z(s)|^2 - \gamma^2|w(s)|^2]ds + V^{\Phi,T}(\xi(T),t) \ : \ \xi(t)=x\} \tag{5.7}$$

where the control $u(\cdot)$ *is piecewise continuous and square integrable, and* ξ, z *are solutions of (2.1) on* $[t,T]$ *with initial condition* $\xi(t)=x$.

Proof. Item (i) follows immediately from the definition (5.1), and item (ii) is a consequence of time invariance.

By Assumption 5.1, upon integration we have

$$\Phi(x) \ge \inf_K \sup_{w\in L_2[t,T]} \{\tfrac{1}{2}\int_t^T [|z(s)|^2 - \gamma^2|w(s)|^2]ds + \Phi(\xi(T)) \ : \ \xi(t)=x\} \tag{5.8}$$

where $u(s)=K(\xi(s))$, and ξ, z are solutions of (2.1). But the RHS of (5.8) is bounded below by $V^{\Phi,T}(x,t)$; hence item (iii).

Next, write $\tau = t_2 - t_1$. Then by dynamic programming and the above items,

$$\begin{aligned} V^{\Phi,T}(x,t_1) &= S_{t_1,T-\tau}V^{\Phi,T}(\cdot,T-\tau)(x) \\ &\le S_{t_1,T-\tau}\Phi(x) \\ &= S_{t_2,T}\Phi(x) \\ &= V^{\Phi,T}(x,t_2) \end{aligned} \tag{5.9}$$

This proves item (iv).

To prove item (v), set $t_1 = t$, $t_2 = t_1+\tau$, where $\tau > 0$. Then by dynamic programming and item (iv)

$$\begin{aligned} V^{\Phi,T}(x,t) &= S_{t,t+\tau}V^{\Phi,T}(\cdot,t+\tau)(x) \\ &\ge S_{t,t+\tau}V^{\Phi,T}(\cdot,t)(x) \end{aligned} \tag{5.10}$$

which is just (5.7). □

Assumption 5.3 *Assume* $V^{\Phi,T}(x,t)$ *is smooth for* $(x,t) \in \mathbf{R}^n \times [0,T]$, *and solves the PDE (4.2) (uniquely) when* Φ *is smooth.*

Theorem 5.4 *Make Assumptions 2.1, 5.1 and 5.3. Let* K_{rh} *be the receding horizon controller as specified by (3.4) above, and let* V_{rh} *be given by (3.6). Then the closed loop system* (G, K_{rh}) *is dissipative, with storage function* V_{rh}. *Further, if* (G, K_{rh}) *is detectable, then* (G, K_{rh}) *is internally stable.*

PROOF. By Lemma 5.2, item (iv), (5.6), we have

$$\frac{\partial V^{\Phi,T}}{\partial t}(x,0) \geq 0,$$

and hence by definition of V_{rh} we have

$$H(x, \nabla V_{rh}(x)) \leq 0 \tag{5.11}$$

Now by (2.5), (2.6) and (4.4), the PDI (5.11) gives

$$\sup_{w \in \mathbf{R}^s} \{\nabla V(A + B_2 K_{rh}) + \tfrac{1}{2}|C_1 + D_{12} K_{rh}|^2 - \gamma^2 \tfrac{1}{2}|w|^2\} \leq 0. \tag{5.12}$$

Integration of (5.12) gives the dissipation inequality (2.8) for $V = V_{rh}$, $u = K_{rh}(\xi)$. □

6 Example

In this section we shall show the viability of the approach for control proposed above. Consider the following system

$$\begin{aligned} \dot{x}(t) &= (1 + 0.25\arctan(|x|^2))Ax(t) + B_1 w(t) + B_2 u(t), \\ z(t) &= C_1 x(t) + D_{12} u(t), \end{aligned} \tag{6.1}$$

with

$$A = \begin{pmatrix} 0.98 & 0.96 \\ 0.18 & 1.16 \end{pmatrix}, \quad B_1 = \begin{pmatrix} 0.87 & 0.28 & 0.46 \\ 0.34 & 0.37 & 0.67 \end{pmatrix}, \quad B_2 = \begin{pmatrix} 0.79 & 0.22 \\ 0.92 & 0.84 \end{pmatrix},$$

$$C_1 = \begin{pmatrix} 0.98 & 0.20 \\ 0.98 & 0.51 \\ 0.16 & 0.40 \end{pmatrix}, \quad D_{12} = \begin{pmatrix} 0.93 & 0.90 \\ 0.45 & 0.30 \\ 0.12 & 0.70 \end{pmatrix}.$$

For the simulations the following perturbation was chosen:

$$w(t) = \sin(2t)/(3 + t^3); \tag{6.2}$$

its graph is shown in Figure 6.1. Further, we have chosen the horizon $T = 1$.

An important point is how to select or compute the function $\Phi(\cdot)$ (Assumption 5.1). In this example we use an approximation of it, which may be useful in general, namely,

$$\Phi(x) = \tfrac{1}{2} x' X_{inf} x, \tag{6.3}$$

where the matrix

$$X_{inf} = \begin{pmatrix} 3.3053 & -2.0712 \\ -2.0712 & 3.4639 \end{pmatrix} \geq 0$$

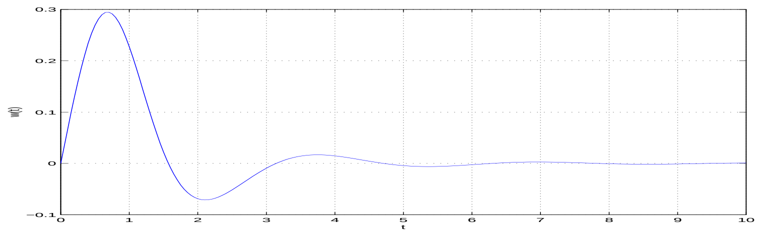

Figure 6.1: Graph of perturbation $w(t)$.

is the solution, in this case with $\gamma = 2.871$, of the Riccati equation:

$$A_{lsf}{}' X_{inf} + X_{inf} A_{lsf} + X_{inf}' B_{lsf} X_{inf} + C_{lsf} = 0,$$

where A_{lsf}, B_{lsf} and C_{lsf} are matrices given by

$$\begin{aligned} A_{lsf} &:= A - B_2 E_1^{-1} D_{12}' C_1, \\ B_{lsf} &:= \gamma^{-2} B_1 B_1' - B_2 E_1^{-1} B_2', \\ C_{lsf} &:= C_1' (I - D_{12} E_1^{-1} D_{12}') C_1. \end{aligned}$$

The heuristic behind this choice is that (6.3) approximates a true storage function in a neighborhood of the origin [15, Section 7.4] (linearization). The matrix X_{inf} can be easily computed with the help of modern control software, for example [5].

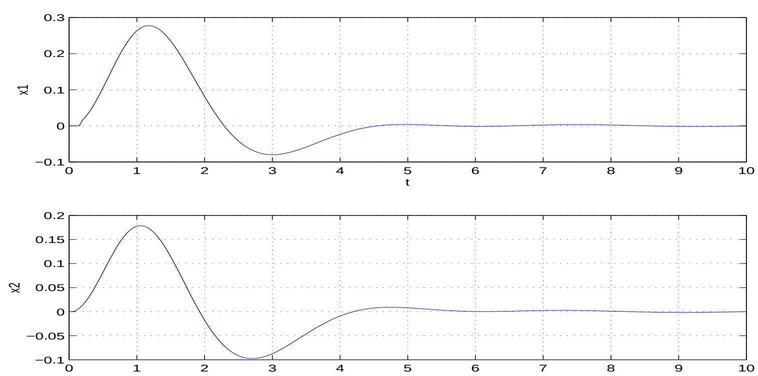

Figure 6.2: Closed loop state $\xi(\cdot)$ trajectory.

Figure 6.2 represents the trajectories of the closed loop (G, K_{rh}) with w given by (6.2). We have used the package *mumus* [4] in the computations associated to (4.5). We see that the approach is feasible and works efficiently. The initial approximations needed in *mumus* were obtained using homotopy. Its convergence was always fast and reliable.

7 Discussion

In this note we make no attempt to weaken the smoothness assumptions for the functions Φ and $V^{\Phi,T}$. For work on relaxing such assumptions in related contexts, see [2] and [10].

The approach we have described relies on the calculation or approximation of the function Φ (a similar assumption is made in the linear case [7]). This is bothersome for several reasons. If such a Φ has been found, then a suitable controller can be computed from it, as described in Section 2. However, the receding horizon construction results in an improvement, since it yields a storage function of lower minimax cost; indeed, $V_a \leq V_{rh} \leq \Phi$, and in fact the value of V_{rh} is monotonically decreasing with respect to the horizon length T. (The value V_a is the optimal minimax cost, and is approached as $T \to \infty$, see (2.10).) Another point is that the calculation or approximation of the function Φ may not be possible, and indeed we seek to avoid completely the solution of a PDE or PDI at each state. In general, it seems that an approximation, such as the one resulting from linearization in Section 6, will suffice. This works well in the examples we have tried to date, and merits further investigation and analysis.

We remark that the receding horizon technique described in [9] does not use a finite function Φ as an end point cost, but instead uses a constraint that the final state be the origin (or a target region [11]). Use of such a constraint does not appear straightforward in the H^∞ context.

We have also experimented with this receding horizon approach for measurement feedback problems under the certainty equivalence conditions of [1]. The results have been encouraging and will be pursued further.

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