Quantum Feedback Networks

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Outline

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References:


Quantum Feedback

- **Open loop** - control actions are predetermined, no feedback is involved.
- **Closed loop** - control actions depend on information gained as the system is operating.
Types of Quantum Feedback:

- **Using measurement**

  The classical measurement results are used by the controller (e.g. classical electronics) to provide a classical control signal.
• Not using measurement

The controller is also a quantum system, and feedback involves a direct flow of quantum information.
Direct interactions

The controller is also a quantum system, and feedback may also include direct interactions.
Quantum Networks I

Quantum networks are ubiquitous

[quantum optics lab - E. Huntington, ADFA/UNSW]
[quantum computing network - (teleportation with loss detection)
- Knill, Laflamme, Milburn, 2001]
The complexity of quantum systems has been considered from a quantum information science perspective, as well as classical electrical and mechanical systems. Classical and quantum optical technologies, as well as artificial neural networks using equivalent circuits, and work out how to build or synthesise systems using realisation approaches.

[Optical laser model - Ralph, Harb, Bachor 1996]
Classical Electrical Networks

[circuit diagram of a classical electronic amplifier]
Modern circuit theory...

• builds on laws of physics (Maxwell, Faraday, Ohm,...)

• evolved to meet the needs of electrical system designers (Thevenin, Kirchhoff,...)

• includes
  - device models
  - rules for interconnection
  - methods for analysis, simplification, and synthesis
For example, Thevenin’s theorem helps engineers simplify complex linear circuits.
For example, realisation techniques help engineers build devices from given specifications (synthesis).
Quantum Networks II

Desirable attributes

• Capture the quantum physics
• Be capable of representing classical components
• Include dissipative mechanisms
  - noise, uncertainty, decoherence (open)
• Preserve canonical structure
  - e.g. commutation relations, energy
• Network of interconnected components should also be a quantum system
  - recursive
• Efficient methods for representation, interconnection, manipulation, and physical realization
• Efficient methods for simplification, analysis and synthesis
Figure of terconnection components coupling generalises (via models mathematical)

$\text{original network} \xrightarrow{} \text{parametric network representation} \xrightarrow{} \text{simplified parametric representation}$

[Representation and simplification of a quantum network]
Elementary network constructs:

Concatenation

\[
G = G_1 \sqcup G_2
\]

[Not parallel]
Series (cascade)

\[ G = G_2 \triangleleft G_1 \]

History:
Gardiner, 1993
Carmichael, 1993
Open systems:

Multichannel open quantum system characterised by parameters

- $H$ is a Hamiltonian (self-adjoint operator)
- $L$ is a vector of field coupling operators
- $S$ is a scattering matrix (self-adjoint matrix of operators)

Shorthand:

$$G = (S, L, H)$$

These parameters, together with field channels specifications, determine the *master equation*, or equivalently, the Heisenburg *quantum stochastic differential equations*.

[Gardiner-Collett 1985
Hudson-Parthasarathy 1984]
Concatenation product

\[ \textbf{G}_1 \boxplus \textbf{G}_2 = ( \begin{pmatrix} S_1 & 0 \\ 0 & S_2 \end{pmatrix}, \begin{pmatrix} L_1 \\ L_2 \end{pmatrix}, H_1 + H_2) \]

Series product

\[ \textbf{G}_2 \triangleleft \textbf{G}_1 = (S_2S_1, L_2 + S_2L_1, H_1 + H_2 + \frac{1}{2i}(L_2^\dagger S_2L_1 - L_1^\dagger S_2^\dagger L_2)) \]
Example: beamsplitter and cavity

\[
B_2 = \tilde{B}_2 = \tilde{A}_2
\]

\[
A_1
\]

\[
A_2
\]

\[
B_1 = \tilde{A}_1
\]

\[
\tilde{B}_1
\]

\[
\text{cavity}
\]

\[
a
\]

\[
d a(t) = (-\frac{\gamma}{2} + i\Delta)a(t)dt - \sqrt{\gamma}dB_1(t)
\]

\[
\tilde{A}_1(t) = \beta A_1(t) - \alpha A_2(t)
\]

\[
\tilde{A}_2(t) = \alpha A_1(t) + \beta A_2(t)
\]

\[
B_1(t) = \tilde{A}_1(t)
\]

\[
B_2(t) = \tilde{A}_2(t)
\]

\[
d\tilde{B}_1(t) = \sqrt{\gamma}a(t)dt + dB_1(t)
\]

\[
d\tilde{B}_2(t) = dB_2(t).
\]
Complete network

\[
G = (S, L, H) = \left(\left(\begin{array}{cc}
\beta & -\alpha \\
\alpha & \beta
\end{array}\right), \left(\begin{array}{c}
\sqrt{\gamma} a \\
0
\end{array}\right), \Delta a^* a\right)
\]
Description in terms of concatenation and series products:

\[
G = (C \boxplus N) \cdot B,
\]

where

\[
C = (1, \sqrt{\gamma}a, \Delta a^*a),
\]
describes the cavity,

\[
N = (1, 0, 0)
\]
is a trivial system (pass-through), and

\[
B = \begin{pmatrix}
\beta & -\alpha \\
\alpha & \beta
\end{pmatrix}, 0, 0
\]
is a representation of the beamsplitter \(S\).
Network manipulations (try to pull beamsplitter through):

\[ G = (C \boxplus N) \triangleleft B = B \triangleleft (C' \boxplus N') \].

Here, the modified cavity is described by the subsystems

\[ C' = (1, \beta^* \sqrt{\gamma} a, \Delta a^* a), \quad N' = (1, -\alpha^* \sqrt{\gamma}, 0). \]
(Principle of Series Connections) The parameters $G_{2-1}$ for the feedback system obtained from $G_1 \oplus G_2$ when the output of the first subcomponent is fed into the input of the second is the series product $G_{2-1} = G_2 \triangleleft G_1$. 

![Diagram](image-url)
Reducible Networks

A reducible quantum network \( \mathcal{N} = (\{G_j\}, K, \{G_j \triangleleft G_k\}) \) consists of

- A reducible decomposition \( G = \bigoplus_j G_j \), where \( S = \text{diag}\{S_1, \ldots, S_n\} \),
- a direct interaction Hamiltonian \( K \) of the form
  \[
  K = \frac{i}{2} \sum_k (N_k^* M_k - M_k^* N_k)
  \]
- a compatible list of field-mediated connections \( \mathcal{L} = \{G_j \triangleleft G_k\} \) such that (i) the field dimensions of the members of each pair are the same, and (ii) each input and each output has at most one connection.
An example of a network that is *not reducible* \cite{Yanagisawa-Kimura2003}

Notes on Linear Quantum Feedback Networks

Original system \( G = (1, L, 0) \) with beamsplitter feedback leads to equivalent system obtained using linear fractional transformation:

\[
G_{eq} = \left( \alpha + \frac{\beta^2}{1 + \alpha}, \frac{\beta}{1 + \alpha}L, -\frac{\text{Im}(\alpha)}{|1 + \alpha|^2}L^*L \right)
\]
Before feedback, the cavity is described by

$$G = (I, \begin{pmatrix} L_1 \\ L_2 \end{pmatrix}, 0) = (1, L_1, 0) \boxplus (1, L_2, 0),$$

and $S = e^{i\theta}$ (phase shift).
After feedback, we have

\[ G_{cl} = (1, L_2, 0) \bowtie (S, 0, 0) \bowtie (1, L_1, 0) \]
\[ = (S, SL_1 + L_2, \frac{1}{2i}(L_2^*SL_1 - L_1^*S^*L_2)). \]
Direct measurement feedback

Controlled Hamiltonian

\[ H_0 + Fc \]

Before feedback, the quantum system is described by

\[ \mathbf{G} = (1, L, H_0) \boxplus (S, 0, 0) \]

where \( S = e^{-iF} \) is unitary.
After feedback, we have

\[ G_{cl} = (S, 0, 0) \triangleleft (1, L, H_0) = (S, SL, H_0) \]

\[ dX = (-i[X, H_0] + \mathcal{L}_{e^{-iF_L}}(X))dt + [L^* e^{iF}, X] e^{-iF} dA + e^{iF} [X, e^{-iF} L] dA^* + (e^{iF} X e^{-iF} - X) d\Lambda. \]
Realistic detection

The quantum system is given by

$$\mathbf{G}_q = (1, L_q, H_q),$$

and the classical detection system is given by the classical stochastic equations

$$dx(t) = \tilde{f}(x(t))dt + g(x(t))dw(t),$$

$$dY(t) = h(x(t))dt + dv(t),$$

[Warszawski-Wiseman-Mabuchi, 2002]
The classical system is equivalent to

\[ \mathbf{G}_c = (1, L_{c1}, H_c) \boxdot (1, L_{c2}, 0) \]

where \( L_{c1} = -ig^T \rho - \frac{1}{2} \nabla^T g \), \( L_{c2} = \frac{1}{2} h \) and \( H_c = \frac{1}{2}(f^T \rho + p^T f) \).

The complete cascade system is

\[
\begin{align*}
\mathbf{G} &= ((1, L_{c1}, H_c) \triangleleft (1, L_q, H_q)) \boxdot (1, L_{c2}, 0) \\
&= (I, \begin{pmatrix} L_1 + L_{c1} \\ L_{c2} \end{pmatrix}, H_q + H_c + \frac{1}{2i}(L_{c1}^* L_q - L_q^* L_{c1}))
\end{align*}
\]
The unnormalized quantum filter for the cascade system is

\[ d\sigma_t(X) = \sigma_t(-i[X, H_q + H_c + \frac{1}{2i}(L_{c1}^*L_q - L_q^*L_{c1})] + \mathcal{L} \begin{pmatrix} L_1 + L_{c1} \\ L_{c2} \end{pmatrix} (X)) dt \]

\[ + \sigma_t(L_{c2}^*X + XL_{c2}) dy. \]

For instance, \( X = X_q \otimes \phi \), where \( \phi \) is a smooth real valued function on \( \mathbb{R}^n \).

Filtered estimate of quantum variables:

\[ \pi_t(X_q) = \sigma_t(X_q)/\sigma_t(1) \]
Conclusion

• Concatenation and series products facilitate quantum network analysis and design (reducible networks).
• Very useful for quantum control
• Allows designers to focus more on systems, less on equations.
• Physical realization a key issue.
• Network paradigm powerful and likely to be helpful for quantum technology.