Effects of Beamforming on the Connectivity of Ad Hoc Networks

Xiangyun Zhou, Haley M. Jones, Salman Durrani and Adele Scott
Department of Engineering, CECS
The Australian National University
Canberra ACT 0200, Australia

Email: Haley.Jones@anu.edu.au

***Abstract*— This paper analyzes the effects of beamforming on the connectivity of wireless ad hoc networks. We study different beamforming techniques using the uniform circular array as the antenna model. In particular, we study centre directed beamforming and greedy beamforming. In centre directed beamforming each node points its main beam toward the geometric centre of the network. The greedy beamforming method allows each node to choose the beamforming direction based on knowledge of other node positions. We investigate the connectivity of each beamforming scheme and compare their performances to that of omnidirectional antennas. The percentages of connection and isolated nodes are used as metrics for connectivity. We also show that greedy beamforming is robust against errors in node position information.**

***Index Terms*— ad hoc networks, antenna arrays, beamforming, connectivity.**

I. INTRODUCTION

Mobile wireless ad hoc networks are an important class of networks that are also known as infrastructureless mobile networks. While infrastructured networks, such as cellular networks, have fixed base stations and a wired core network connection, ad hoc networks do not have these features. In ad hoc networks all nodes have the ability to move rapidly and can be connected dynamically in an arbitrary manner. While cellular networks have a single hop connection between a mobile user and the base station, two nodes in an ad hoc network are usually connected by multiple hops which is similar to large scale computer networks.

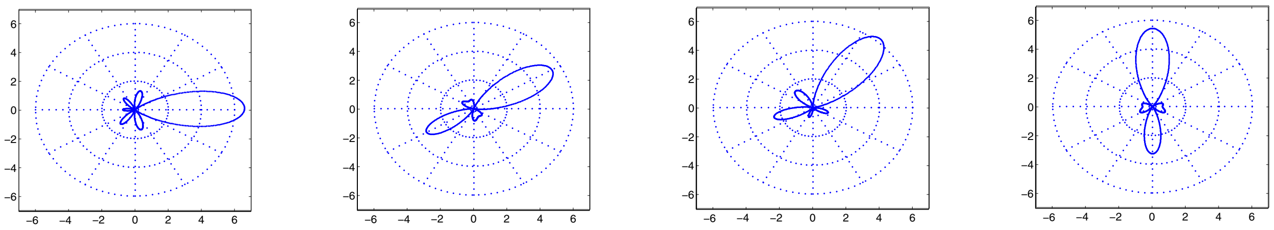
A common strategy in wireless ad hoc networks is to model the nodes using omnidirectional antennas. Recently, there has been an increasing interest in the use of directional antennas in ad hoc networks. Directional antennas have the ability to concentrate most of their radiated power towards a specific direction. Therefore they can provide larger transmission and reception ranges without increasing power usage. Many papers have investigated medium access techniques with directional antennas [1]. Work has also been done in the areas of neighbour discovery techniques [2], new routing protocols [1] and network capacity [3]. A complete ad hoc networking system including cohesive multilayer design was studied in [4].

Until recently, little attention has been paid to connectivity which is a fundamental property of ad hoc networks. Since the network is connected in a multihop manner, every single link between any two nodes contributes to the connectivity of the entire network. Some papers have investigated the relationship between connectivity and node transmission range based on omnidirectional antennas [5], [6]. It has been shown that beamforming using smart antennas can significantly improve the connectivity [7]. Different beamforming techniques have also been proposed for application to ad hoc networks. The use of randomized beamforming has been studied in [7]. The randomized beamforming technique allows each node in the network to direct its main beam in a direction from a uniform distribution on $[0, 2\pi)$. This simple technique does not require knowledge about location of neighbouring nodes, and it is shown to give significant improvement in the connectivity of ad hoc networks.

In this paper we propose two beamforming methods, one called *centre directed beamforming* and the other called *greedy beamforming*. In centre directed beamforming all nodes orientate their main beam towards the geometric centre of the network, assuming the location of the centre is known. Greedy beamforming allows each node to choose the direction of its main beam based on knowledge of the locations of other nodes, such that the maximum number of one hop connections for the node is achieved. In the greedy beamforming scheme, each node assumes that others are equipped with omnidirectional antennas with known locations and performs a simple calculation to decide the direction of the main beam which maximizes its local connectivity. We study the connectivity of an ad hoc network using both techniques. For greedy beamforming we consider the cases of nodes having perfect and imperfect knowledge of the positions of other nodes. We show that centre directed beamforming has certain advantages over random beamforming, and greedy beamforming outperforms both random and centre directed beamforming.

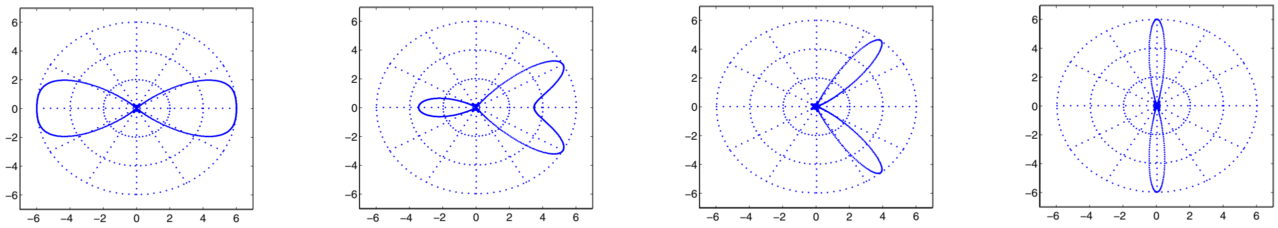
The rest of this paper is organized as follows. In Section II the antenna models used are presented. In Section III the network model for the wireless links and the metrics used to measure network connectivity are presented. In Section IV we investigate the characteristics of different beamforming techniques. In Section V we analyze the results on characteristics of different antenna models, and discuss the connectivity results. Finally, conclusions are drawn in Section VI.

1-4244-0741-9/07/$20.00 © 2007 IEEE



$(a)\ \phi_o = 0^o$ $(b)\ \phi_o = 30^o$ $(c)\ \phi_o = 50^o$ $(d)\ \phi_o = 90^o$

Fig. 1. Gain patterns of a UCA for different main beam angles with 6 antenna elements. ϕ_o is the direction of main beam



$(a)\ \phi_o = 0^o$ $(b)\ \phi_o = 30^o$ $(c)\ \phi_o = 50^o$ $(d)\ \phi_o = 90^o$

Fig. 2. Gain patterns of a ULA for different main beam angles with 6 antenna elements. ϕ_o is the direction of main beam

II. ANTENNA MODEL

A. Antenna Gain

Without loss of generality, we assume plane wave propagation. Thus only the far field of the antenna is important and the antenna can be treated as a point source. The angle from the x-axis in the xy-plane is $\phi \in [0, 2\pi]$, and the angle from z-axis is $\theta \in [0, \pi]$.

Assuming lossless antennas, the antenna gain is defined as [8]

$$g(\theta,\phi) = \frac{u(\theta,\phi)}{\frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} u(\theta,\phi)\,\sin(\theta)\,d\theta\,d\phi} \tag{1}$$

where $u(\theta,\phi)$ is the radiation intensity of the antenna in a given direction (θ,ϕ), defined as power per unit solid angle. The radiation intensity is related to power density, P_r, by [8]

$$u(\theta,\phi) = r^2 P_r(\theta,\phi) \tag{2}$$

where r is the radius of the observation sphere.

Antenna gain can also be written in terms of electric field strength, E. The relationship between the electric field and the power density is given by [8]

$$P_r(\theta,\phi) = \frac{1}{2}\frac{\mid E(\theta,\phi)\mid^2}{Z_o} \tag{3}$$

where Z_o is the intrinsic impedance of free space. Therefore we can express the antenna gain in terms of the electric field as

$$g(\theta,\phi) = \frac{\mid E(\theta,\phi)\mid^2}{\frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi}\mid E(\theta,\phi)\mid^2\,\sin(\theta)\,d\theta\,d\phi}. \tag{4}$$

An omnidirectional antenna has constant electric field in all directions, hence (4) gives $g(\theta,\phi) = 1,\ \forall(\theta,\phi)$.

B. Antenna Array

Two practical antenna models are considered, being the Uniform Circular Array (UCA) and the Uniform Linear Array (ULA). Beamforming is achieved by phase shifting each antenna element in the array such that its main beam points towards the desired direction.

Uniform circular arrays place the antenna elements on a circle with radius a. The electric field of a UCA using identical omnidirectional antennas can be expressed as [9]

$$E(\theta,\phi) = \sum_{n=1}^{N} E_o \exp[jka\sin(\theta)\cos(\phi-\phi_n) + j\alpha_n] \tag{5}$$

where N is the number of antenna elements in the array, E_o is the electric field pattern of the omnidirectional antennas, $k = \frac{2\pi}{\lambda}$, $\phi_o = \frac{2\pi n}{N}$, and α_n is the phase shift of the nth element. For the conventional cophasal excitation, [9]

$$\alpha_n = -ka\sin(\theta_o)\cos(\phi_o - \phi_n) \tag{6}$$

where (θ_o, ϕ_o) are the angles of the desired main beam. Substituting (5) into (4), we can calculate the antenna gain for any azimuthal angle, ϕ, for a UCA.

Uniform linear arrays place the antenna elements along a line, with a distance d between adjacent elements. The electric field of a ULA using identical omnidirectional antennas can be expressed as [8]

$$E(\phi) = \frac{\sin(\frac{n\psi(\phi)}{2})}{\sin(\frac{\psi(\phi)}{2})} \tag{7}$$

where $\psi(\phi)$ is the phase difference between adjacent elements in the direction ϕ, which is related to d by

$$\psi(\phi) = \frac{2\pi d}{\lambda}\cos(\phi) + \delta \tag{8}$$

where δ is the progressive phase shift of adjacent elements, due to the physical placements of the elements, and can be derived from the angle of the main beam as

$$\delta = -\frac{2\pi d}{\lambda}\cos(\phi_o). \tag{9}$$

Substituting (7) into (4), we can calculate the antenna gain for any azimuthal angle, ϕ, for a ULA.

In this paper, we only consider the azimuthal plane, by setting $\theta = \theta_o = \frac{\pi}{2}$. The gain pattern of a UCA with 6 antenna elements is shown in Fig. 1 for different directions of main beam. We observe that the maximum gain varies slightly about the number of antenna elements, N, as the direction of the main beam changes. But the 3dB width of the main beam is independent of the direction of the main beam. (The 3dB width is defined to be the span of gains in the main beam where the gains exceed half of the maximum gain). We also investigate the effect of increasing the number of antenna elements in the array. Results show that the maximum gain always stays around N, but the 3dB width of the main beam does not change with N.

The gain pattern of a ULA with 6 antenna elements is shown in Fig. 2 for different angles of main beam. The maximum gain is exactly equal to N, regardless of the direction of the main beam. Unlike the UCA, the 3dB beam width varies significantly as the angle of the main beam changes, reaching its maximum at $\phi = 0^o, 180^o$, and its minimum at $\phi = \pm 90^o$.

These gain results show that UCAs have several advantages over ULAs. UCAs are more likely to achieve a larger 3dB beam width than that achieved by ULAs when the angle of main beam is randomly chosen. The level of the side beams in a UCA is also stronger than that of a ULA [7]. Therefore, we use a UCA antenna model in our simulation.

III. CONNECTIVITY

A. Network Connection Model

We use the large scale path loss model to determine whether there is a connection between two given nodes. Following [5], [7], we assum that one node transmits a signal with power p_t, and another receives it with power, p_r. Hence the path loss, or signal power attenuation, in dB is given by [7], [10]

$$PL(\mathrm{dB}) = 10\log\frac{p_t}{p_r} = 10\log\frac{1}{g_t g_r}\left(\frac{s}{1\,\mathrm{m}}\right)^{\alpha} \tag{10}$$

where g_t is the antenna gain of the transmitting node in the direction of transmission, g_r is the antenna gain of the receiving node in the direction of reception, s is the distance between the two nodes, and α is the path loss exponent which ranges from 2.7 to 3.5 in an urban outdoor environment [10]. We also define a threshold path loss PL_o, i.e. two nodes can establish connection if the path loss of the signal transmission between them is smaller than or equal to PL_o.

B. Connectivity Metrics

One measure of the level of connectivity is the path probability, or P(path). It is defined as the probability of two randomly chosen nodes in a random ad hoc network topology being connected either via a single hop or multihop path. It is calculated as the statistical average of percentage of connected node pairs as [7]

$$\begin{aligned} P(\text{path}) &= E\left\{\frac{\#\ \text{connected node pairs}}{\#\ \text{node pairs}}\right\} \\ &= E\left\{\frac{\sum_{i=1}^{\nu}\frac{1}{2}n_i(n_i-1)}{\frac{1}{2}n(n-1)}\right\} \end{aligned} \tag{11}$$

where # denotes "number of", n_i is the number of nodes in the ith subnetwork, n is the total number of nodes in the entire network, and ν is the number of subnetworks. A subnetwork is a group of nodes which are interconnected with each other but isolated from all other nodes in the network.

Another important metric for connectivity is probability of isolation, or P(isolation). It is defined as the probability that a randomly chosen node does not have any connections to other nodes. It is calculated as the statistical average of percentage of isolated nodes in a random ad hoc network topology as

$$P(\text{isolation}) = E\left\{\frac{\#\ \text{isolated nodes}}{\#\ \text{node}}\right\}. \tag{12}$$

We will use these metrics to compare the connectivity performance of various beamforming techniques, in Section V.

IV. BEAMFORMING

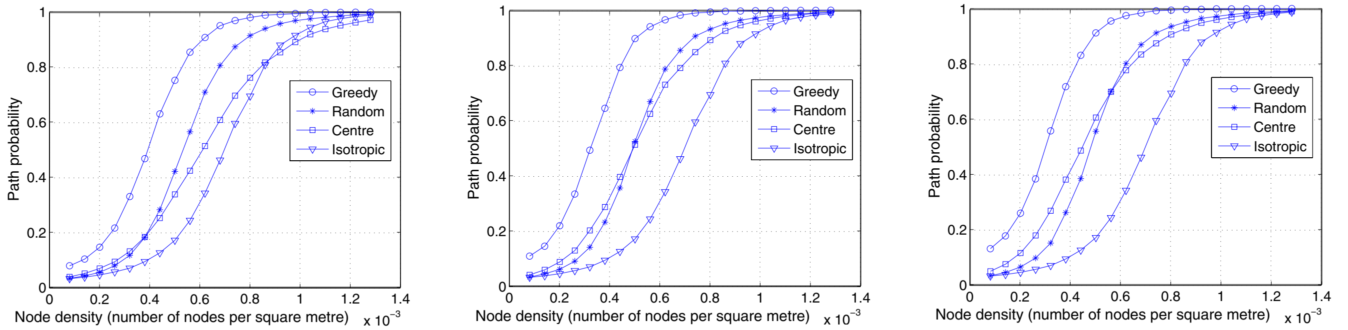
We investigate three different beamforming techniques. These are random beamforming, centre directed beamforming and greedy beamforming.

In the centre directed beamforming scenario, every node directs its main beam towards the geometric centre of the network. We assume that the centre position is known by all nodes. For nodes near the centre, connection can be easily established if any two nodes on opposite sides of the centre are located within the width of each other's main beam. We say two nodes are facing each other if their main beams are pointing towards each other or, more precisely, if they are located within the width of each other's main beam. Therefore we expect that the nodes near the centre have high probability of being connected together via their main beams. The size of the high connectivity area is proportional to the gains of the main beam. However, nodes far from the network centre have no chance of facing each other, resulting in poor connectivity.

On the other hand, the probability of two nodes facing each other in the random beamforming scenario is independent of the distance from the centre. Therefore, the connectivity performance of random beamforming is almost the same throughout the network. The border effect is a minor drawback of random beamforming [7]: nodes near the border of the network may happen to steer their main beams outside the network area. Hence their main beams become useless to the network, and they may become isolated. Both the centre beamforming and random beamforming techniques have low complexity as they do not require knowledge of node positions in the network.

In the greedy beamforming scheme each node chooses the direction of its main beam based on its knowledge of the locations of other nodes. This beamforming scheme also

1-4244-0741-9/07/$20.00 © 2007 IEEE



(a) number of antenna elements $N = 4$ (b) number of antenna elements $N = 8$ (c) number of antenna elements $N = 12$

Fig. 3. Path probability P(path) of uniformly distributed nodes on 250000 m^2 rectangular area with path loss exponent $\alpha = 3$ and threshold path loss $PL_o = 50$dB for different beamforming schemes.

has relatively low complexity, as each node does not require knowledge of the actual gain patterns of its neighbouring nodes. Its goal is to achieve maximum local connectivity. Local connectivity (i.e. the number of neighbours) is an important characteristic of a node in an ad hoc network. Nodes in ad hoc networks are very likely to fail, be switched off or suffer from a depleted battery [5], and a direct link between two nodes in ad hoc networks is often unstable because of node mobility. Therefore a high level of local connectivity can provide robustness against node failure and stable connectivity in the network. However, a high level of local connectivity may also introduce a large amount of interference between neighbouring nodes. This problem can be minimized by careful channel planning and good medium access control. Therefore, greedy beamforming is suitable for ad hoc networks.

However, the assumption of perfect positioning in the greedy beamforming scheme is an ideal case. For example, a large scale wireless sensor network is deployed for environmental monitoring purposes. Each sensor is distributed to a specified position but thrown from an aircraft. Therefore its position is close to the specified position with some small error compared to the area of the entire network. Imperfect positioning also occurs in ad hoc networks which are already connected. In a connected network, each node may periodically update its positioning information of all known nodes and pass it to neighbouring nodes. Due to the high mobility of ad hoc networks, some positioning information received by neighbours after some time interval will be incorrect. The position error is generally small if nodes are close to each other and not moving too quickly or randomly. On the other hand, the error may turn out to be significant if the time between the initial position identification of one node and the reception of this information by another node is too long, which usually happens when the nodes are far apart. Additionally, hardware technology constraints on antenna beamforming may introduce errors in beam direction. This effectively results in small position errors for nodes nearby and large errors for nodes that are further away. Therefore, we also study the effects of imperfect positioning on network connectivity.

V. RESULTS

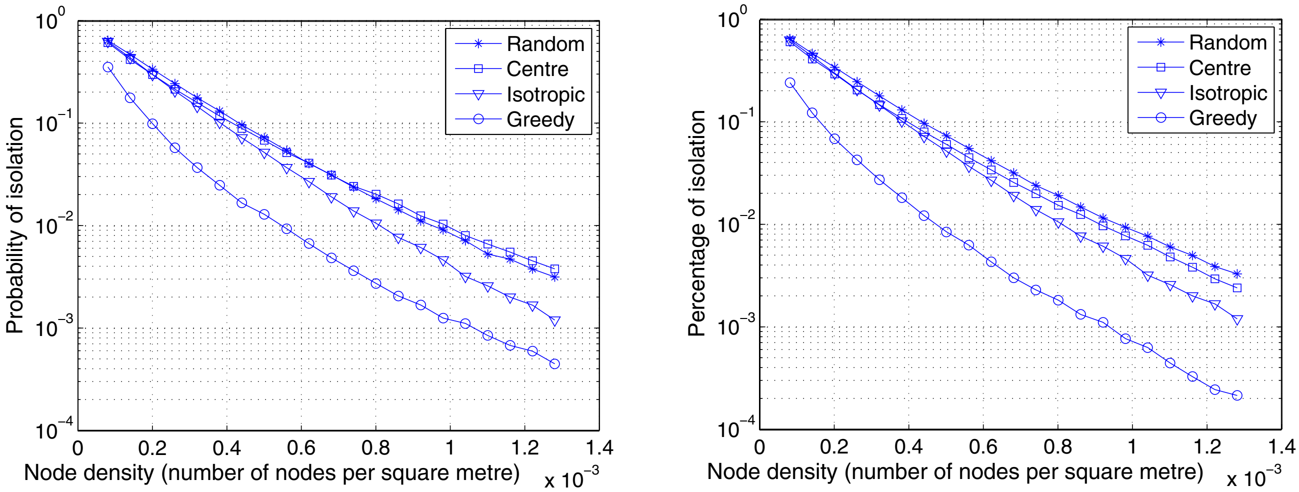
A. Path Probability

Our simulations were carried out in Matlab. In the simulation we distribute nodes uniformly, at random, on a square with side length 500 m. The threshold path loss PL_o is fixed at 50 dB as a commonly used value in ad hoc networks [7], and the path loss exponent α is chosen to be 3 as a typical value in an urban outdoor environment.

Fig. 3 shows the simulation results for path probability, P(path), for different beamforming techniques and omnidirectional antennas. We can see that the overall connectivity achieved by beamforming is much higher than for an isotropic antenna when the node density is not too high. This is because beamforming increases the transmission and reception range of each node. As a result, connections can be established over long distances. At high node densities, P(path) in all scenarios converges to unity because the distances between nodes are very small. It is not surprising that greedy beamforming outperforms the other two beamforming schemes to a large extent and, hence, results in the highest connectivity. This shows that a high level of local connectivity can achieve a high level of global connectivity.

It can also been seen that, with beamforming, P(path) increases as the number of antenna elements, N increases. We have already shown that the maximum gain of the main beam is approximately equal to N, and the width of main beam is independent of N. As N increases, the main beam of each node can reach a greater distance from the node, while its width stays the same. Therefore the main beam can cover a large area, which results in improvement in connectivity.

The comparison between random beamforming and centre directed beamforming in Fig. 3 shows some interesting results. Centre directed beamforming outperforms random beamforming at low node densities while random beamforming performs better at high node densities. There is also a trend that centre beamforming overtakes random beamforming as N increases. At low node densities, the utilization of main beams is crucial for network connectivity, since nodes are usually far apart. Although a number of node pairs in a random beamforming network are facing each other, connection may not be estab-



(a) number of antenna elements $N = 4$

(b) number of antenna elements $N = 8$

Fig. 4. Probability of isolation P(isolation) of uniformly distributed nodes on 250000 m^2 rectangular area with path loss exponent $\alpha = 3$ and threshold path loss $PL_o = 50$dB for different beamforming schemes.

lished if the distances between node pairs are too large. For established connections, most are local connections, which cannot contribute to the global connectivity. In the centre directed beamforming scenario, higher connectivity still exists near the centre provided the number of nodes in this area is not too small. Therefore, centre directed beamforming generally outperforms random beamforming at low node densities.

As the node density increases, side beams become more and more important. Connection between two closely located nodes can be formed by a main beam and a side beam. This effect can significantly increase the connectivity in the random beamforming scenario. In centre directed beamforming networks the size of the high connectivity area does not change with node density. Additionally the connectivity in areas far from the centre remains at a low level until connection between node pairs can be easily formed by side beams, which requires high node densities.

We have seen that the gains in the main beam increases linearly with the number of antenna elements N. This results in a significant increase in the size of the high connectivity area in centre beamforming networks. This has a direct improvement effect on global connectivity. Although random beamforming also benefits from an increase in gain, the improvement on the global connectivity is less significant than that with centre beamforming. Hence, centre directed beamforming outperforms random beamforming for large N, and low to moderate node density.

B. Probability of Isolation

Isolated nodes cannot communicate with other nodes and are thus useless for the connectivity of the network. Fig. 4 shows simulated isolation results for greedy, random, and centre beamforming methods, as well as for the omnidirectional scenario. Clearly greedy beamforming significantly reduces P(isolation), while random and centre beamforming increase P(isolation). This is in agreement with earlier results in [7]. The difference from the isotropic case is most noticeable at high node densities. We observe that, with beamforming, P(isolation) reduces as the number of antenna elements N increases. We also notice that centre directed beamforming outperforms random beamforming for large N (e.g. $N = 8$).

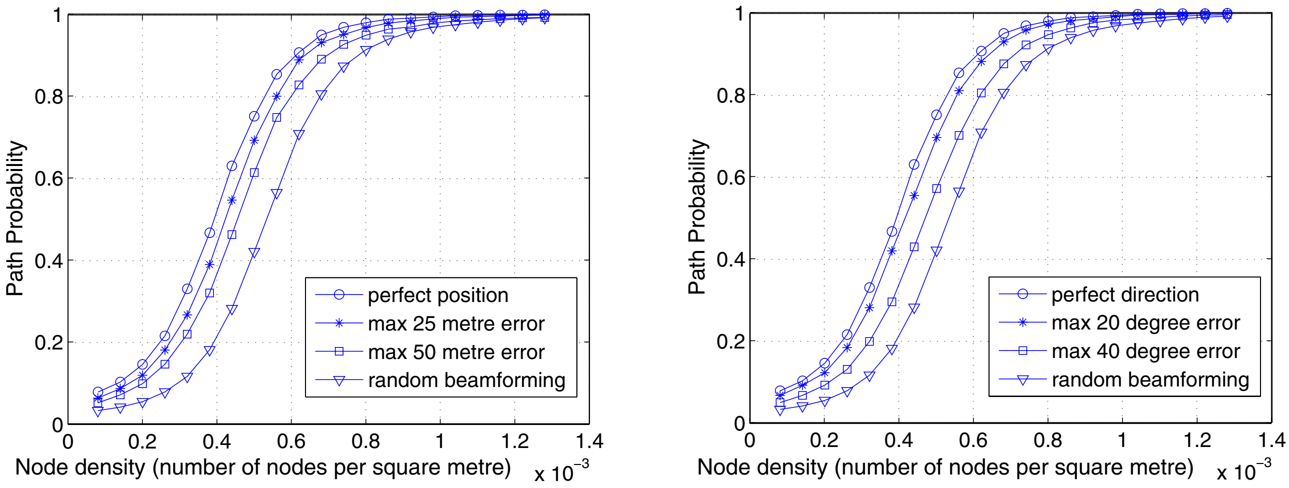
The results can be explained by the above described properties of each beamforming network. In greedy beamforming, each node tries to maximize the number of connected neighbours, hence isolation rarely happens. Nodes near the border of a random beamforming network suffer from the border effect, and hence have high probability of becoming isolated. In centre directed beamforming, nodes located far from the centre of the network have no chance of facing each other, thus they have high probability of being isolated as well.

C. Greedy Beamforming with Imperfect Positioning

We have already shown that greedy beamforming with accurate knowledge of node positions significantly improves network connectivity, at the same time reducing the percentage of isolated nodes. However, the assumption of perfect positioning is generally impractical. Thus, we also investigate the performance of greedy beamforming on connectivity with imperfect positioning.

Firstly, we introduce position error with uniform distribution to each node. Figure 5(a) shows the simulation results on connectivity for greedy beamforming. We include the random beamforming scenario as a reference. The level of connectivity decreases as position error gets larger. It is found that greedy beamforming with relatively large error (e.g. maximum of ±50 m) still outperforms random beamforming.

We also introduce error in the direction of the main beam. The direction error (in degrees) is uniformly distributed with specified maximum degree error. Figure 5(b) shows the simulation results for connectivity for greedy beamforming. We also include the random beamforming scenario as a reference. As previously, the connectivity decreases as the direction error



(a) number of antenna elements $N = 4$ (b) number of antenna elements $N = 4$

Fig. 5. Path probability P(path) of greedy beamforming with uniformly distributed nodes on 250000 m^2 rectangular area with pathloss exponent $\alpha = 3$ and threshold pathloss $PL_o = 50$dB. The positioning errors introduced in (a) are uniform distributed random error expressed in metres. The errors introduced in (b) are direction errors with uniform distribution. The random beamforming scenario is included as a reference.

increases. Compared to random beamforming, the connectivity in a greedy beamforming network is higher even for a large degree of error (e.g. maximum of 40 degree error).

Figure 5 shows the results for $N = 4$. Similar trends are observed for $N = 8$. In both scenarios (i.e. error in position and error in beam direction), greedy beamforming is robust against positioning error. It outperforms the other beamforming techniques provided the positioning error is not too large. Due to imperfect knowledge of node positions, some connections to neighbouring nodes fail. Hence the level of local connectivity decreases, also resulting in a decrease in global connectivity. However, greedy beamforming can maintain a reasonable level of local connectivity, enabling the entire network to remain at a high level of connectivity.

VI. CONCLUSION

In this paper, we have investigated the effects of different beamforming techniques on ad hoc networks connectivity. Both random beamforming and centre directed beamforming do not require knowledge of node positions, hence they have low complexity. Comparing the two schemes, centre directed beamforming performs better at low node densities while random beamforming performs better at high node densities. As the number of antenna elements increases, centre directed beamforming tends to outperform random beamforming at high node densities as well. However, these two beamforming techniques have a negative effect on network connectivity as they tend to increase the percentage of isolated nodes.

The application of greedy beamforming further increases the network connectivity to a large extent. Unlike the two other beamforming methods, it significantly reduces the percentage of isolated nodes. We also show that greedy beamforming is robust against quite significant errors in node position information. Thus it is a very practical beamforming technique. The complexity is still relatively low as nodes do not require knowledge of gain patterns of other nodes.

The results are based on large scale path loss channel model, which does not account for small scale fading. Further work could investigate connectivity in fading channels.

REFERENCES

[1] P. Mohapatra and S. V. Krishnamurthy, *Ad Hoc Networks: Technologies and Protocols*. Springer, 2005.

[2] M. E. Steenstrup, “Neighbor discovery among mobile nodes equipped with smart antennas,” in *Scandinavian Workshop on Wireless Ad-hoc Networks*, May 2003.

[3] A. Spyropoulos and C. S. Raghavendra, “Capacity bounds for ad-hoc networks using directional antennas,” in *IEEE ICC*, May 2003.

[4] R. Ramanathan, J. Redi, C. Santivanez, D. Wiggins, and S. Polit, “Ad hoc networking with directional antennas: a complete system solution,” *IEEE Journal on Selected Areas in Communications*, vol. 23, pp. 496–506, March 2003.

[5] C. Bettstetter, “On the connectivity of ad hoc networks,” *The Computer Journal*, vol. 47, pp. 462–447, July 2004.

[6] P. Santi and D. M. Blough, “The critical transmitting range for connectivity in sparse wireless ad hoc networks,” *IEEE Trans. Mobile Computing*, vol. 2, pp. 25–39, March 2003.

[7] C. Bettstetter, C. Hartmann, and C. Moser, “How does randomized beamforming improve the connectivity of ad hoc networks?” *IEEE International Conference on Communications*, vol. 5, pp. 3380–3385, May 2005.

[8] J. D. Kraus, *Antennas*. McGraw-Hill Book, 1950.

[9] M. T. Ma, *Theory and Application of Antenna Arrays*. John Wiley and Sons, 1974.

[10] T. S. Rappaport, *Wireless Communications: Principles and Practice*. Prentice Hall, 2002.