Quantum Networks: Modelling, Analysis, and Control

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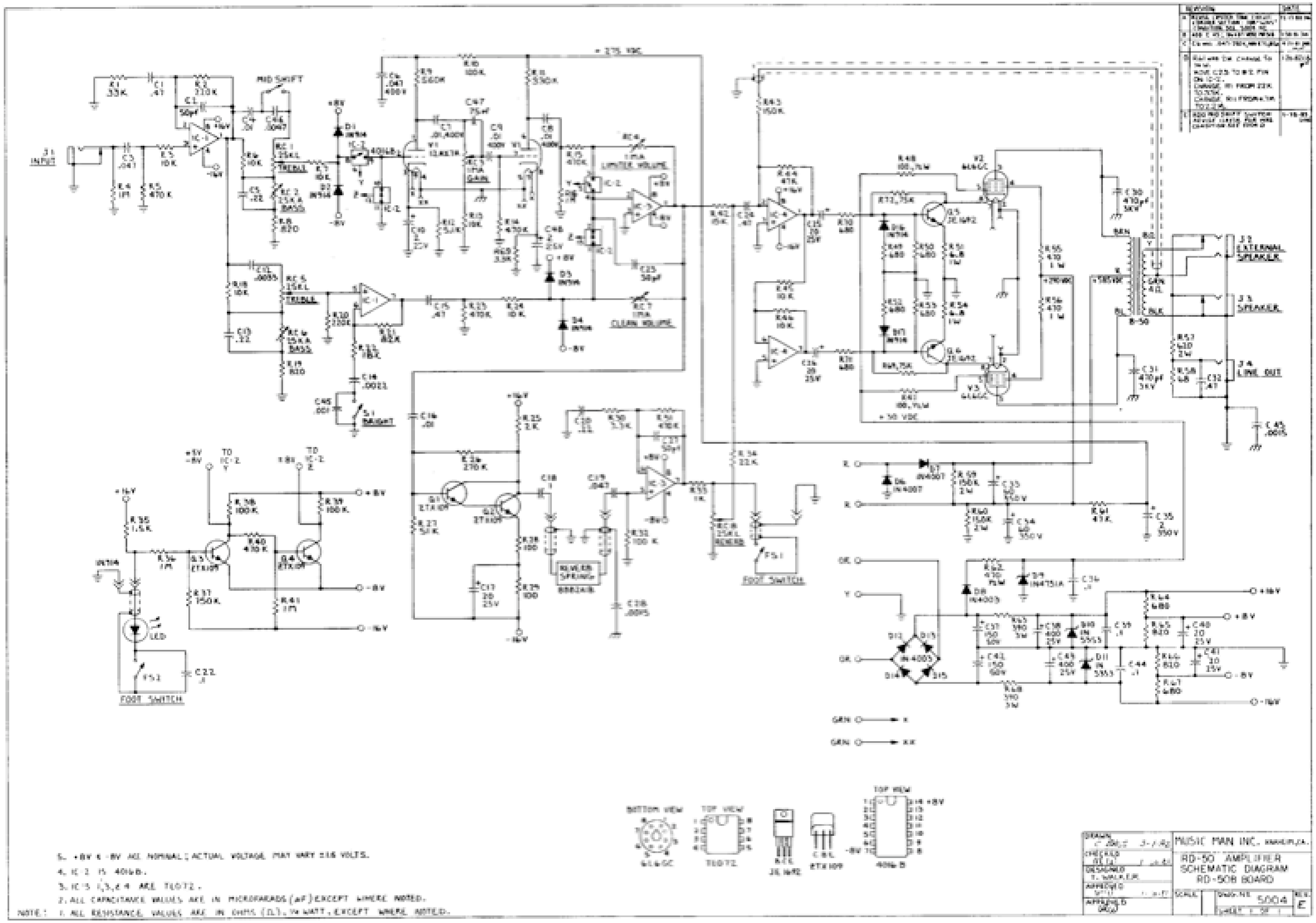
Outline

- Wiring things up
- Quantum Dissipative Systems
- Feedback Control by Interconnection

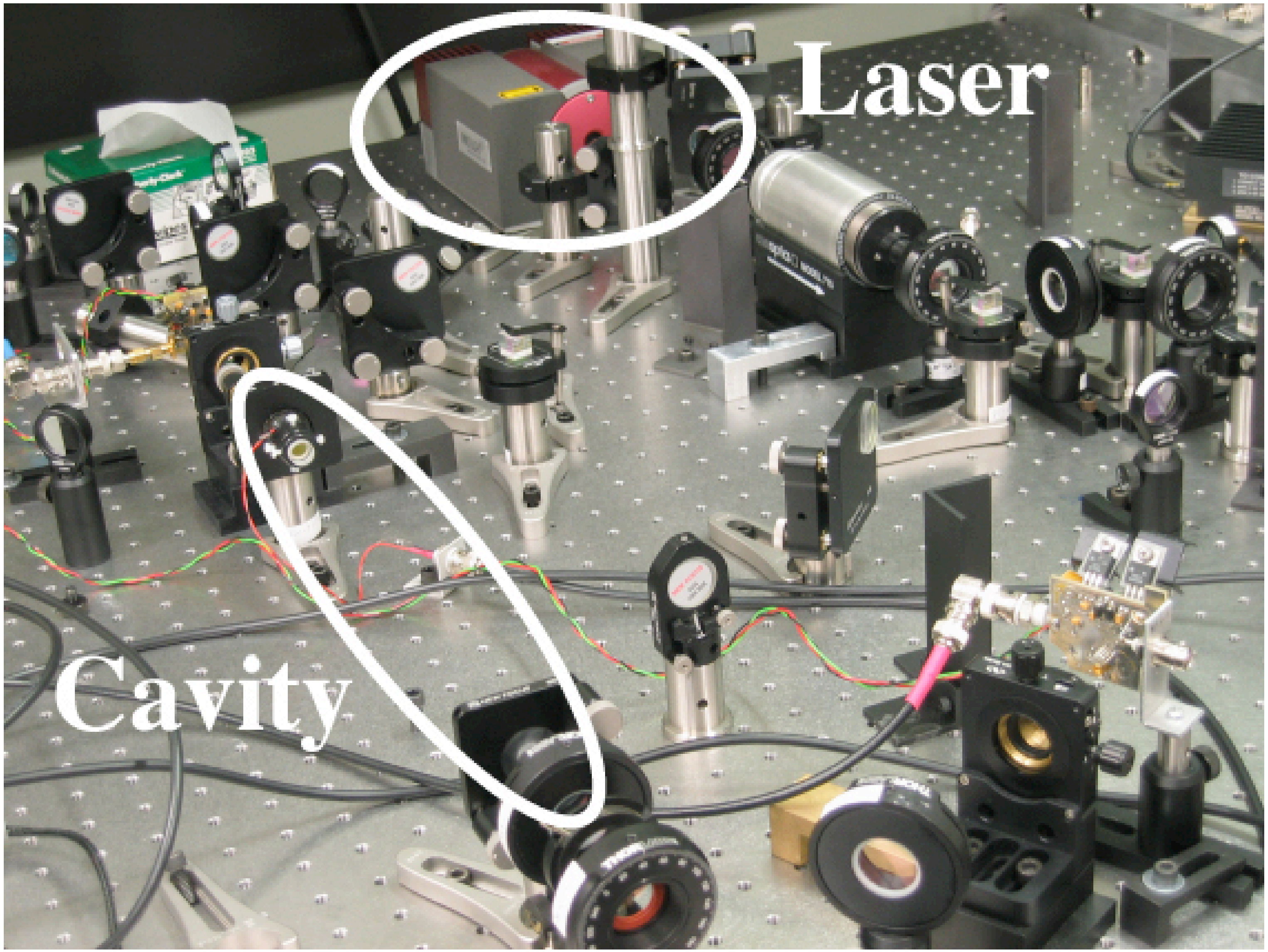
References:

- J. Gough and M.R. James, *The Series Product and Its Application to Quantum Feedforward and Feedback Networks*, quant-ph/0707.0048, submitted to IEEE TAC.
- M.R. James and J. Gough, *Quantum Dissipative Systems and Feedback Control by Interconnection*, quant-ph/0707.1074, submitted to IEEE TAC.
- M.R. James, H.I. Nurdin and I.R. Petersen, *H-Infinity Control of Linear Quantum Stochastic Systems*, quant-ph/0703150, accepted, IEEE TAC.

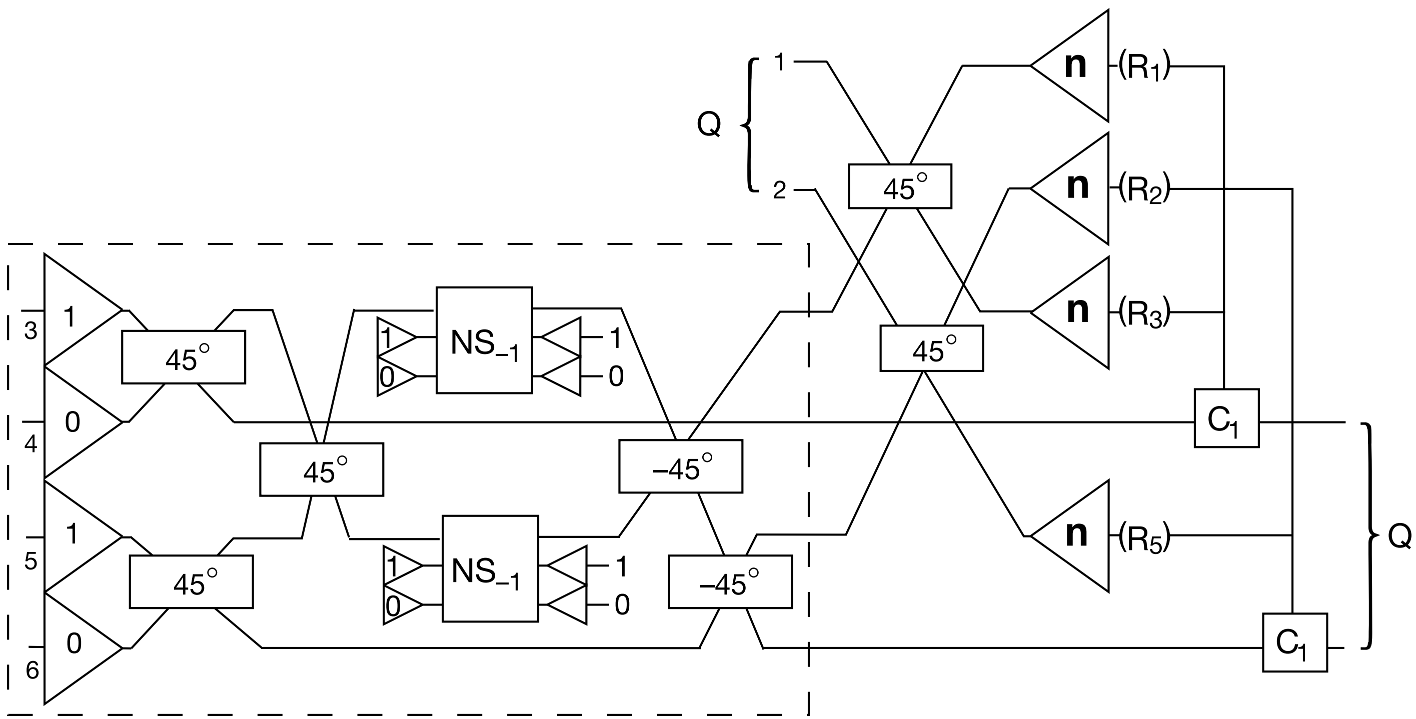
Wiring Things Up



[circuit diagram of a classical electronic amplifier]



[quantum optics lab - E. Huntington, ADFA/UNSW]



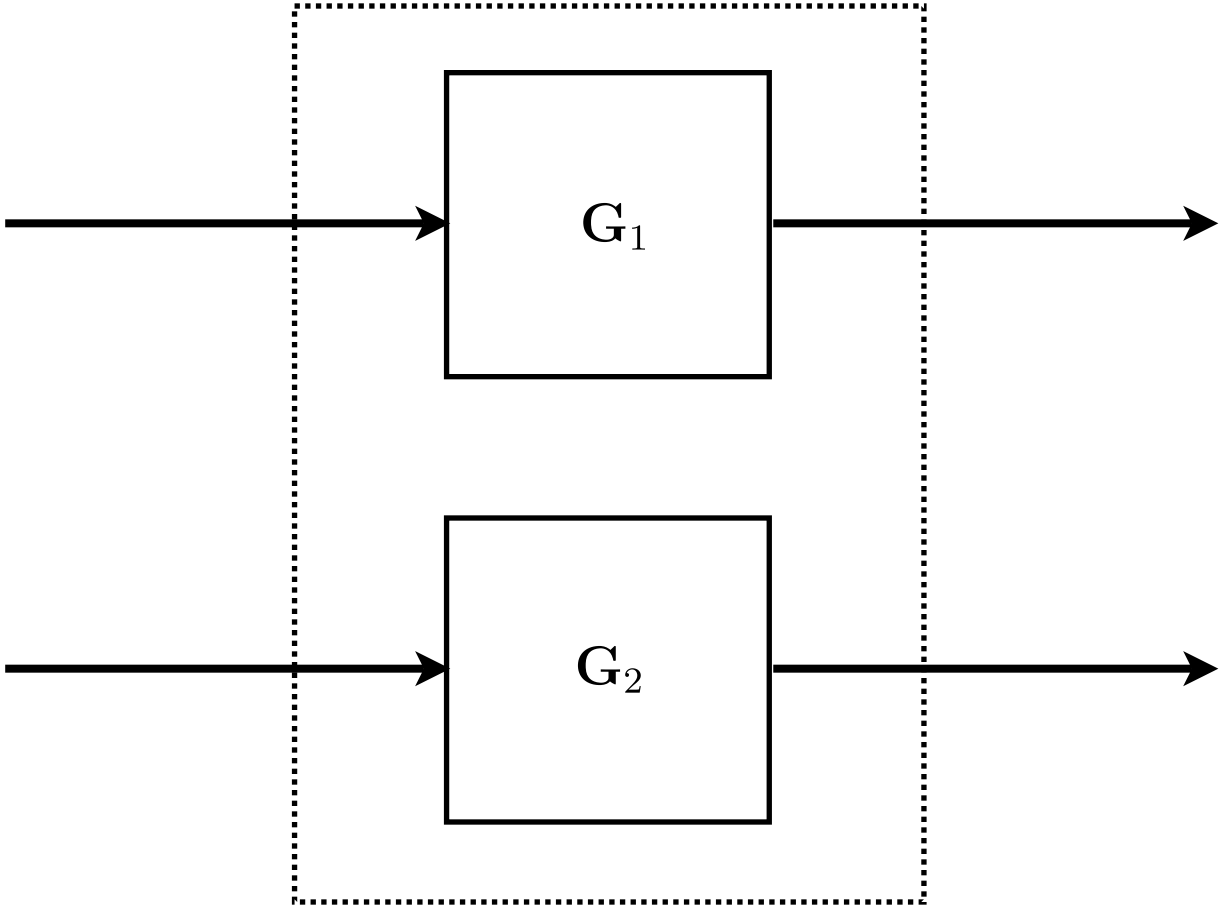
[quantum computing network - (teleportation with loss detection) - Knill, Laflamme, Milburn, 2001]

Quantum network models - desirable attributes

- Capture the quantum physics
- Be capable of representing classical components
- Include dissipative mechanisms
  - noise, uncertainty, decoherence
- Preserve canonical structure
  - e.g. commutation relations, energy
- Network of interconnected components should also be a quantum system
  - recursive
- Efficient methods for representation, interconnection, manipulation, and physical realization
- Efficient methods for analysis and synthesis

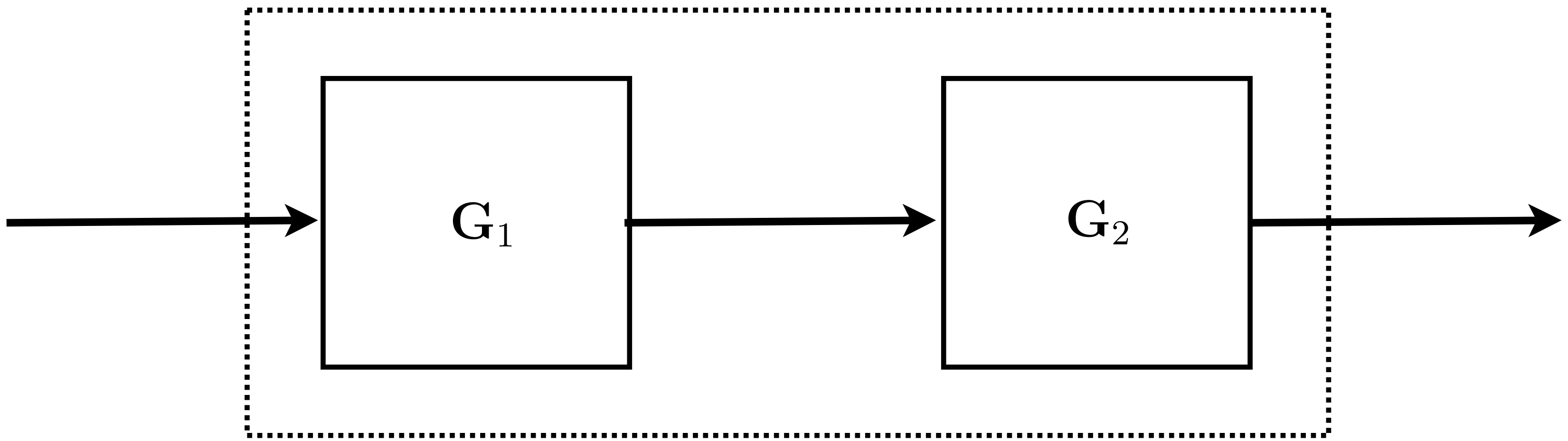
Elementary network constructs:

Concatenation



$$G = G_1 \boxplus G_2$$

Series (cascade)



$$\mathbf{G} = \mathbf{G}_2 \triangleleft \mathbf{G}_1$$

History:
Gardiner, 1993
Carmichael, 1993

Mathematical definitions:

Multichannel open quantum system:

$$\mathbf{G} = \begin{pmatrix} -\frac{1}{2}\mathbf{L}^{\dagger}\mathbf{L} - iH & -\mathbf{L}^{\dagger}\mathbf{S} \\ \mathbf{L} & \mathbf{S} - \mathbf{I} \end{pmatrix}$$

where

- H is a Hamiltonian (self-adjoint operator)
- $\mathbf{L}$ is a vector of field coupling operators
- $\mathbf{S}$ is a scattering matrix (self-adjoint matrix of operators)

Shorthand:

$$\mathbf{G} = (\mathbf{S}, \mathbf{L}, H)$$

Concatenation product

$$\mathbf{G}_1 \boxplus \mathbf{G}_2 = \left(\begin{pmatrix} \mathbf{S}_1 & 0 \\ 0 & \mathbf{S}_2 \end{pmatrix}, \begin{pmatrix} \mathbf{L}_1 \\ \mathbf{L}_1 \end{pmatrix}, H_1 + H_2 \right)$$

Series product

$$\mathbf{G}_2 \triangleleft \mathbf{G}_1 = \mathbf{G}_1 + \mathbf{G}_2 + \mathbf{G}_2 \mathbf{\Pi} \mathbf{G}_1$$

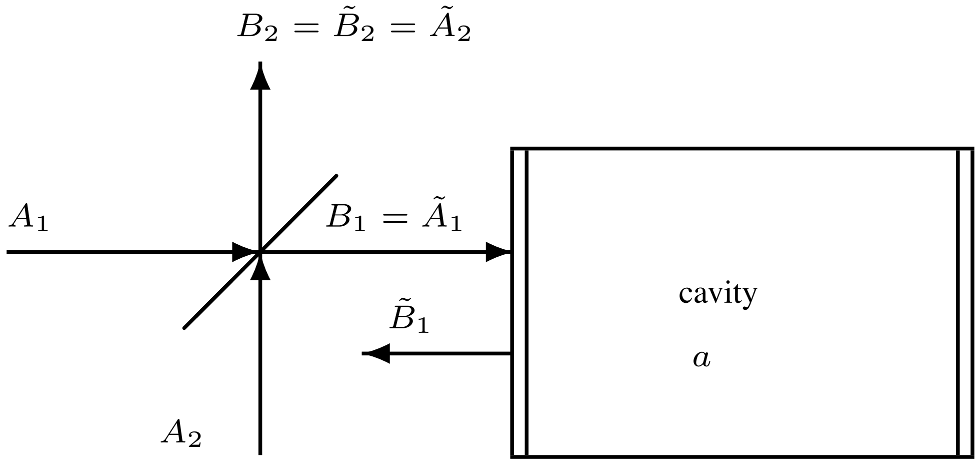
where

$$\mathbf{\Pi} := \left(\begin{array}{c|c} 0 & 0 \\ \hline 0 & \mathbf{I} \end{array} \right),$$

or using the shorthand,

$$\mathbf{G}_2 \triangleleft \mathbf{G}_1 = (\mathbf{S}_2\mathbf{S}_1, \quad \mathbf{L}_2 + \mathbf{S}_2\mathbf{L}_1, \quad H_1 + H_2 + \frac{1}{2i}(\mathbf{L}_2^\dagger \mathbf{S}_2 \mathbf{L}_1 - \mathbf{L}_1^\dagger \mathbf{S}_2^\dagger \mathbf{L}_2))$$

Example: beamsplitter and cavity



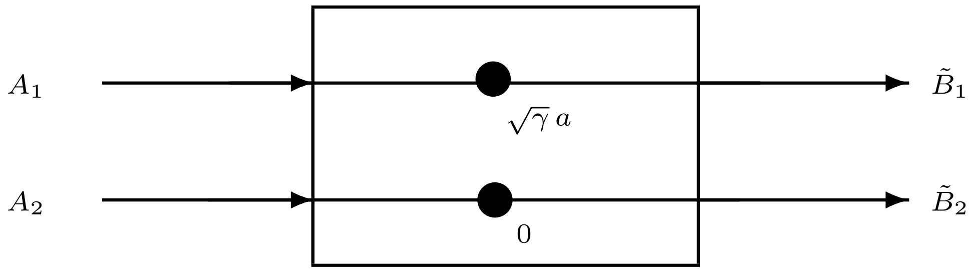
$$
\begin{aligned}
da(t) &= (-\frac{\gamma}{2} + i\Delta)a(t)dt - \sqrt{\gamma}\, dB_1(t) \\
\tilde{A}_1(t) &= \beta A_1(t) - \alpha A_2(t) \\
\tilde{A}_2(t) &= \alpha A_1(t) + \beta A_2(t) \\
B_1(t) &= \tilde{A}_1(t) \\
B_2(t) &= \tilde{A}_2(t) \\
d\tilde{B}_1(t) &= \sqrt{\gamma} a(t)dt + dB_1(t) \\
d\tilde{B}_2(t) &= dB_2(t).
\end{aligned}
$$

Complete network

$$\mathbf{G} = \left(\begin{array}{r|c} -\frac{1}{2}\mathbf{L}^\dagger\mathbf{L} - iH & -\mathbf{L}^\dagger\mathbf{S} \\ \hline \mathbf{L} & \mathbf{S} - \mathbf{I} \end{array}\right) = \left(\begin{array}{c|cc} -\frac{\gamma}{2}a^*a - i\Delta a^*a & -\sqrt{\gamma}\,\beta a^* & \sqrt{\gamma}\,\alpha a^* \\ \hline \sqrt{\gamma}\,a & \beta - 1 & -\alpha \\ 0 & \alpha & \beta - 1 \end{array}\right)$$

or using the shorthand

$$\mathbf{G} = (\mathbf{S}, \mathbf{L}, H) = \left(\begin{pmatrix} \beta & -\alpha \\ \alpha & \beta \end{pmatrix}, \begin{pmatrix} \sqrt{\gamma}\,a \\ 0 \end{pmatrix}, \Delta a^* a \right)$$



Description in terms of concatenation and series products:

$$\mathbf{G} = (\mathbf{C} \boxplus \mathbf{N}) \triangleleft \mathbf{B},$$

where $\mathbf{C}$ is a matrix of cavity parameters

$$\mathbf{C} = \left(\begin{array}{c|c} -\frac{\gamma}{2}a^*a - i\Delta a^*a & -\sqrt{\gamma}\,a^* \\ \hline \sqrt{\gamma}\,a & 0 \end{array}\right),$$

$$\mathbf{N} = \left(\begin{array}{c|c} 0 & 0 \\ \hline 0 & 0 \end{array}\right),$$

is a trivial system (pass-through), and

$$\mathbf{B} = \left(\begin{array}{c|cc} 0 & 0 & 0 \\ \hline 0 & \beta - 1 & -\alpha \\ 0 & \alpha & \beta - 1 \end{array}\right),$$

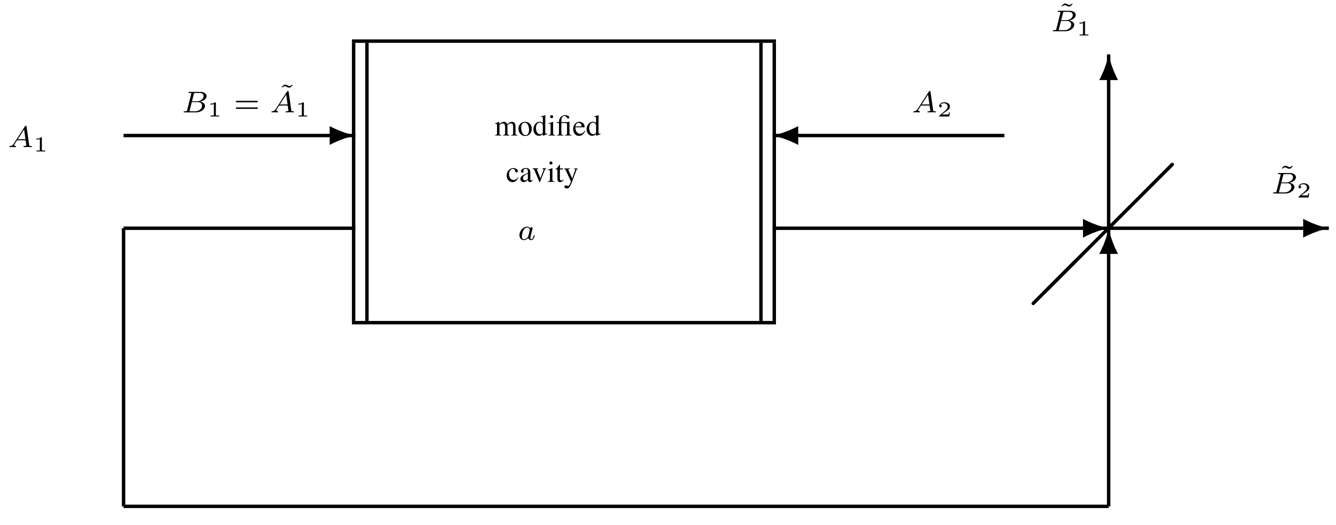
is a representation of the beamsplitter $\mathbf{S}$.

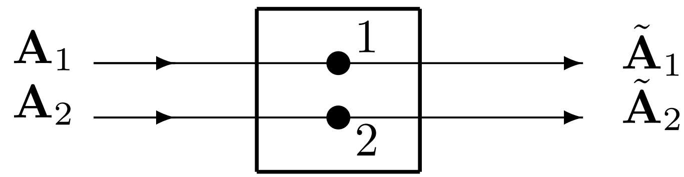
Network manipulations (try to pull beamsplitter through):

$$\mathbf{G} = (\mathbf{C} \boxplus \mathbf{N}) \triangleleft \mathbf{B} = \mathbf{B} \triangleleft (\mathbf{C}' \boxplus \mathbf{N}').$$

Here, the modified cavity is described by the subsystems

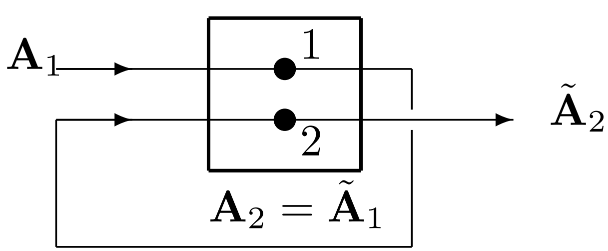
$$\mathbf{C}' = \left(\begin{array}{c|c} -\frac{\gamma}{2}|\beta|^2 a^* a - i\Delta a^* a & -\beta\sqrt{\gamma}\, a^* \\ \hline \beta^* \sqrt{\gamma}\, a & 0 \end{array} \right), \quad \mathbf{N}' = \left(\begin{array}{c|c} -\frac{\gamma}{2}|\alpha|^2 a^* a & \alpha\sqrt{\gamma}\, a^* \\ \hline -\alpha^* \sqrt{\gamma}\, a & 0 \end{array} \right).$$

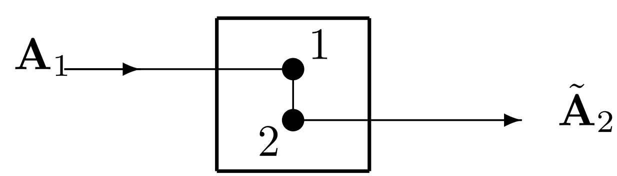




Theorem

(Principle of Series Connections) *The generator* $\mathbf{G}_{2\leftarrow 1}$ *for the feedback system obtained from* $\mathbf{G}_1 \boxplus \mathbf{G}_2$ *when the output of the first subcomponent is fed into the input of the second is the series product* $\mathbf{G}_{2\leftarrow 1} = \mathbf{G}_2 \lhd \mathbf{G}_1$.





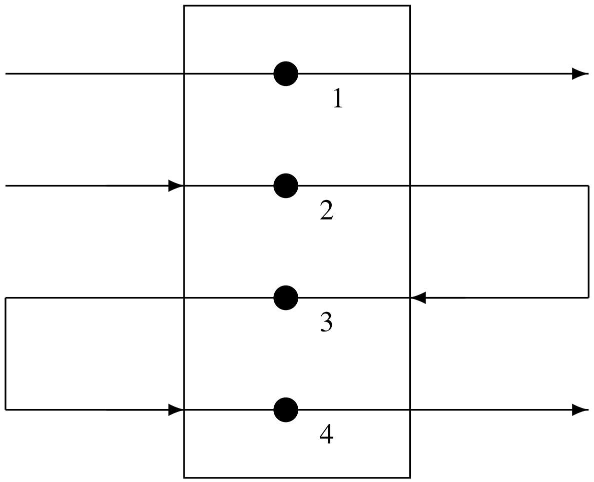
Reducible Networks

A *reducible quantum network* $\mathscr{N} = (\{\mathbf{G}_j\}, K, \{\mathbf{G}_j \triangleleft \mathbf{G}_k\})$ consists of

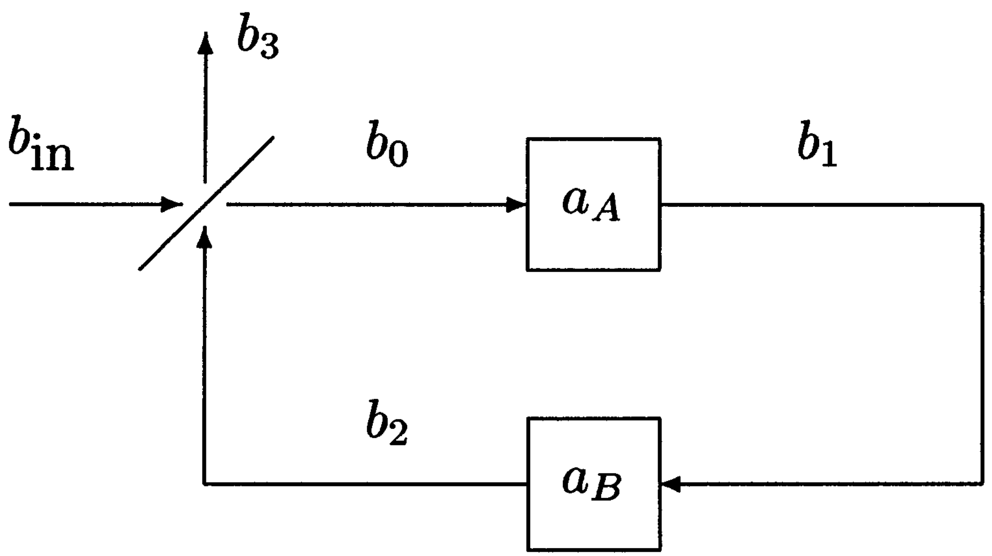
- A reducible decomposition $\mathbf{G} = \boxplus_j \mathbf{G}_j$, where $\mathbf{S} = \text{diag}\{\mathbf{S}_1, \ldots, \mathbf{S}_n\}$,
- a direct interaction Hamiltonian K of the form

$$K = i \sum_k (N_k^* M_k - M_k^* N_k)$$

- a compatible list of field-mediated connections $\mathscr{L} = \{\mathbf{G}_j \triangleleft \mathbf{G}_k\}$ such that (i) the field dimensions of the members of each pair are the same, and (ii) each input and each output has at most one connection.



An example of a network that is *not reducible*

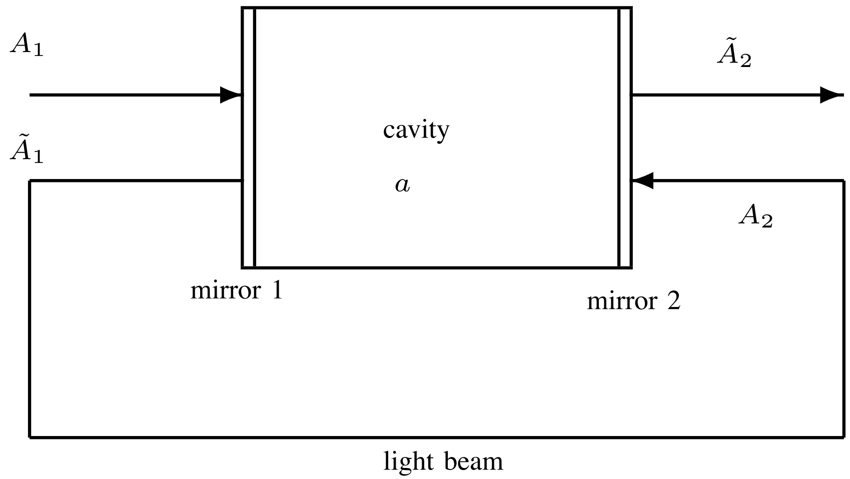


[Yanagisawa-Kimura, 2003]

Examples

All-optical feedback

[Wiseman-Milburn, 1994]



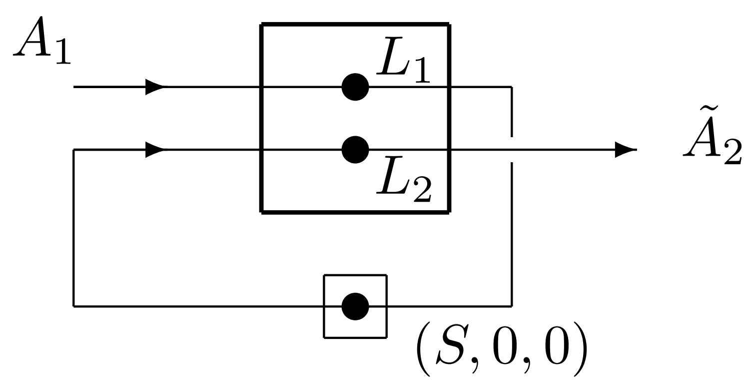
Before feedback, the cavity is described by

$$\mathbf{G} = (\mathbf{I}, \begin{pmatrix} L_1 \\ L_2 \end{pmatrix}, 0) = (1, L_1, 0) \boxplus (1, L_2, 0),$$

and $S = e^{i\theta}$ (phase shift).

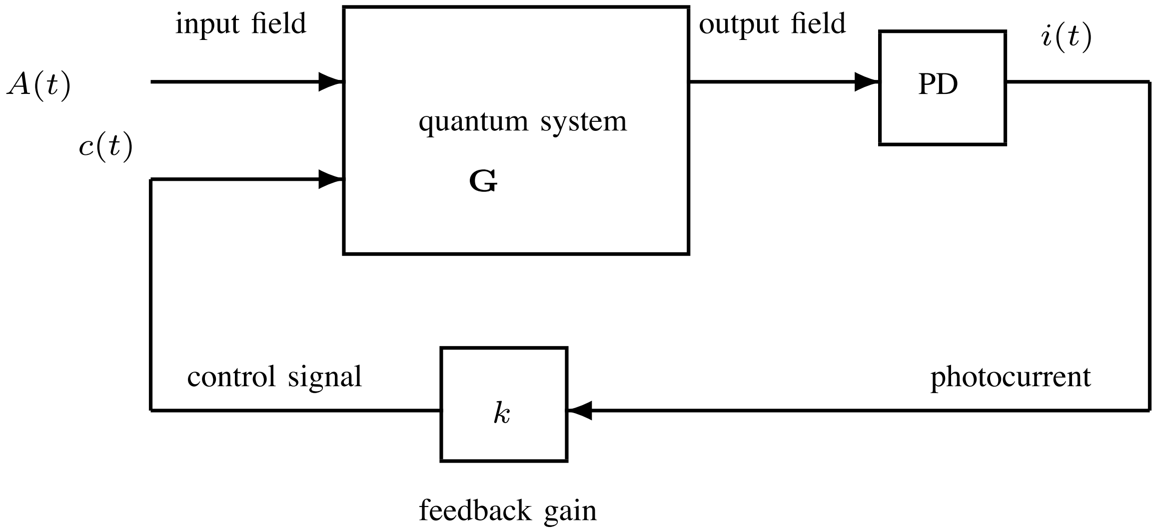
After feedback, we have

$$
\begin{aligned}
\mathbf{G}_{cl} &= (1, L_2, 0) \triangleleft (S, 0, 0) \triangleleft (1, L_1, 0) \\
&= (S, SL_1 + L_2, \frac{1}{2i}(L_2^* S L_1 - L_1^* S^* L_2)).
\end{aligned}
$$



Direct measurement feedback

[Wiseman, 1994]



Controlled Hamiltonian

$$H_0 + Fc$$

Before feedback, the quantum system is described by

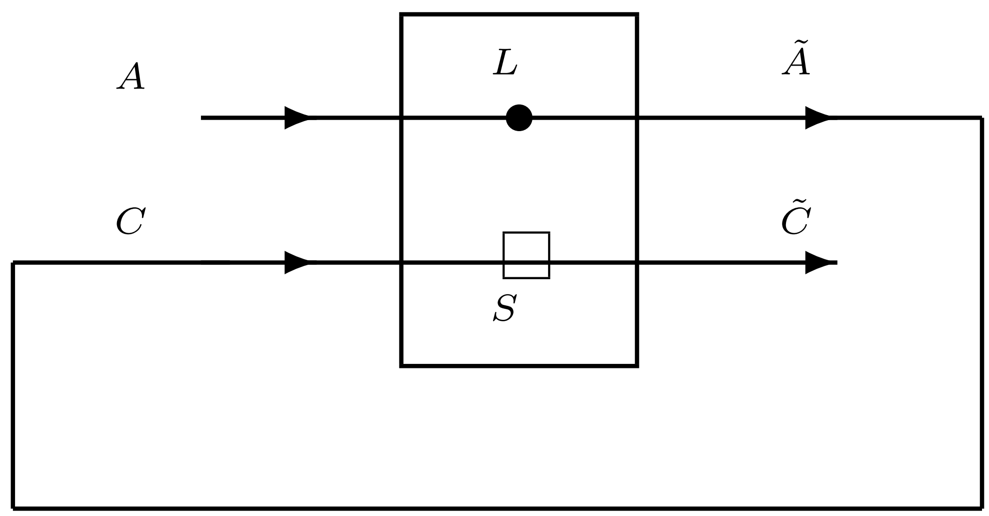
$$\mathbf{G} = (1, L, H_0) \boxplus (S, 0, 0)$$

where $S = e^{-iF}$ is unitary.

After feedback, we have

$$\mathbf{G}_{cl} = (S, 0, 0) \triangleleft (1, L, H_0) = (S, SL, H_0)$$

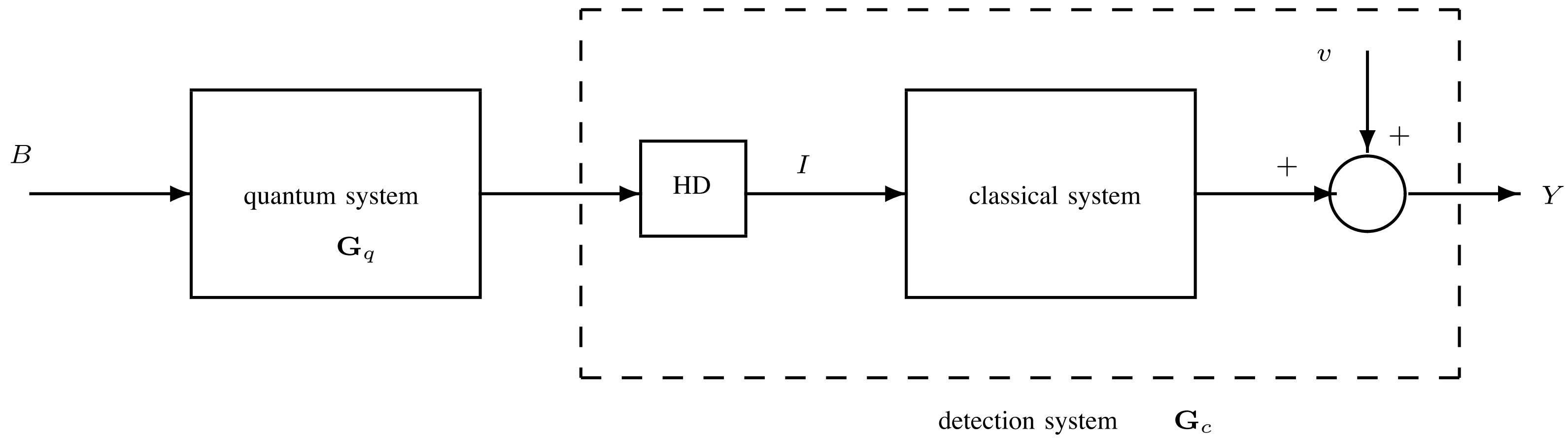
$$dX = (-i[X, H_0] + \mathcal{L}_{e^{-iF}L}(X))dt + [L^* e^{iF}, X]e^{-iF} dA + e^{iF}[X, e^{-iF}L]dA^* + (e^{iF} X e^{-iF} - X)d\Lambda.$$



(can also do quadrature measurement)

Realistic detection

[Warszawski-Wiseman-Mabuchi, 2002]



The quantum system is given by

$$\mathbf{G}_q = (1, L_q, H_q),$$

and the classical detection system is given by the classical stochastic equations

$$\begin{aligned} dx(t) &= \tilde{f}(x(t))dt + g(x(t))dw(t), \\ dY(t) &= h(x(t))dt + dv(t), \end{aligned}$$

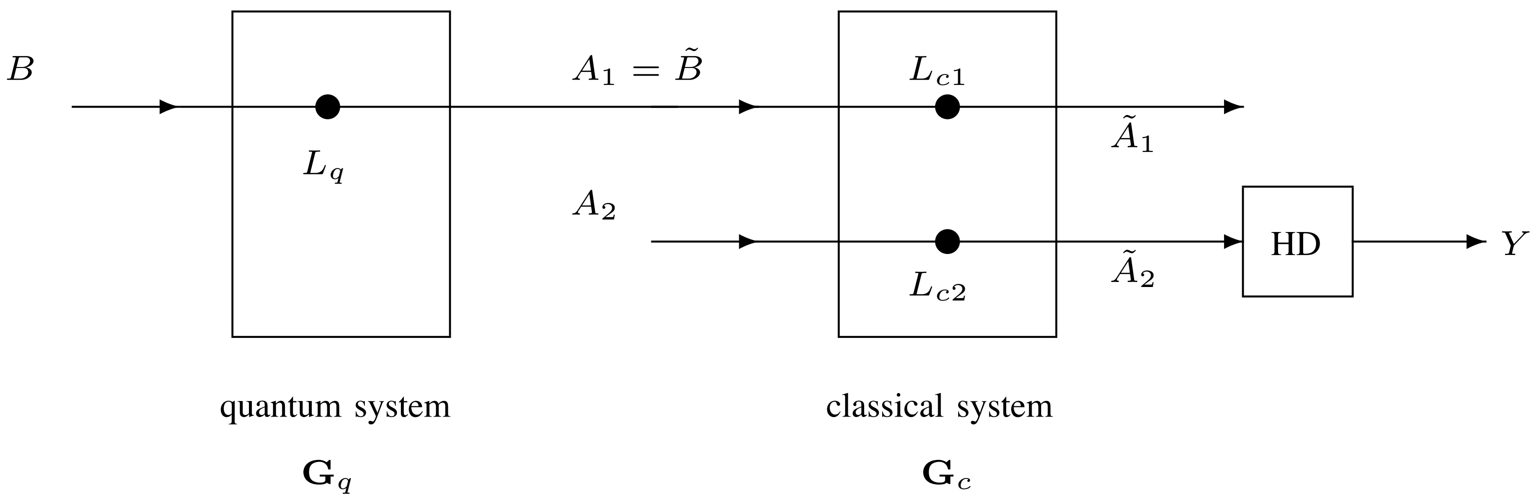
The classical system is equivalent to

$$\mathbf{G}_c = (1, L_{c1}, H_c) \boxplus (1, L_{c2}, 0)$$

where $L_{c1} = -ig^T p - \frac{1}{2}\nabla^T g$, $L_{c2} = \frac{1}{2}h$ and $H_c = \frac{1}{2}(f^T p + p^T f)$.

The complete cascade system is

$$\begin{aligned} \mathbf{G} &= ((1, L_{c1}, H_c) \triangleleft (1, L_q, H_q)) \boxplus (1, L_{c2}, 0) \\ &= (\mathbf{I}, \begin{pmatrix} L_1 + L_{c1} \\ L_{c2} \end{pmatrix}, H_q + H_c + \frac{1}{2i}(L_{c1}^* L_q - L_q^* L_{c1})) \end{aligned}$$



The unnormalized quantum filter for the cascade system is

$$
\begin{aligned}
d\sigma_t(X) &= \sigma_t(-i[X, H_q + H_c + \frac{1}{2i}(L_{c1}^* L_q - L_q^* L_{c1})] + \mathcal{L}_{\begin{pmatrix} L_1 + L_{c1} \\ L_{c2} \end{pmatrix}}(X))dt \\
&\quad + \sigma_t(L_{c2}^* X + X L_{c2})dy.
\end{aligned}
$$

For instance, $X = X_q \otimes \phi$, where ϕ is a smooth real valued function on $\mathbb{R}^n$.

Filtered estimate of quantum variables:

$$\pi_t(X_q) = \sigma_t(X_q)/\sigma_t(1)$$

Quantum Dissipative Systems

Combine perspectives from

- quantum physics
  - damping, commutation relations
  - quantum noise

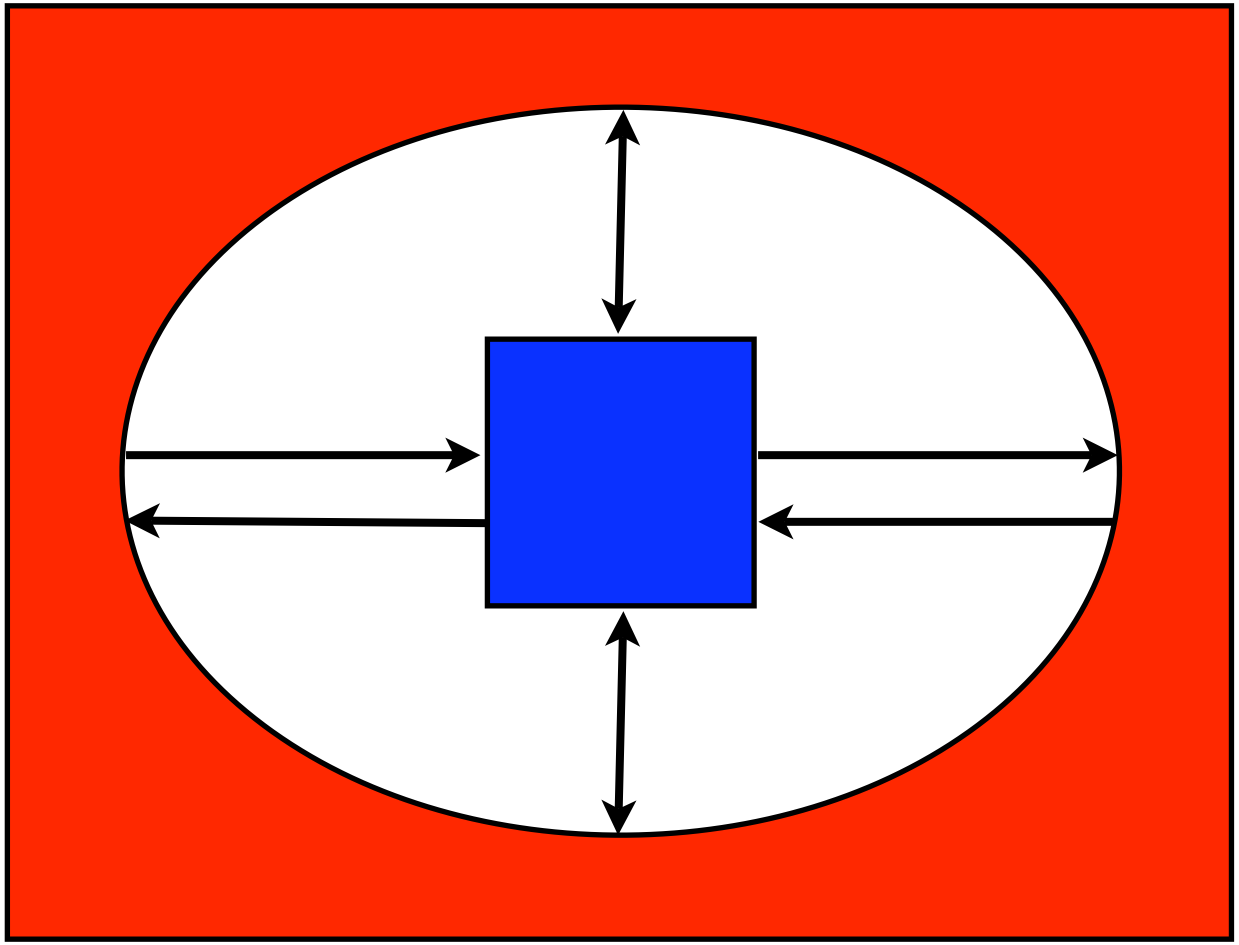
  (e.g. Gardiner-Collett, 1985, etc)

- control theory
  - behaviours
  - signals, disturbances, uncertainty
  - passivity, gain

  (e.g. Zames, 1965, Willems, 1972, 1997, etc)

in order to develop analysis and synthesis tools.

The plant is the system of interest, interacting with its environment.



Environment can include infinite heat baths, as well as other systems - *a network*.

Given two reducible systems $\mathbf{P} = \boxplus_j \mathbf{P}_j$ and $\mathbf{W} = \boxplus_{j'} \mathbf{W}_{j'}$, an interaction Hamiltonian

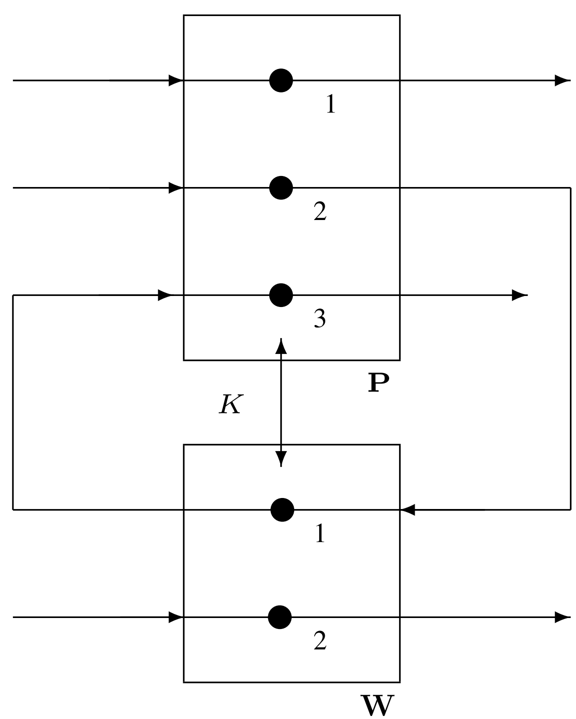
$$K = -i \sum_k (N_k^* M_k - M_k^* N_k),$$

where $N_k \in \mathscr{A}_{\mathbf{P}}$, $M_k \in \mathscr{A}_{\mathbf{G}}$, and a list of series connections

$$\mathscr{S} = \{\mathbf{W}_k \triangleleft \mathbf{P}_j,\, \mathbf{P}_{k'} \triangleleft \mathbf{W}_{j'}\},$$

one can form a network

$$\mathbf{N} = \mathbf{P} \wedge \mathbf{W}.$$



We call $\mathbf{W}$ an *exosystem*, and keeping the interconnection structure fixed, we let $\mathbf{W}$ vary in a *class* $\mathscr{W}$ of exosystems.

Lindblad generator for a system $\mathbf{G} = (\mathbf{S}, \mathbf{L}, H)$:

$$\mathcal{G}_{\mathbf{G}}(X) = \mathcal{L}_{\mathbf{L}}(X) - i[X, H]$$

where

$$\mathcal{L}_{\mathbf{L}}(X) = \frac{1}{2}\mathbf{L}^{\dagger}[X, \mathbf{L}] + \frac{1}{2}[\mathbf{L}^{\dagger}, X]\mathbf{L}.$$

Then

$$\mathbb{E}_s[X(t)] = X(s) + \int_s^t \mathbb{E}_s\left[\mathcal{G}_{\mathbf{G}}(X(r))\right] dr$$

for all $t \geq s$.

Plant

$$\mathbf{P} = (\mathbf{S}, \mathbf{L}, H)$$

Exosystem

$$\mathbf{W} = (\mathbf{R}, \mathbf{w}, D) \in \mathscr{W}$$

Supply rate

$$r_{\mathbf{P}}(\mathbf{W}) \in \mathscr{A}_{\mathbf{P}} \otimes \mathscr{A}_{ex}$$

a self-adjoint symmetrically ordered function of the exosystem parameters, depending on the plant parameters.

We say that the plant $\mathbf{P}$ is *dissipative* with supply rate r with respect to a class $\mathscr{W}$ of exosystems if there exists a non-negative system observable $V \in \mathscr{A}_{\mathbf{P}}$ such that

$$\mathbb{E}_0 \left[V(t) - V - \int_0^t r(\mathbf{W})(s) ds \right] \leq 0$$

for all exosystems $\mathbf{W} \in \mathscr{W}$ and all $t \geq 0$.

Infinitesimal characterization

The plant $\mathbf{P}$ is *dissipative* with supply rate r with respect to a class $\mathscr{W}$ of exosystems if and only if there exists a non-negative system observable $V \in \mathscr{A}_{\mathbf{P}}$ such that

$$\mathcal{G}_{\mathbf{P}\wedge\mathbf{W}}(V) - r(\mathbf{W}) \leq 0$$

for all exosystem parameters $\mathbf{W} \in \mathscr{W}$.

Special case from now on:

$$\mathbf{P} \wedge \mathbf{W} = \mathbf{P} \triangleleft \mathbf{W}$$

and

$$\mathbf{W} = (\mathbf{I}, \mathbf{L}, H)$$

Consider

$$\mathbf{W} = (\mathbf{R}, \mathbf{w}, -i(\mathbf{v}^\dagger \mathbf{K} - \mathbf{K}^\dagger \mathbf{v}))$$

where $\mathbf{v}$ commutes with plant variables and $K_P \in \mathscr{A}_{\mathbf{P}}$.

Open quantum systems are dissipative with respect to the "natural" supply rate

$$\begin{aligned} r_0(\mathbf{W}) &= \mathcal{G}_{\mathbf{P} \triangleleft \mathbf{W}}(V_0) \\ &= \mathcal{L}_{\mathbf{w}}(V_0) + \mathcal{L}_{\mathbf{L}}(V_0) + \begin{pmatrix} \mathbf{w}^\dagger & \mathbf{v}^\dagger \end{pmatrix} \mathbf{Z} + \mathbf{Z}^\dagger \begin{pmatrix} \mathbf{w} \\ \mathbf{v} \end{pmatrix}, \end{aligned}$$

where $V_0 \geq 0$ commutes with H.

passivity (energy flow into plant)

dissipation due to quantum noise

dissipation due to exosystem

$$\mathbf{Z} = [V_0, \begin{pmatrix} \mathbf{L} \\ \mathbf{K} \end{pmatrix}]$$

Transformation under series product

Let $\mathbf{P}_1$ and $\mathbf{P}_2$ be dissipative with respect to supply rates $r_{\mathbf{P}_1}(\mathbf{W})$ and $r_{\mathbf{P}_2}(\mathbf{W})$, storage functions V_1 and V_2, and exosystem classes $\mathscr{W}_1$ and $\mathscr{W}_2$ respectively.

The series system $\mathbf{P}_2 \triangleleft \mathbf{P}_1$ is dissipative with storage function $V_1 + V_2$ and supply rate

$$\boxed{r_{\mathbf{P}_2 \triangleleft \mathbf{P}_1}(\mathbf{W}) = r_{\mathbf{P}_1}(\mathbf{P}_2' \triangleleft \mathbf{W}) + r_{\mathbf{P}_2}(\mathbf{P}_1 \triangleleft \mathbf{W}),}$$

with respect to the exosystem class

$$\mathscr{W} = \{\mathbf{W} \ : \ \mathbf{P}_2' \triangleleft \mathbf{W} \in \mathscr{W}_1 \text{ and } \mathbf{P}_1 \triangleleft \mathbf{W} \in \mathscr{W}_2\},$$

where

$$\mathbf{P}_2' = (\mathbf{S}_1^\dagger \mathbf{S}_2 \mathbf{S}_1,\ \mathbf{S}_1^\dagger (\mathbf{S}_2 - \mathbf{1}) \mathbf{L}_1 + \mathbf{S}_1^\dagger \mathbf{L}_2,\ H_2 + \mathrm{Im}\left\{\mathbf{L}_2^\dagger (\mathbf{S}_2 + \mathbf{1}) \mathbf{L}_1 - \mathbf{L}_1^\dagger \mathbf{S}_2 \mathbf{L}_1\right\}).$$

Example

Open harmonic oscillator (e.g optical cavity)

$$\mathbf{P} = (1, \sqrt{\gamma} a, \omega a^* a)$$

Let $V_0 = H/\omega = a^* a$.

$$r_0(\mathbf{W}) = \mathcal{G}_{\mathbf{P} \triangleleft \mathbf{W}}(V_0) \;\; = \;\; -\gamma a^* a - \sqrt{\gamma}(w^* a + a^* w) + \mathcal{L}_w(V_0) - i[V_0, D]$$

By completion of squares the supply rate can be re-written

$$r_0(\mathbf{W}) = -(\sqrt{\gamma}\, a + w)^*(\sqrt{\gamma}\, a + w) + w^* w + \mathcal{L}_w(V_0) - i[V_0, D]$$

and hence the system has gain 1 relative to the output quantity $\sqrt{\gamma}\, a + w$ and commuting inputs w.

Note that if we include ground state energy and write $V = a^* a + \frac{1}{2} = q^2 + p^2$ (here $q = a + a^*$, $p = -i(a - a^*)$, then passivity and gain holds but with $\lambda = \gamma > 0$.

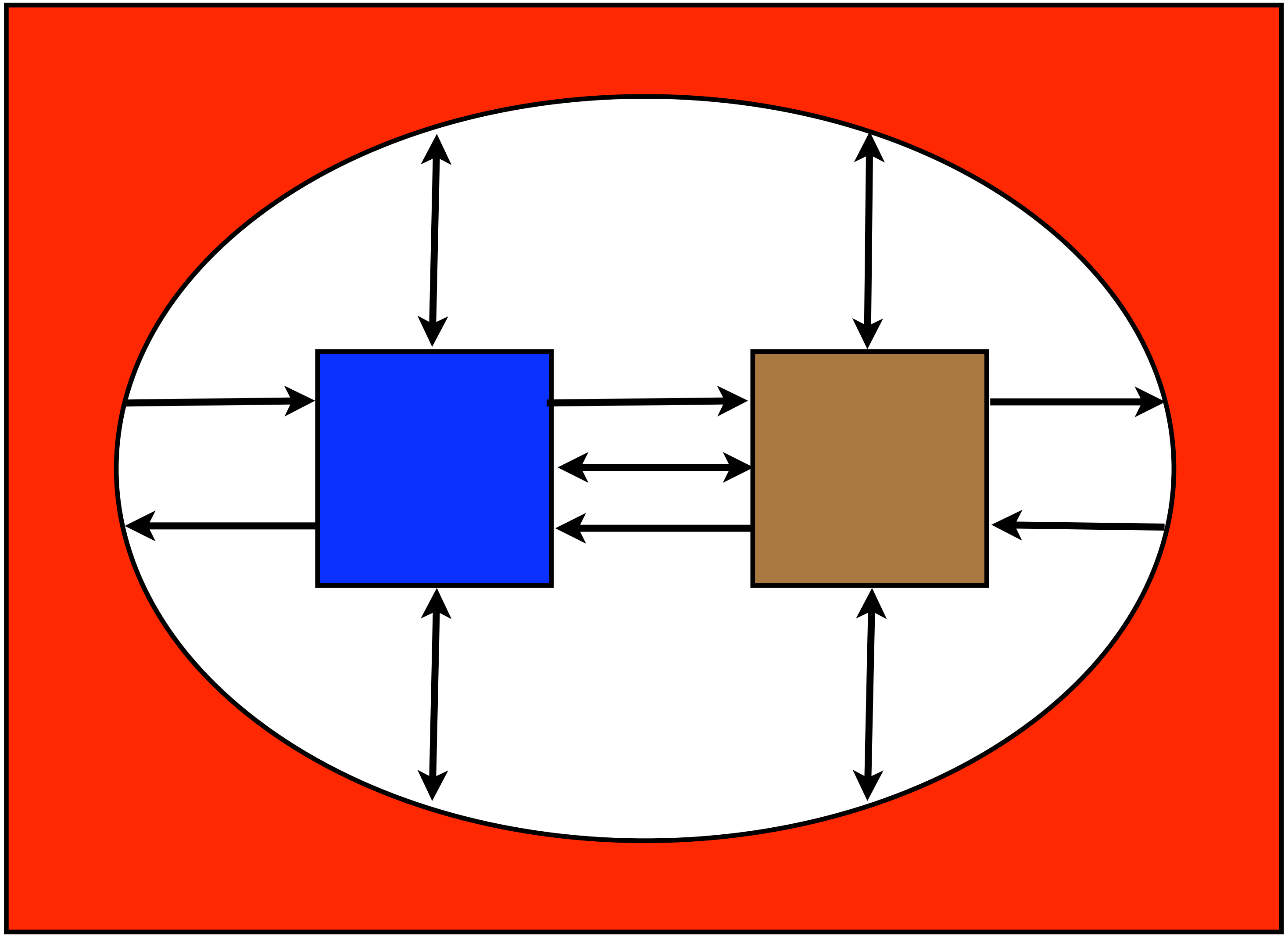
Feedback Control by Interconnection

- Inspired by behavioural ideas of J.C. Willems and energy-based design methods for classical mechanical systems (e.g. robotics)

  (e.g. Ortega and Spong, 1989, etc)

- Control design as network design
- Controller may be classical, quantum, or a mixture of the two
- Design focusses on the physical structure
- Interconnections can be field-mediated and/or direct interactions
- Covers standard problems of stabilization, regulation, robustness

The plant and the controller may interact with their environment.



Methodology

Specify the control objectives by encoding them in

- a non-negative observable $V_d \in \mathscr{A}_{\mathbf{P}} \otimes \mathscr{A}_{\mathbf{C}}$,
- a supply rate $r_d(\mathbf{W})$,
- and a class of exosystems $\mathscr{W}_d$ for which a network $(\mathbf{P} \wedge \mathbf{C}) \wedge \mathbf{W}$ is well defined.

One then seeks to find, if possible, a controller $\mathbf{C}$ such that

$$\mathcal{G}_{(\mathbf{P} \wedge \mathbf{C}) \wedge \mathbf{W}}(V_d) - r_d(\mathbf{W}) \leq 0$$

for all exosystem parameters $\mathbf{W} \in \mathscr{W}_d$.

Example

Cavity $\mathbf{P} = (1, a, 0)$ with vacuum input.

Wish to maintain steady-state photon number $\alpha^* \alpha$.

Consider simple direct plant-controller interaction

$$\mathbf{C} = (1, 0, -i(K_P^* \nu - \nu^* K_P)) \, ,$$

where K_p is a plant operator and ν is a complex number, both to be chosen.

Closed loop system

$$\mathbf{P} \wedge \mathbf{C} = \mathbf{P} \boxplus \mathbf{C}.$$

We set

$$V_d = (a - \alpha)^* (a - \alpha) = a^* a - \alpha^* a - a^* \alpha + \alpha^* \alpha,$$

and for a positive real number c,

$$r_d(\mathbf{W}) = -c V_d,$$

with $\mathscr{W}_d = \{(\_, \_, 0)\}$, which consists only of the trivial exosystem.

The design problem is to select, if possible, K_P, a plant operator, and ν, a complex number, such that

$$\mathcal{G}_{\mathbf{P}\boxplus\mathbf{C}}(V_d) + cV_d \leq 0$$

for suitable $c > 0$. We choose $K_P = a$.

Evaluate LHS, and set $c = 1/2$, $\nu = -\alpha/2$.

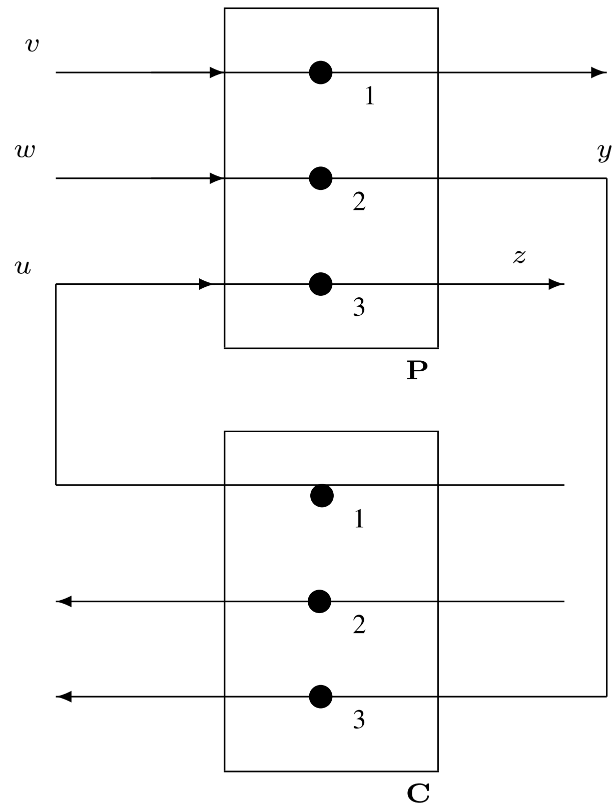
Physically, this control design corresponds to a classical energy source connected to the cavity, such as when the vacuum field is replaced by a coherent field (signal plus noise), i.e. a laser beam.

Example

H-Infinity Control

[James-Nurdin-Petersen, 2006]

The control objective is to reduce the gain from input w to output z by an appropriate choice of controller $\mathbf{C}$.



Plant

$$\begin{aligned}\mathbf{P} &= \mathbf{P}_1 \boxplus \mathbf{P}_2 \boxplus \mathbf{P}_3 \\ &= (1, \sqrt{\kappa_1}\, a,\, 0) \boxplus (1, \sqrt{\kappa_2}\, a,\, 0) \boxplus (1, \sqrt{\kappa_3}\, a,\, 0),\end{aligned}$$

Controller

$$\mathbf{C} = \mathbf{C}_1 \boxplus \mathbf{C}_2 \boxplus \mathbf{C}_3.$$

The plant-controller network is

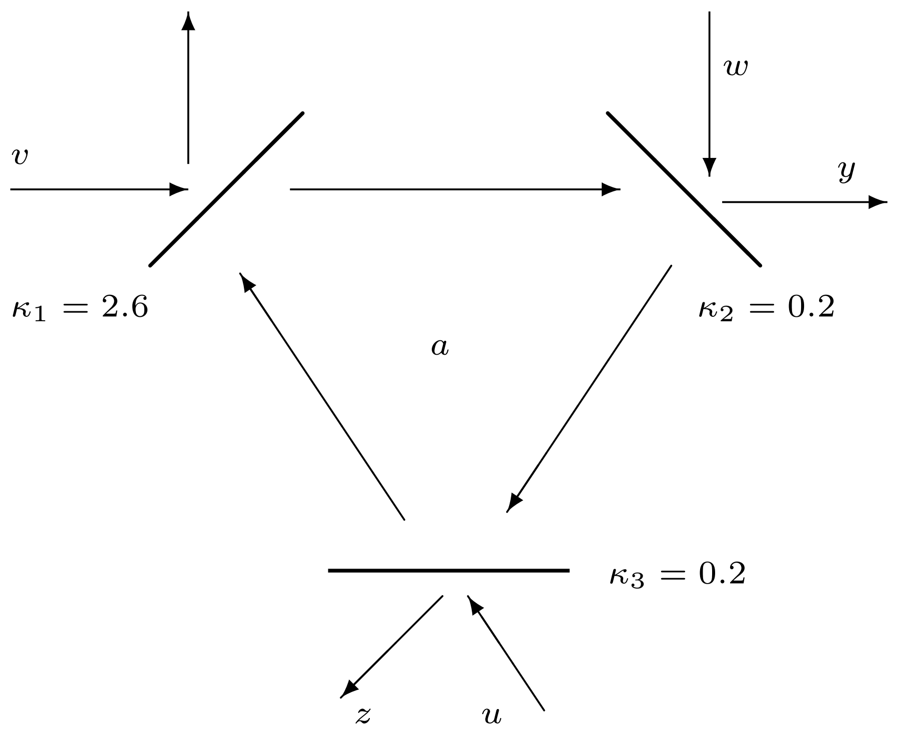
$$\mathbf{P} \wedge \mathbf{C} = \mathbf{P}_1 \boxplus (\mathbf{C}_3 \triangleleft \mathbf{P}_2) \boxplus (\mathbf{P}_3 \triangleleft \mathbf{C}_1) \boxplus \mathbf{C}_2.$$

The supply rate is

$$r(\mathbf{W}) = g^2 w^* w - (\sqrt{\kappa_3}\, a + w)^* (\sqrt{\kappa_3}\, a + w)$$

for exosystems $\mathbf{W} \in \mathscr{W}_d$, where

$$\mathscr{W}_d = \{\mathbf{W} = (1,0,0) \boxplus (1,w,0) \boxplus (1,0,0) \boxplus (1,0,0) \;\;:\;\; w \text{ commutes with } \mathscr{A}_{\mathbf{P}} \otimes \mathscr{A}_{\mathbf{C}}\}\,.$$



Plant, an optical cavity.

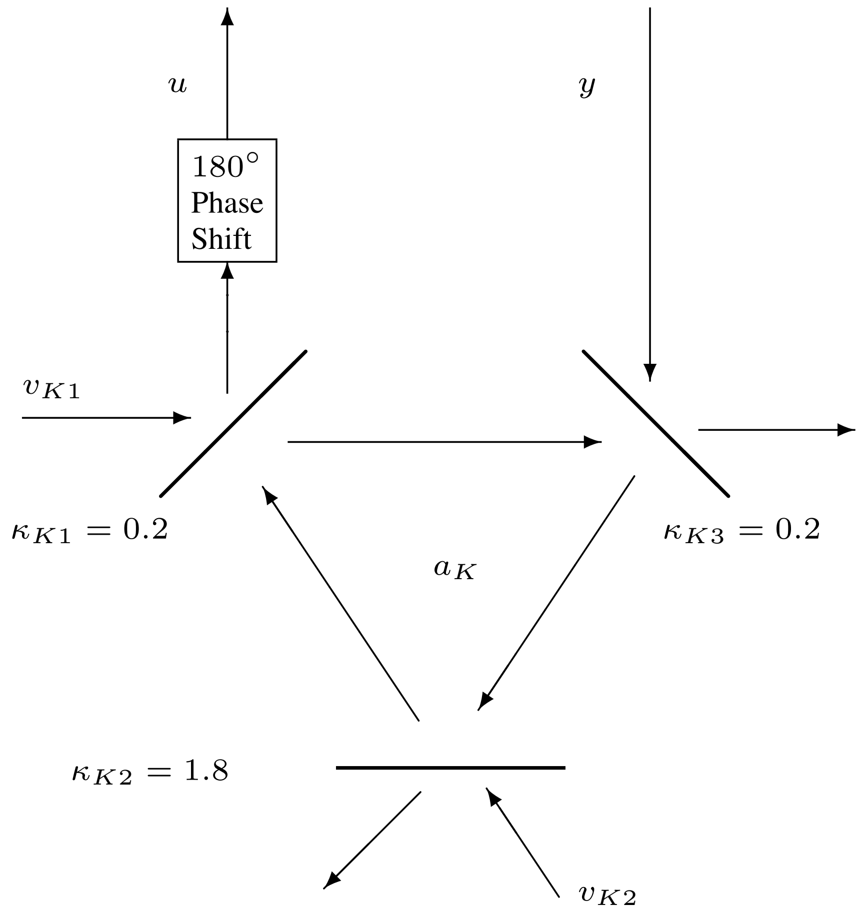
For the plant parameters $\kappa_1 = 2.6$, $\kappa_2 = 0.2$, $\kappa_3 = 0.2$, a controller was realized as a cavity with annihilation operator b:

$$\mathbf{C} = (-1, -\sqrt{0.2}\, b,\ 0) \boxplus (1, \sqrt{1.8}\, b,\ 0) \boxplus (1, \sqrt{0.2}\, b,\ 0).$$

This construction had two steps:

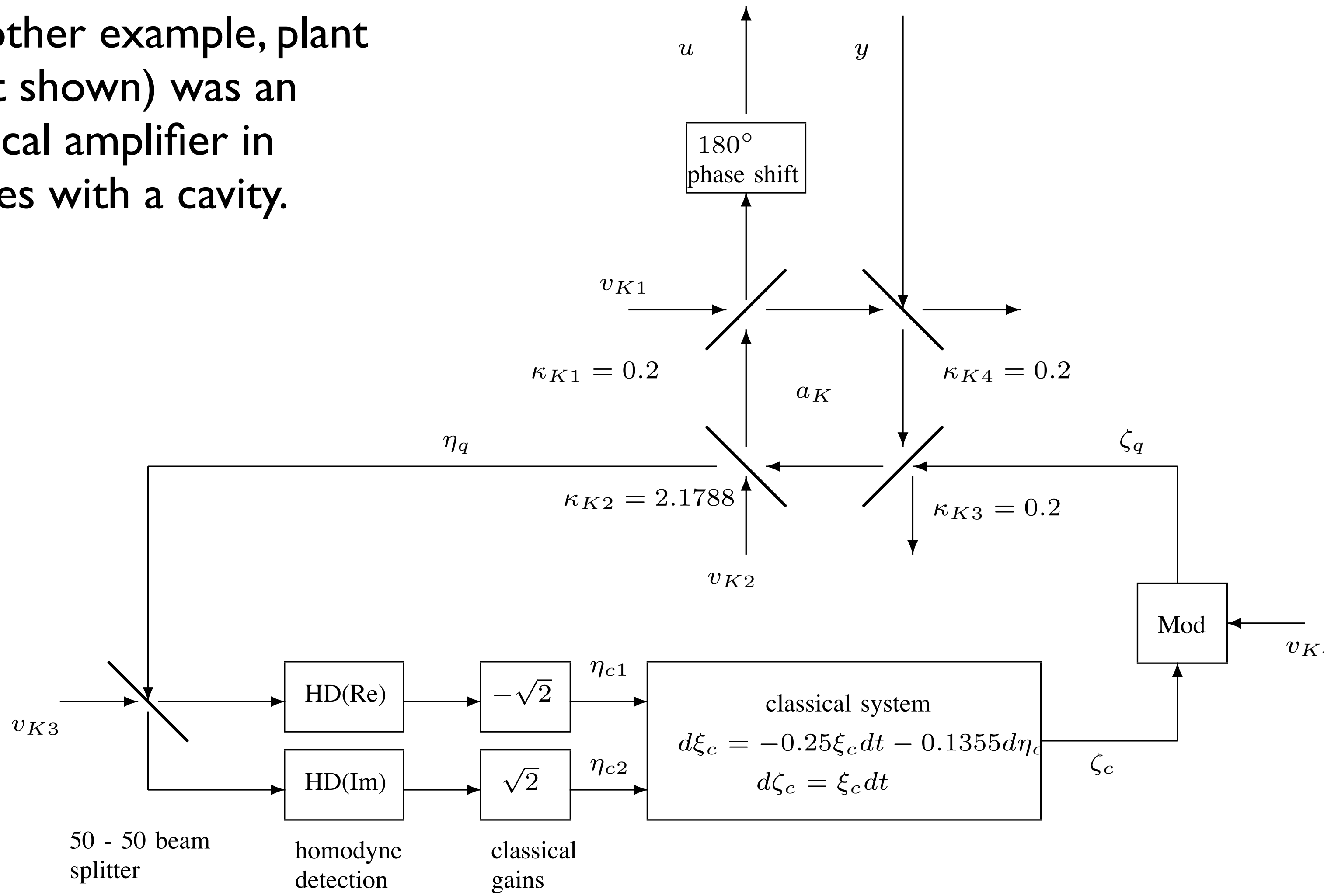
1. Evaluation of quadratic forms with respect to Gaussian states, and using some classical results. This gives *part, not all*, of the solution.

2. Completing the design by adding field couplings to ensure commutation relations preserved. This is algebraic.



Controller, specified to be quantum, realized as a cavity.

Another example, plant (not shown) was an optical amplifier in series with a cavity.



Here, the controller, specified to have quantum and classical degrees of freedom.

Conclusion

- Concatenation and series products facilitate quantum network analysis and design (reducible networks).
- Combined quantum physics and control engineering perspectives on dissipation.
- Control design by interconnection based on direct physical description.
- Controllers may be quantum, classical, or a mixture.
- Allows designers to focus more on systems, less on equations.
- Physical realization a key issue.
- Network paradigm powerful and likely to be helpful for quantum technology.