Quantum Feedback Networks

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Outline

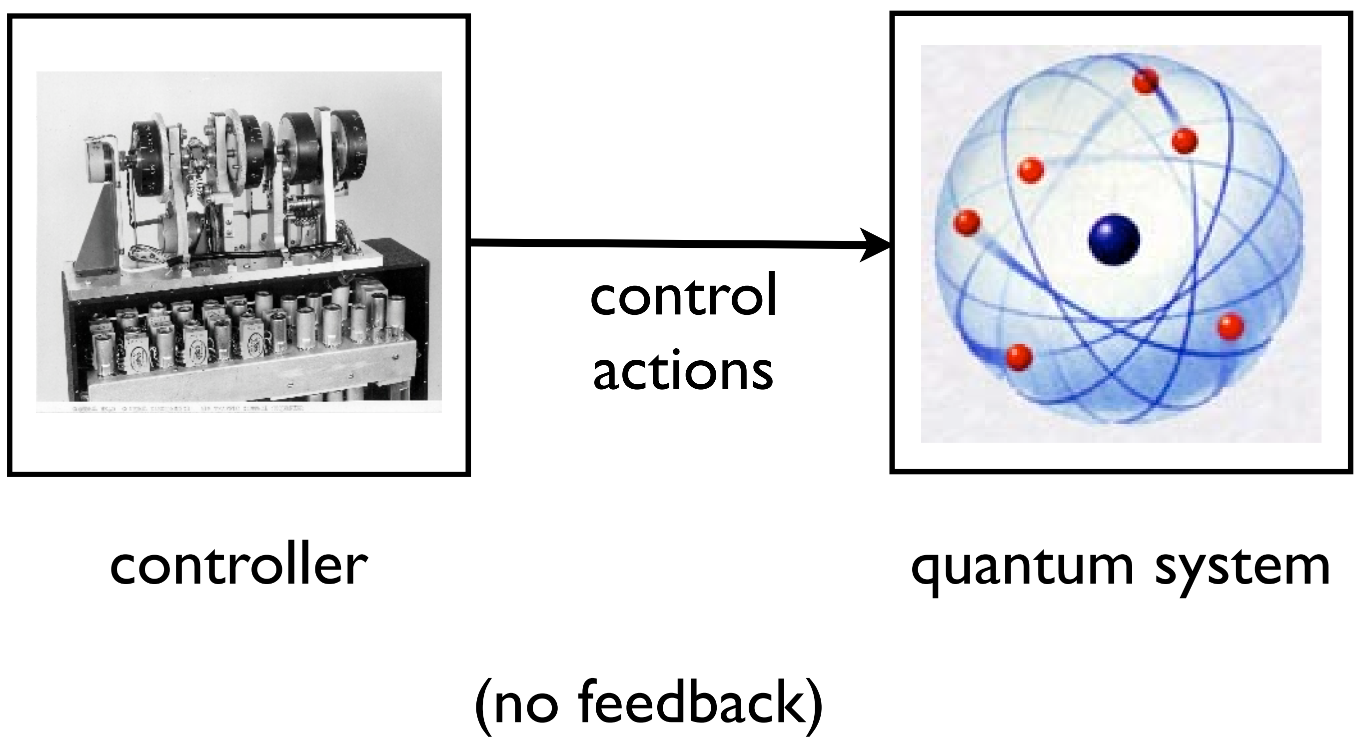
- Quantum Feedback
- Quantum Networks - I
- Classical Electrical Networks
- Quantum Networks - II
- Examples in Quantum Control

References:

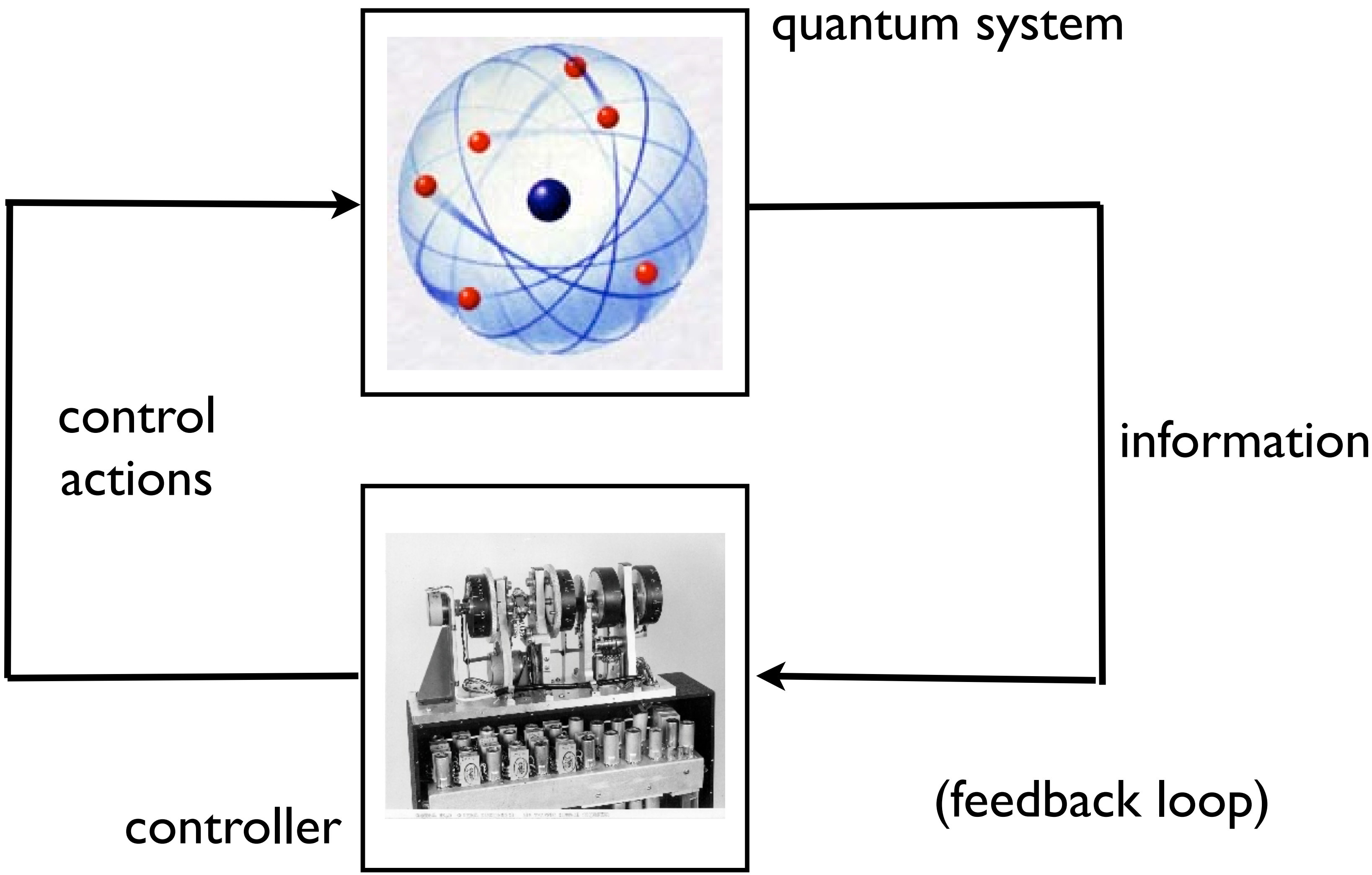
- J. Gough and M.R. James, *The Series Product and Its Application to Quantum Feedforward and Feedback Networks*, quant-ph/0707.0048.
- M.R. James and J. Gough, *Quantum Dissipative Systems and Feedback Control by Interconnection,* quant-ph/0707.1074.
- M.R. James, H.I. Nurdin and I.R. Petersen, *H-Infinity Control of Linear Quantum Stochastic Systems,* quant-ph/0703150, accepted, IEEE TAC.

Quantum Feedback

- <u>Open loop</u> - control actions are predetermined, no feedback is involved.



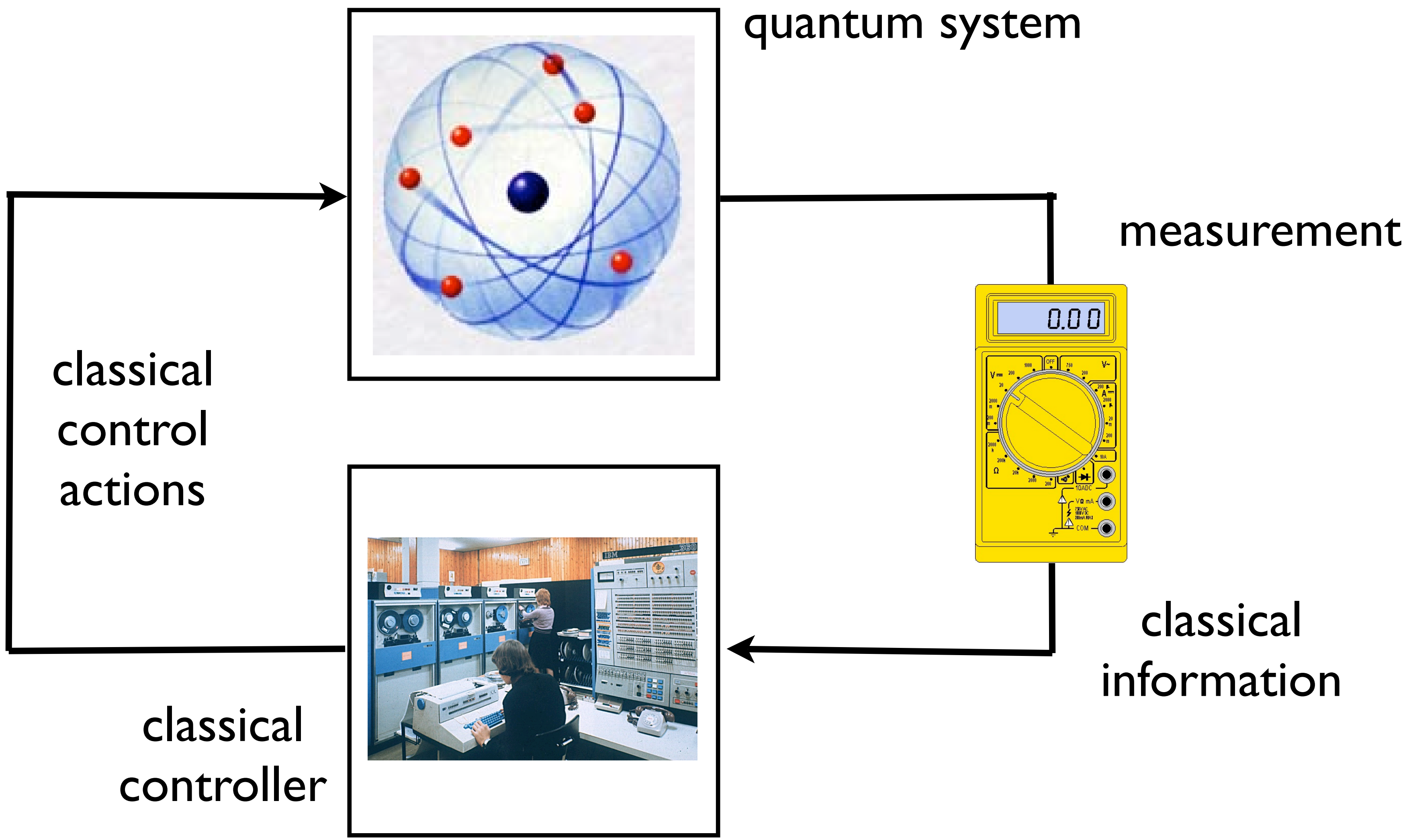
- <u>Closed loop</u> - control actions depend on information gained as the system is operating.



Types of Quantum Feedback:

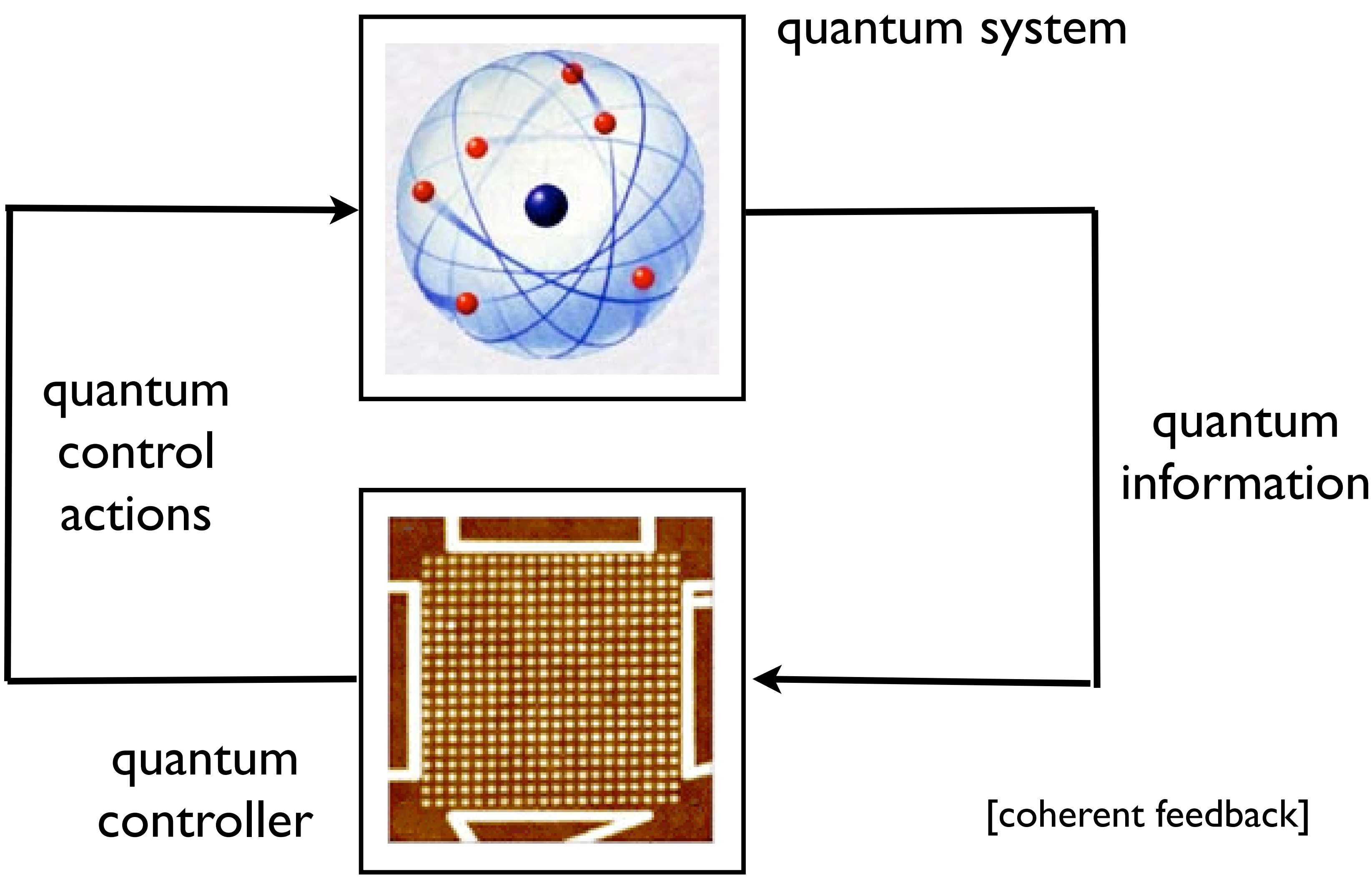
- Using measurement

The classical measurement results are used by the controller (e.g. classical electronics) to provide a classical control signal.



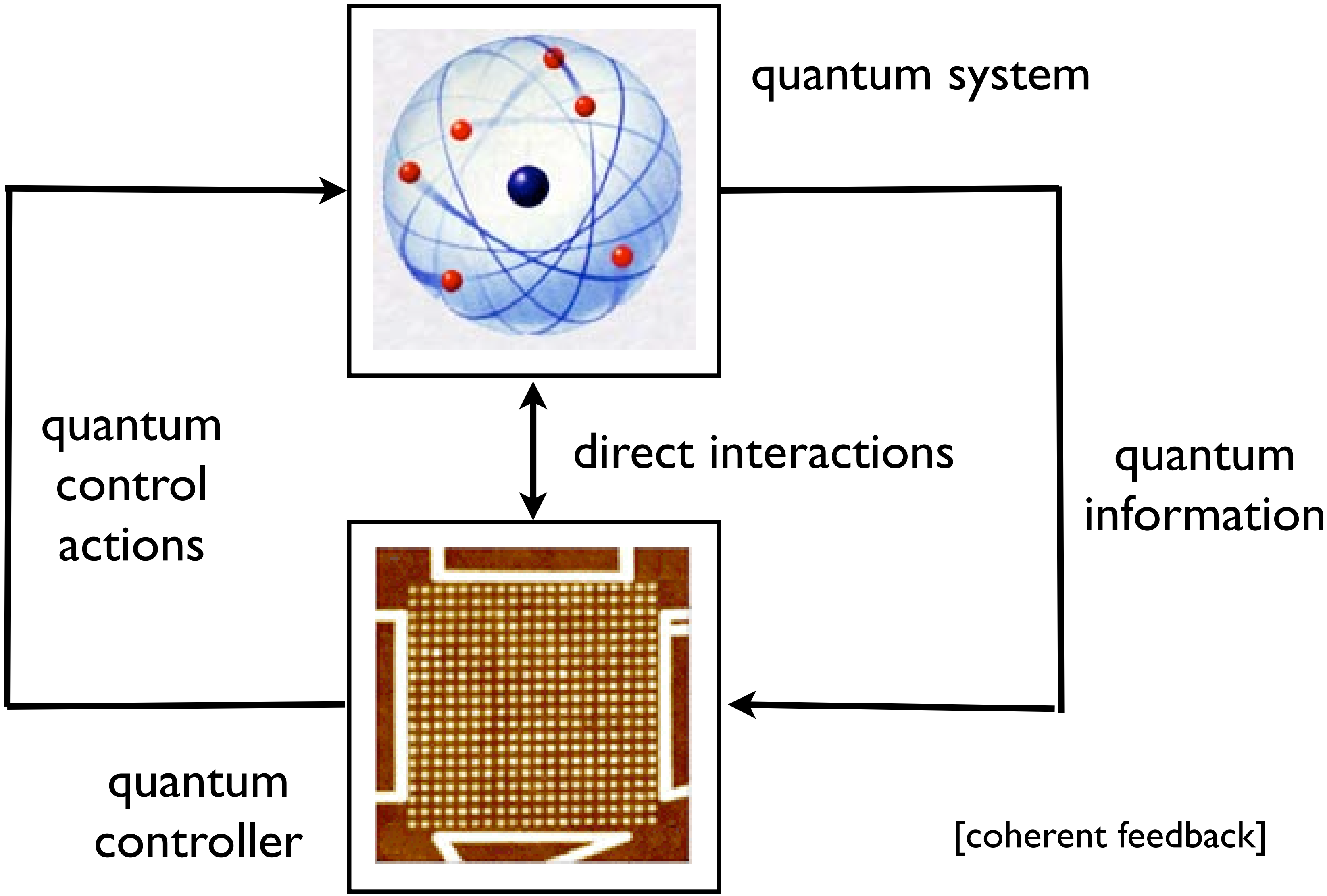
- Not using measurement

The controller is also a quantum system, and feedback involves a direct flow of quantum information.



- # Direct interactions

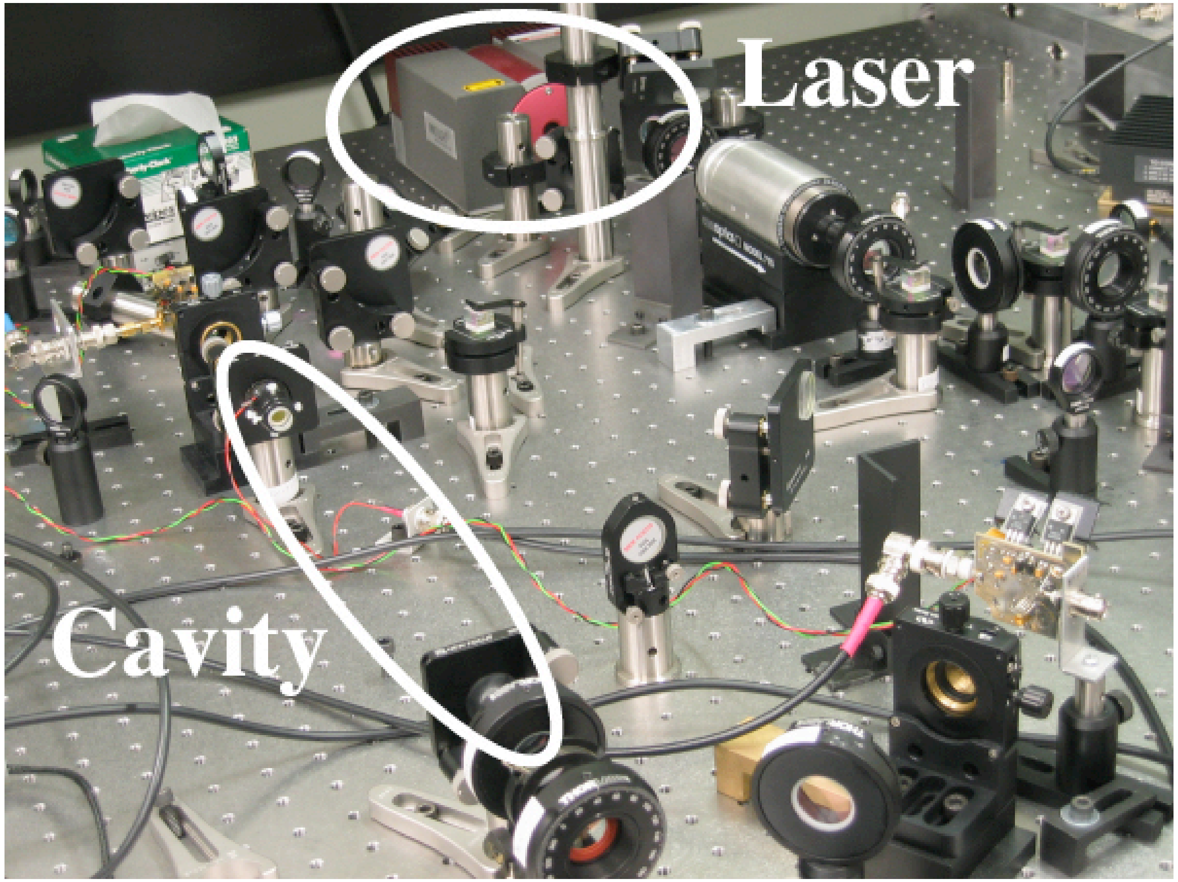
The controller is also a quantum system, and feedback may also include direct interactions.



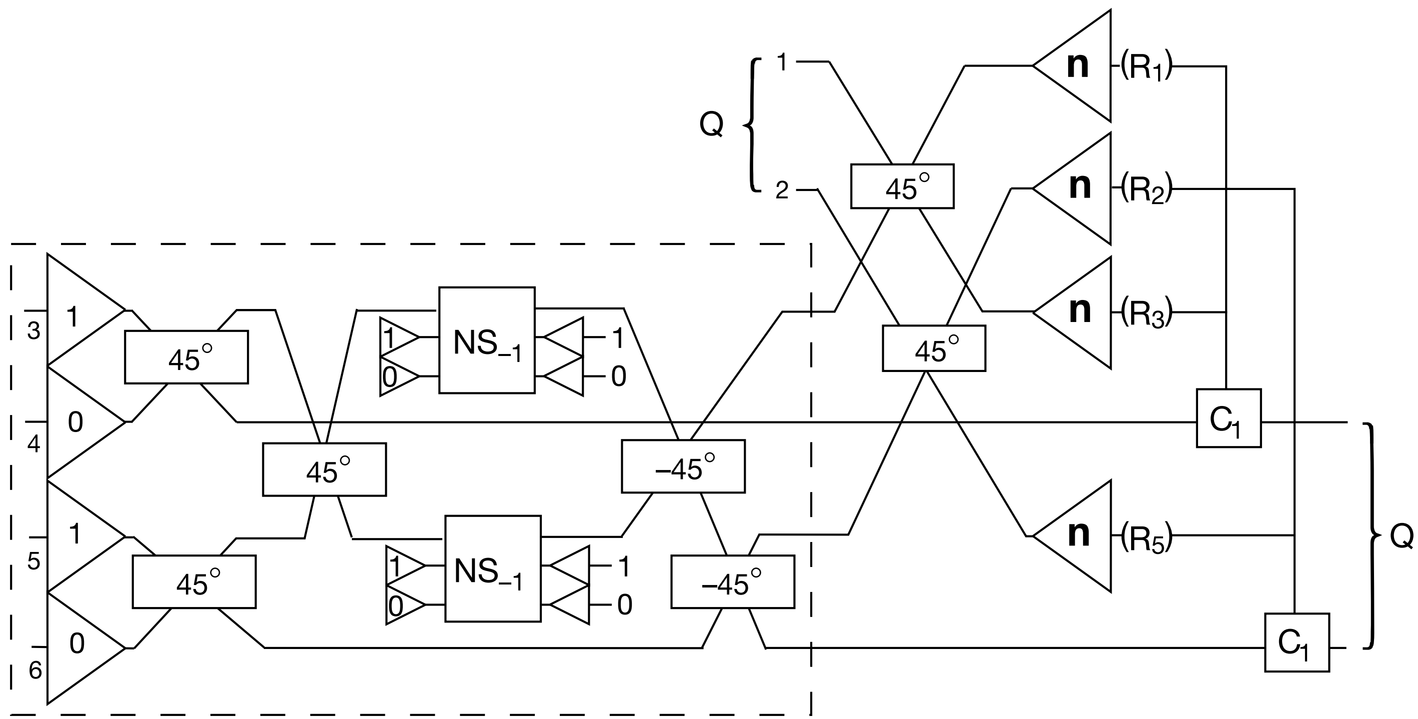
[coherent feedback]

Quantum Networks I

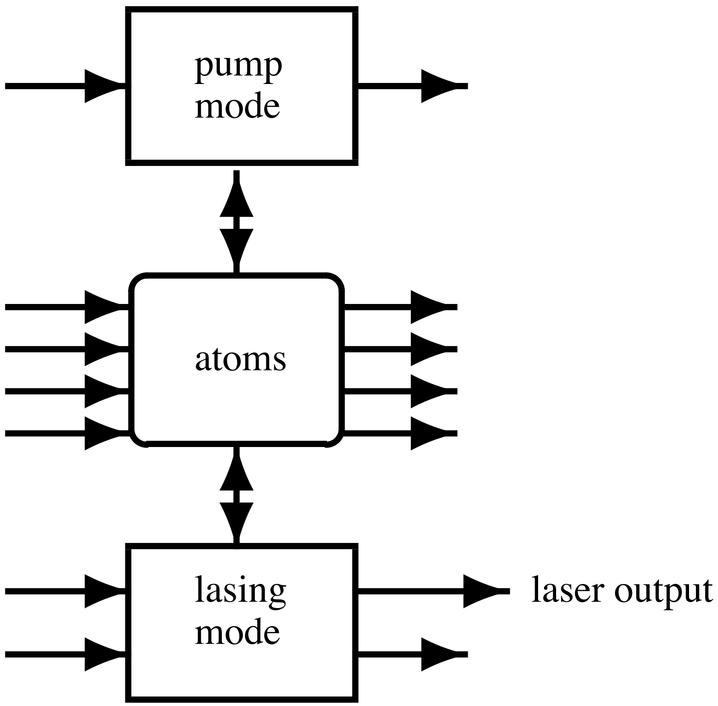
Quantum networks are ubiquitous



[quantum optics lab - E. Huntington, ADFA/UNSW]

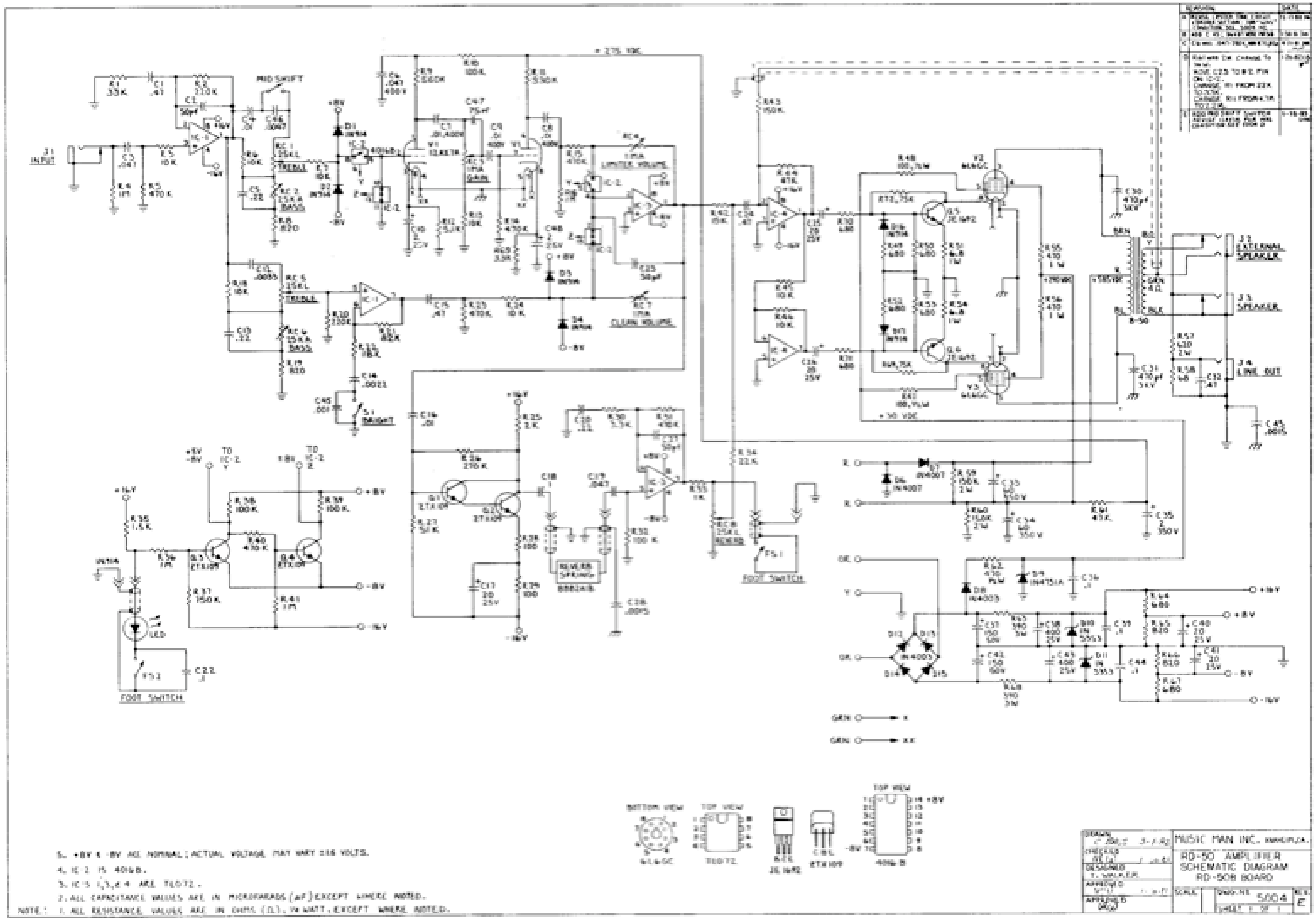


[quantum computing network - (teleportation with loss detection)
- Knill, Laflamme, Milburn, 2001]



[Optical laser model - Ralph, Harb, Bachor 1996]

Classical Electrical Networks

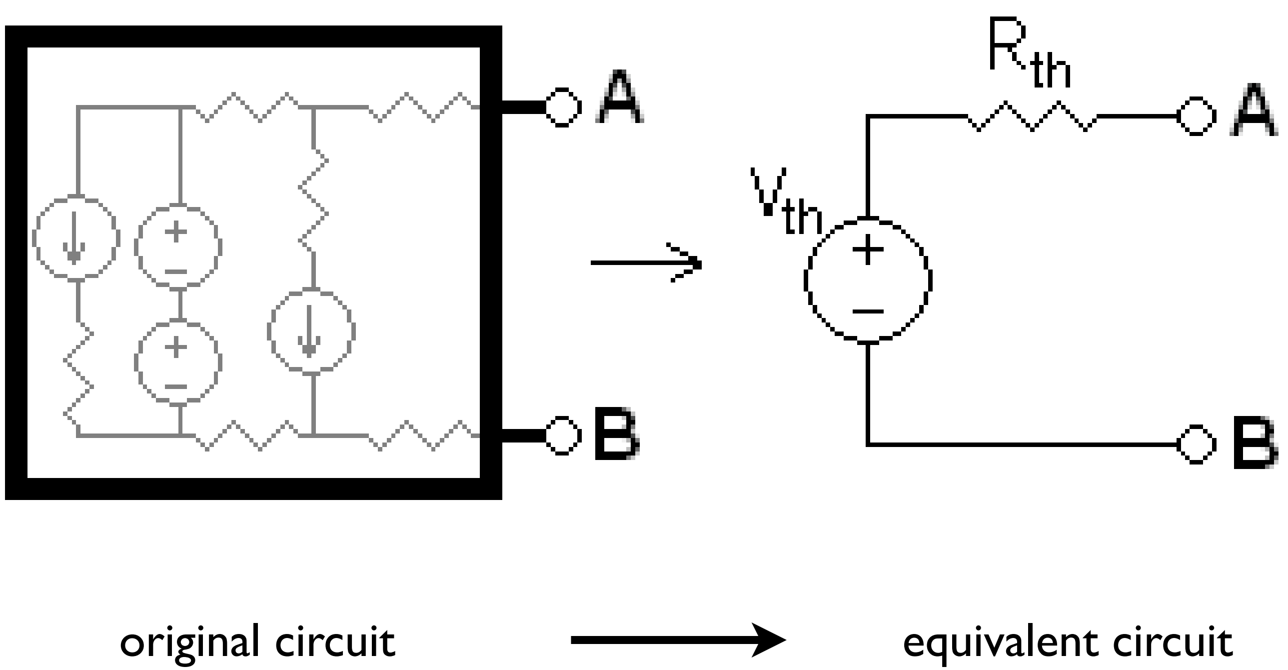


[circuit diagram of a classical electronic amplifier]

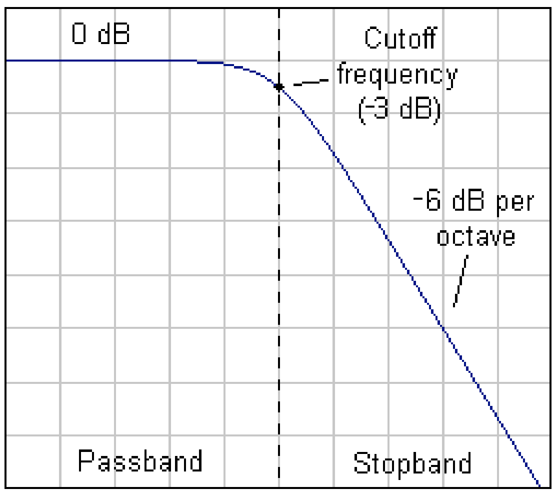
Modern circuit theory...

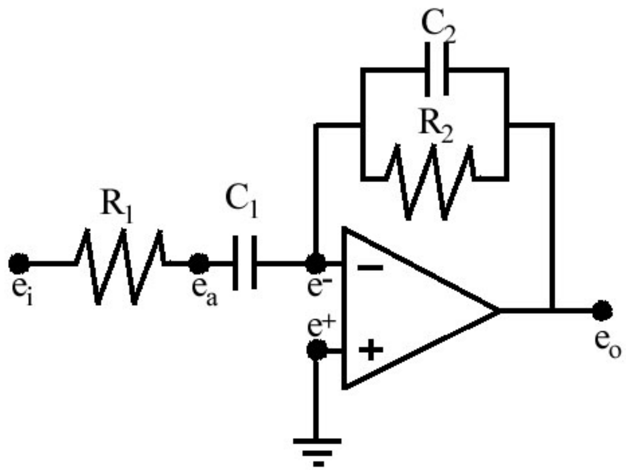
- builds on laws of physics (Maxwell, Faraday, Ohm,...)
- evolved to meet the needs of electrical system designers (Thevenin, Kirchhoff,...)
- includes
  - device models
  - rules for interconnection
  - methods for analysis, simplification, and synthesis

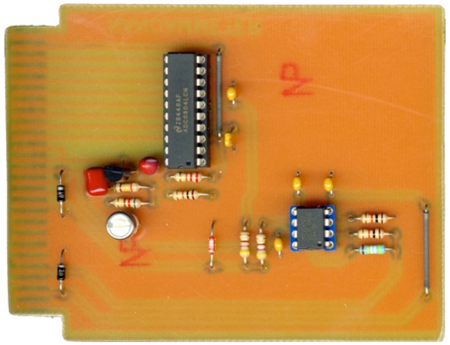
For example, Thevenin's theorem helps engineers simplify complex linear circuits.



For example, realisation techniques help engineers build devices from given specifications (synthesis).





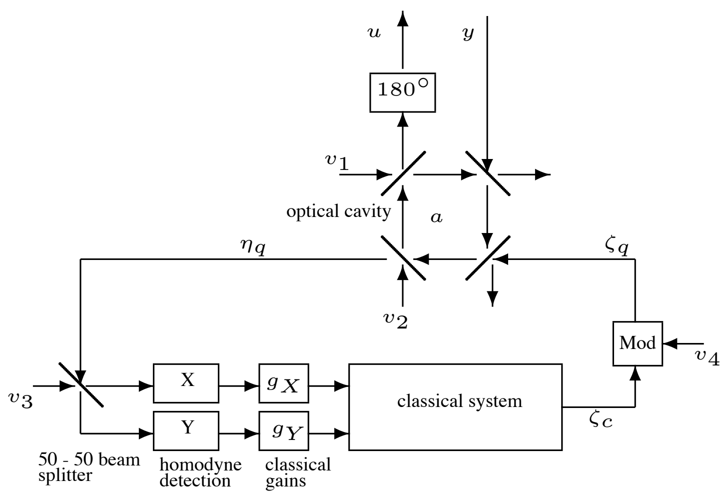


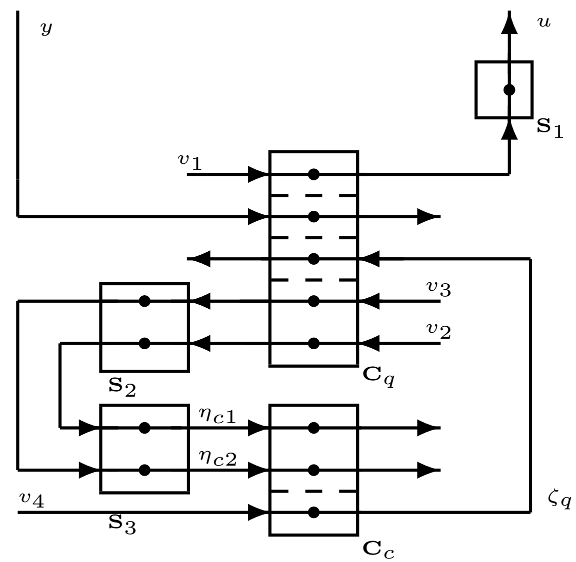
specifications → circuit diagram → physical device

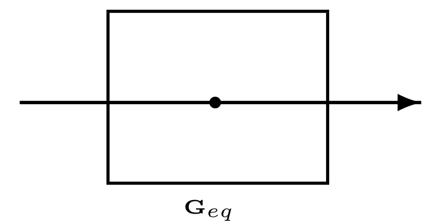
Quantum Networks II

Desirable attributes

- Capture the quantum physics
- Be capable of representing classical components
- Include dissipative mechanisms
  - noise, uncertainty, decoherence (open)
- Preserve canonical structure
  - e.g. commutation relations, energy
- Network of interconnected components should also be a quantum system
  - recursive
- Efficient methods for representation, interconnection, manipulation, and physical realization
- Efficient methods for simplification, analysis and synthesis





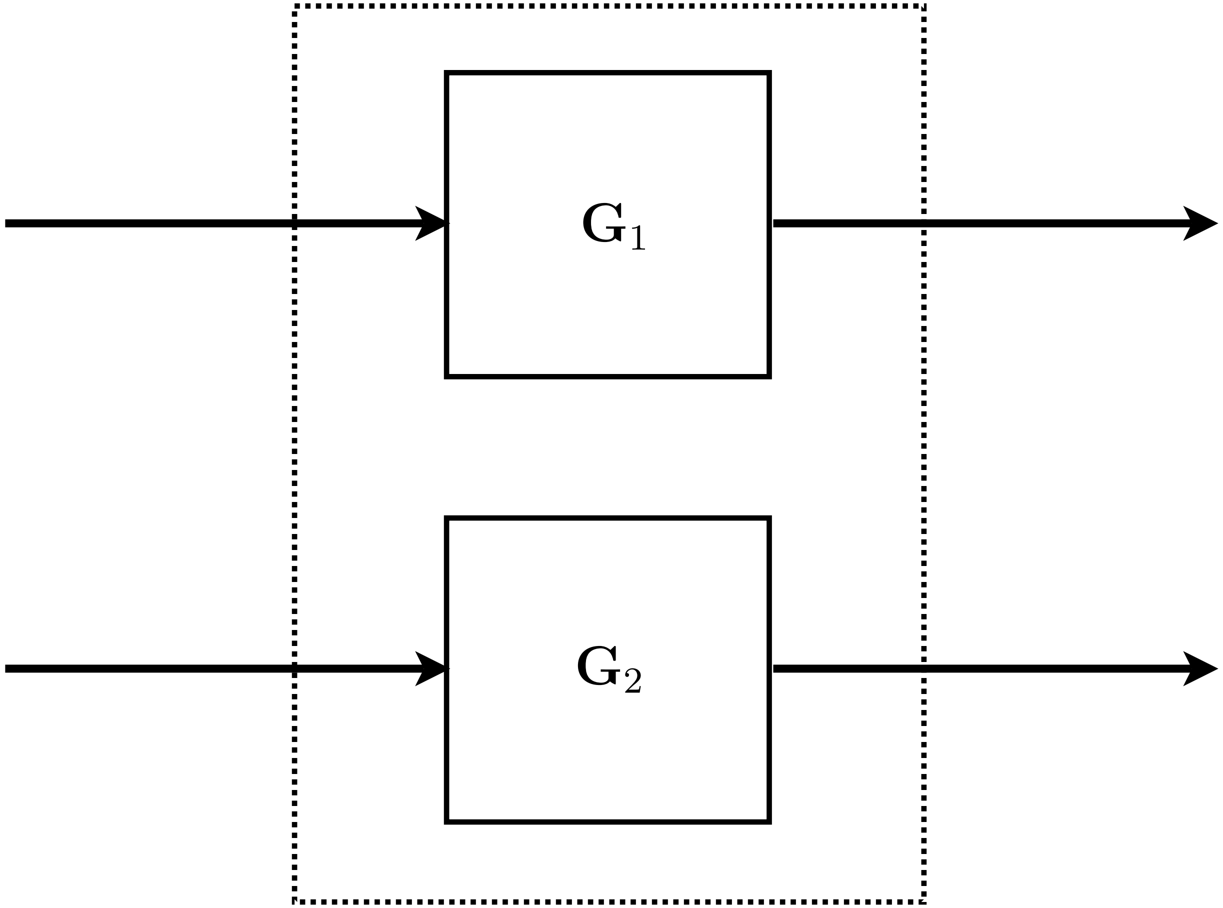


original network → parametric network representation → simplified parametric representation

[Representation and simplification of a quantum network]

Elementary network constructs:

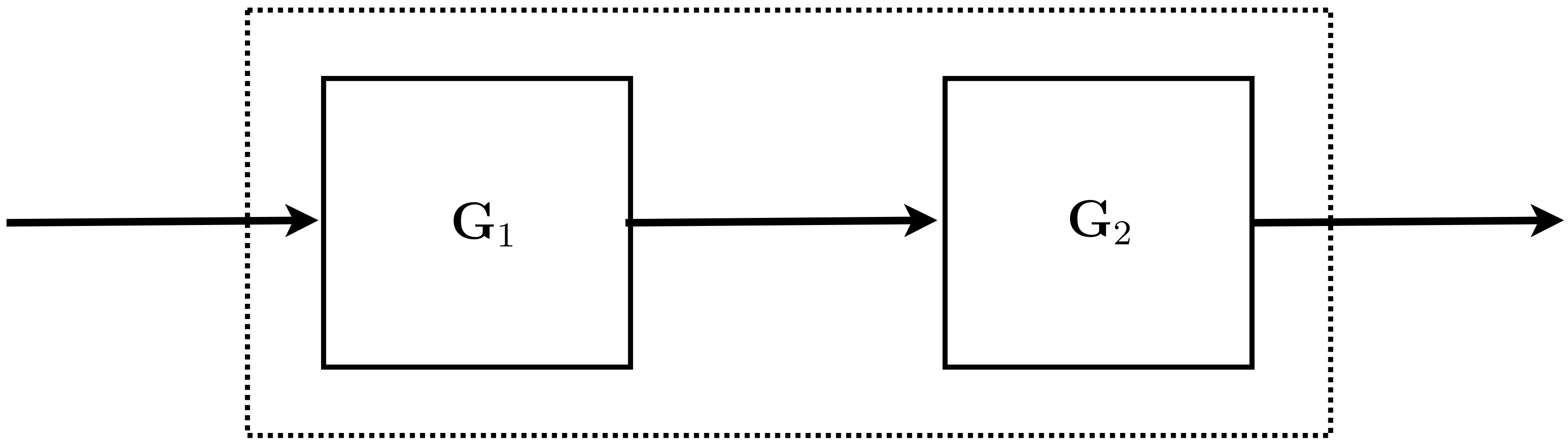
Concatenation



$$\mathbf{G} = \mathbf{G}_1 \boxplus \mathbf{G}_2$$

[*Not* parallel]

Series (cascade)



$$\mathbf{G} = \mathbf{G}_2 \triangleleft \mathbf{G}_1$$

History:
Gardiner, 1993
Carmichael, 1993

Open systems:

Multichannel open quantum system characterised by parameters

- H is a Hamiltonian (self-adjoint operator)
- $\mathbf{L}$ is a vector of field coupling operators
- $\mathbf{S}$ is a scattering matrix (self-adjoint matrix of operators)

Shorthand:

$$\mathbf{G} = (\mathbf{S}, \mathbf{L}, H)$$

These parameters, together with field channels specifications, determine the *master equation*, or equivalently, the Heisenburg *quantum stochastic differential equations*.

[Gardiner-Collett 1985
Hudson-Parthasarathy 1984]

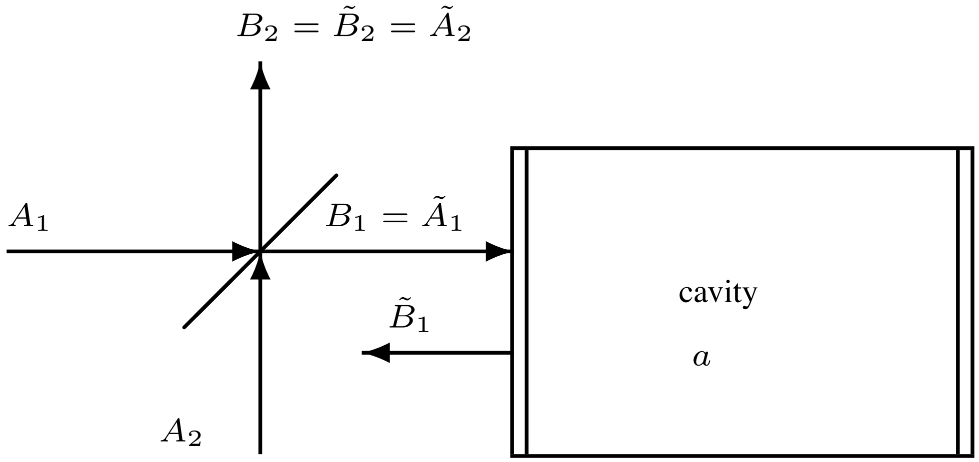
Concatenation product

$$\mathbf{G}_1 \boxplus \mathbf{G}_2 = \left(\begin{pmatrix} \mathbf{S}_1 & 0 \\ 0 & \mathbf{S}_2 \end{pmatrix}, \begin{pmatrix} \mathbf{L}_1 \\ \mathbf{L}_2 \end{pmatrix}, H_1 + H_2 \right)$$

Series product

$$\mathbf{G}_2 \triangleleft \mathbf{G}_1 = \left(\mathbf{S}_2\mathbf{S}_1, \quad \mathbf{L}_2 + \mathbf{S}_2\mathbf{L}_1, \quad H_1 + H_2 + \frac{1}{2i}(\mathbf{L}_2^\dagger \mathbf{S}_2 \mathbf{L}_1 - \mathbf{L}_1^\dagger \mathbf{S}_2^\dagger \mathbf{L}_2)\right)$$

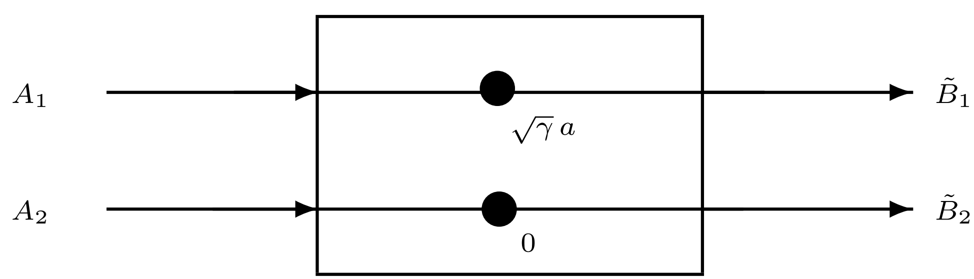
Example: beamsplitter and cavity



$$
\begin{aligned}
da(t) &= (-\frac{\gamma}{2} + i\Delta)a(t)dt - \sqrt{\gamma}\, dB_1(t) \\
\tilde{A}_1(t) &= \beta A_1(t) - \alpha A_2(t) \\
\tilde{A}_2(t) &= \alpha A_1(t) + \beta A_2(t) \\
B_1(t) &= \tilde{A}_1(t) \\
B_2(t) &= \tilde{A}_2(t) \\
d\tilde{B}_1(t) &= \sqrt{\gamma} a(t)dt + dB_1(t) \\
d\tilde{B}_2(t) &= dB_2(t).
\end{aligned}
$$

Complete network

$$\mathbf{G} = (\mathbf{S}, \mathbf{L}, H) = \left(\begin{pmatrix} \beta & -\alpha \\ \alpha & \beta \end{pmatrix}, \begin{pmatrix} \sqrt{\gamma}\, a \\ 0 \end{pmatrix}, \Delta a^* a \right)$$



Description in terms of concatenation and series products:

$$\mathbf{G} = (\mathbf{C} \boxplus \mathbf{N}) \triangleleft \mathbf{B},$$

where

$$\mathbf{C} = (1, \sqrt{\gamma}\, a, \Delta a^* a),$$

describes the cavity,

$$\mathbf{N} = (1, 0, 0)$$

is a trivial system (pass-through), and

$$\mathbf{B} = (\left(\begin{array}{cc} \beta & -\alpha \\ \alpha & \beta \end{array} \right), 0, 0)$$

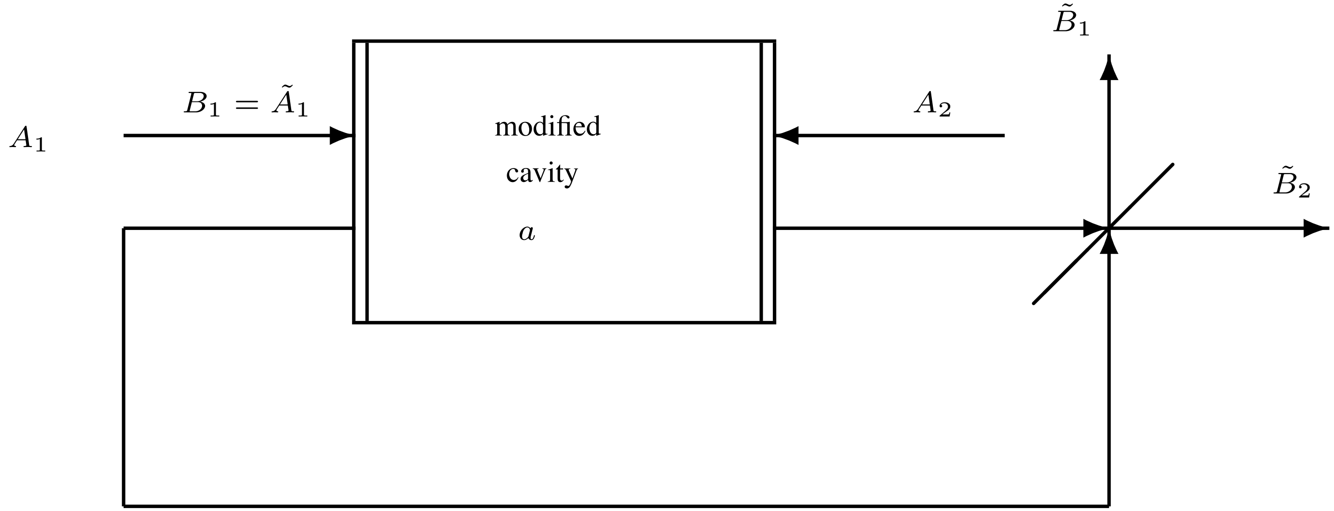
is a representation of the beamsplitter **S**.

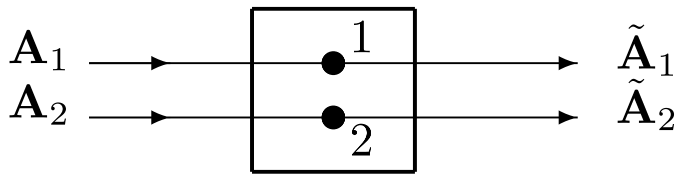
Network manipulations (try to pull beamsplitter through):

$$\mathbf{G} = (\mathbf{C} \boxplus \mathbf{N}) \triangleleft \mathbf{B} = \mathbf{B} \triangleleft (\mathbf{C}' \boxplus \mathbf{N}').$$

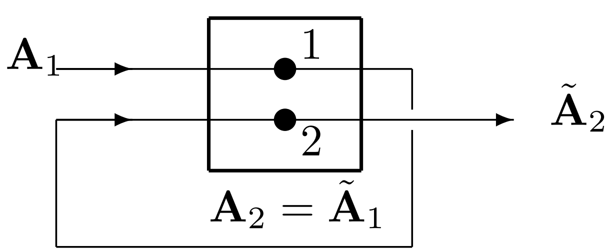
Here, the modified cavity is described by the subsystems

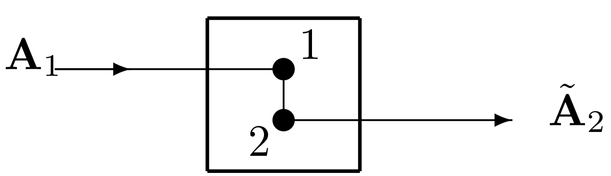
$$\mathbf{C}' = (1,\, \beta^* \sqrt{\gamma}\, a,\, \Delta a^* a), \quad \mathbf{N}' = (1,\, -\alpha^* \sqrt{\gamma}, 0).$$





(Principle of Series Connections) *The parameters* $\mathbf{G}_{2\leftarrow 1}$ *for the feedback system obtained from* $\mathbf{G}_1 \boxplus \mathbf{G}_2$ *when the output of the first subcomponent is fed into the input of the second is the series product* $\mathbf{G}_{2\leftarrow 1} = \mathbf{G}_2 \triangleleft \mathbf{G}_1$.





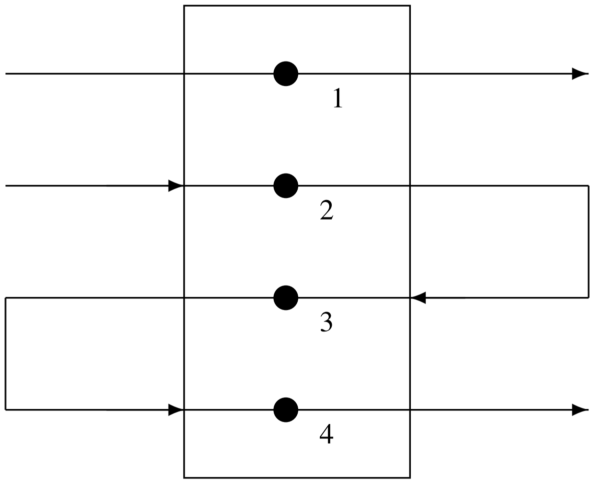
Reducible Networks

A *reducible quantum network* $\mathscr{N} = (\{\mathbf{G}_j\}, K, \{\mathbf{G}_j \triangleleft \mathbf{G}_k\})$ consists of

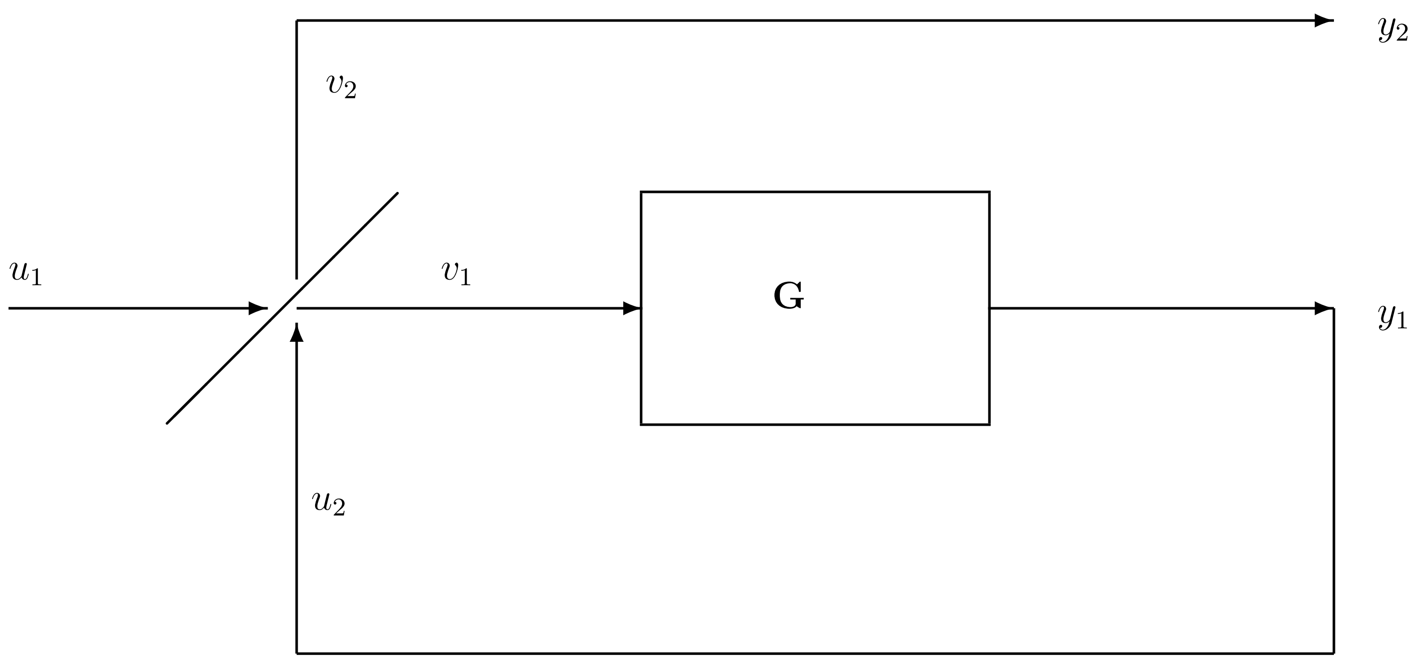
- A reducible decomposition $\mathbf{G} = \boxplus_j \mathbf{G}_j$, where $\mathbf{S} = \mathrm{diag}\{\mathbf{S}_1, \ldots, \mathbf{S}_n\}$,
- a direct interaction Hamiltonian K of the form

$$K = i \sum_k (N_k^* M_k - M_k^* N_k)$$

- a compatible list of field-mediated connections $\mathscr{L} = \{\mathbf{G}_j \triangleleft \mathbf{G}_k\}$ such that (i) the field dimensions of the members of each pair are the same, and (ii) each input and each output has at most one connection.



An example of a network that is *not reducible* [Yanagisawa-Kimura, 2003]



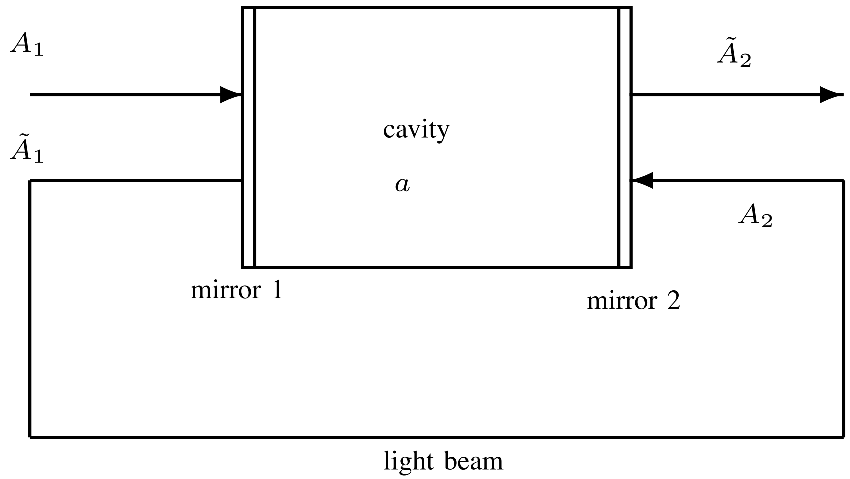
Original system $\mathbf{G} = (1, L, 0)$ with beamsplitter feedback leads to equivalent system obtained using linear fractional transformation:

$$\mathbf{G}_{eq} = (\alpha + \frac{\beta^2}{1+\alpha}, \ \frac{\beta}{1+\alpha}L, \ -\frac{\mathrm{Im}(\alpha)}{|1+\alpha|^2}L^*L)$$

Examples in Quantum Control

All-optical feedback

[Wiseman-Milburn, 1994]



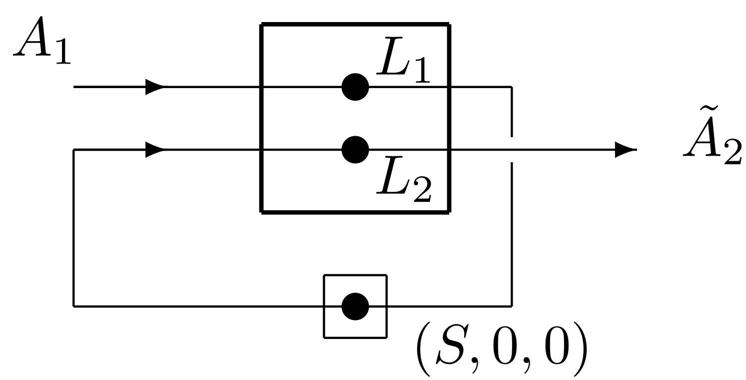
Before feedback, the cavity is described by

$$\mathbf{G} = (\mathbf{I}, \begin{pmatrix} L_1 \\ L_2 \end{pmatrix}, 0) = (1, L_1, 0) \boxplus (1, L_2, 0),$$

and $S = e^{i\theta}$ (phase shift).

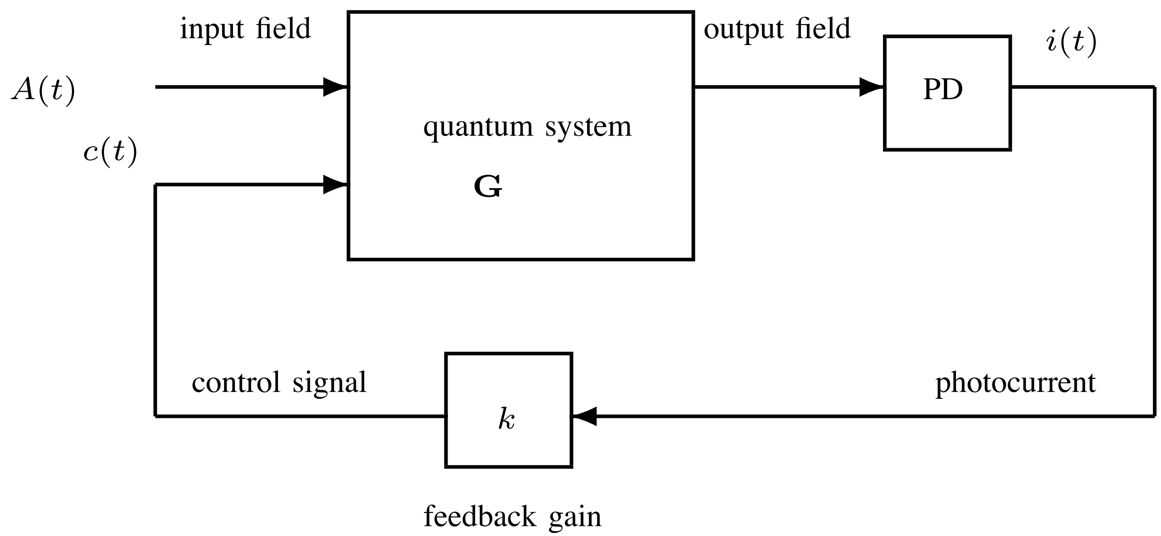
After feedback, we have

$$\begin{aligned}\mathbf{G}_{cl} &= (1, L_2, 0) \triangleleft (S, 0, 0) \triangleleft (1, L_1, 0) \\ &= (S, SL_1 + L_2, \frac{1}{2i}(L_2^* S L_1 - L_1^* S^* L_2)).\end{aligned}$$



Direct measurement feedback

[Wiseman, 1994]



Controlled Hamiltonian

$$H_0 + Fc$$

Before feedback, the quantum system is described by

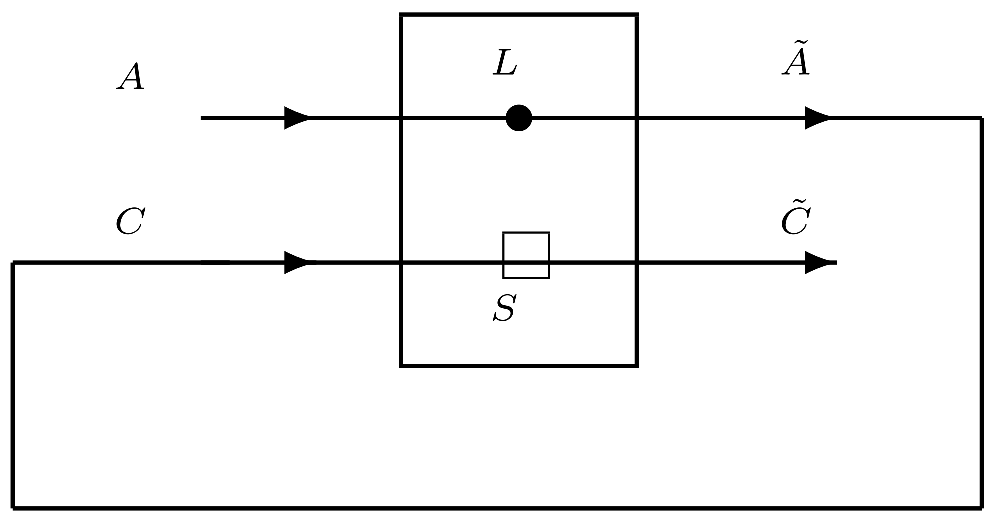
$$\mathbf{G} = (1, L, H_0) \boxplus (S, 0, 0)$$

where $S = e^{-iF}$ is unitary.

After feedback, we have

$$\mathbf{G}_{cl} = (S,0,0) \triangleleft (1,L,H_0) = (S, SL, H_0)$$

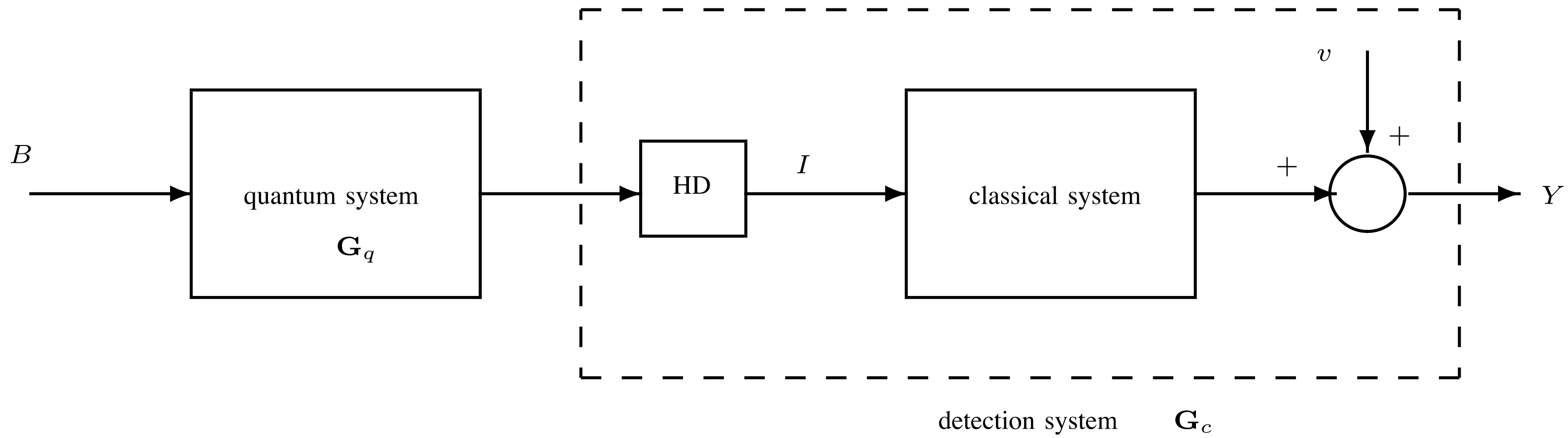
$$dX = (-i[X,H_0]+\mathcal{L}_{e^{-iF}L}(X))dt+[L^* e^{iF}, X]e^{-iF}dA+e^{iF}[X,e^{-iF}L]dA^*+(e^{iF}Xe^{-iF}-X)d\Lambda.$$



(can also do quadrature measurement)

Realistic detection

[Warszawski-Wiseman-Mabuchi, 2002]



The quantum system is given by

$$\mathbf{G}_q = (1, L_q, H_q),$$

and the classical detection system is given by the classical stochastic equations

$$\begin{aligned} dx(t) &= \tilde{f}(x(t))dt + g(x(t))dw(t), \\ dY(t) &= h(x(t))dt + dv(t), \end{aligned}$$

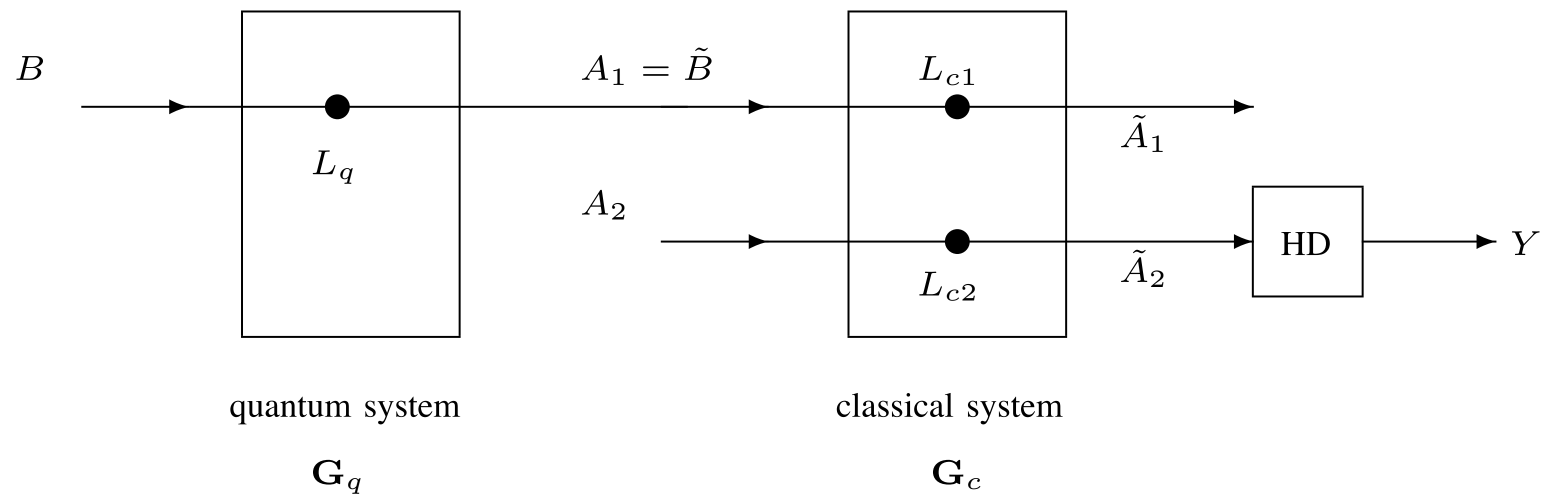
The classical system is equivalent to

$$\mathbf{G}_c = (1, L_{c1}, H_c) \boxplus (1, L_{c2}, 0)$$

where $L_{c1} = -ig^T p - \frac{1}{2}\nabla^T g$, $L_{c2} = \frac{1}{2}h$ and $H_c = \frac{1}{2}(f^T p + p^T f)$.

The complete cascade system is

$$\begin{aligned} \mathbf{G} &= ((1, L_{c1}, H_c) \lhd (1, L_q, H_q)) \boxplus (1, L_{c2}, 0) \\ &= (\mathbf{I}, \begin{pmatrix} L_1 + L_{c1} \\ L_{c2} \end{pmatrix}, H_q + H_c + \frac{1}{2i}(L_{c1}^* L_q - L_q^* L_{c1})) \end{aligned}$$



The unnormalized quantum filter for the cascade system is

$$
\begin{aligned}
d\sigma_t(X) &= \sigma_t(-i[X, H_q + H_c + \frac{1}{2i}(L_{c1}^* L_q - L_q^* L_{c1})] + \mathcal{L}_{\begin{pmatrix} L_1 + L_{c1} \\ L_{c2} \end{pmatrix}}(X))dt \\
&\quad + \sigma_t(L_{c2}^* X + X L_{c2})dy.
\end{aligned}
$$

For instance, $X = X_q \otimes \phi$, where ϕ is a smooth real valued function on $\mathbb{R}^n$.

Filtered estimate of quantum variables:

$$\pi_t(X_q) = \sigma_t(X_q)/\sigma_t(1)$$

Conclusion

- Concatenation and series products facilitate quantum network analysis and design (reducible networks).
- Very useful for quantum control
- Allows designers to focus more on systems, less on equations.
- Physical realization a key issue.
- Network paradigm powerful and likely to be helpful for quantum technology.