ELECTROMAGNETISM SUMMARY

➤ Transmission line transformers
➤ Skin depth
➤ Matching circuits
➤ Noise
➤ Link Budgets
Rules for Transmission Line Transformers

➤ Always wind the windings in multifilar fashion.

➤ Can use either toroidal or linear or whatever shaped ferrites. Toroidal ferrites are usually the best.

➤ The dot on the transformer diagram points to one end of the wires at one end of the transformer.

➤ The voltage drop across all windings must be same. **WHY?**

➤ The currents in the same direction in the windings must sum to zero. **WHY?**

➤ Respects phase delays along the transmission line when doing its sums?
Transmission Line Transformer 180° Hybrid

![Diagram of a transmission line transformer with 180° Hybrid configuration. The diagram shows two input voltages V1 and V2, two input currents I1 and I2, and two output voltages V+ and V-. The transformer has a central V symbol indicating the phase shift.]
The Linear Phase Shift Combiner.
The Magic-T (Wilkinson)
The Magic-T Analysis

\[ I_2 + I_4 + I_3 = 0 \]  
\[ \text{but} \ I_3 = 0 \]

\[ I_1 + I_4 = I_2 = -I_4 \Rightarrow I_4 = -I_1/2 \]

\[ V/2 = I_2 Z_L \Rightarrow \frac{V}{I_2} = 2Z_L \Rightarrow \frac{V}{I_1} = Z_L = 75\Omega \]

\[ V/2 = -I_4 Z_L \Rightarrow \frac{V}{I_4} = -2Z_L \Rightarrow \frac{V}{I_2} = 2Z_L \]
Skin Depth

Electromagnetic waves, j, E, B, ... only penetrate a distance $\delta$ into a metal. Check the magnitude of $\delta$ in lab and web exercises.

The wave equation for metals simplifies to...

$$\frac{\partial^2 E_y(z)}{\partial z^2} = j\omega \sigma \mu_0 E_y(z)$$

The solution...

$$E_y(z) = \exp\left(-\frac{1}{\delta} + j \frac{\delta}{j} z\right)$$

where $\delta$ the **skin depth** is given by...

$$\delta = \sqrt{\frac{2}{\omega \sigma \mu_0}}$$
Skin Depth

Incident electric field, $E$

Metal-air interface

$j(0) = \sigma E(0)$

Metal

$\delta$

$z$
Impedance per Square

- By integrating the formula for the electric field inside a metal,
  \[ E_y(z) = \exp\left( -\frac{1 + \frac{j}{\delta}}{z} \right) \]
  to find the current per unit width \( I_s \) we defined the impedance per square as
  \[ Z_s = \frac{E_y(0)}{I_s} = \frac{1 + \frac{j}{\sigma\delta}}{\sigma} = \sqrt{\frac{\pi\mu_0 f}{\sigma}} (1 + j) \]

- For a wire of radius, \( a \), length \( L \) and circumference \( 2\pi a \), we obtain
  \[ Z = \frac{L}{2\pi a} Z_s \]
Use Q to Design Matching Networks

➤ The formula for \( Q \) depends on whether we imagine the \( R \) to be in series with or in parallel with the reactance. Just an issue of convenience.

➤ \( R \) in series with \( X \), then \( Q = \frac{X}{R_s} \)

➤ \( R \) in parallel with \( X \), then \( Q = \frac{R_p}{X} \)

➤ Notice that \( R_s \) is not the same as \( R_p \) but they are related (an exercise).
Analysis of T and Pi Networks

➤ Choose Q.

➤ Consider the T or Pi network to be a pair of back to back L networks.

➤ The virtual resistance in a Pi network must be smaller than those on the source and load.
PI Matching Example

Match 75Ω source to the 1 kΩ / 100 nH load with a Q of 10.
PI Matching Example

Split into two halves with $R_V$ in the middle.
Example Noise Power Calculation.

➤ Consider the following receiver chain which is typical of that in a wireless receiver.

➤ The noise figure of the mixer and filter (both passive devices with the given insertion losses) is 11dB.

➤ Find the overall noise figure of the receiver
Example Noise Power Calculation. (Contd)

➤ The noise factor of the amplifier is 2 ($=10\log_{10}(3)$).

➤ The noise figure of the mixer and filter is 11 dB and so the noise factor is 12.6 ($=10\log_{10}(11)$). Thus,

\[ F_{TOT} = F_1 + \frac{F_2 - 1}{G_1} = 2 + \frac{12.6 - 1}{10} = 3.16. \]

➤ Finally we obtain

\[ F_{TOT} = 10\log_{10}(3.16) = 5\, dB. \]
Receiver Noise Calculations

➤ The thermal noise added to a signal when passing through a system is given by,

\[ N_o = k_B T B \]

➤ In dBm

\[ N_o = 10 \log_{10} \left( \frac{k_B T B}{1 \times 10^{-3}} \right) \]

➤ If \( N_o \) and the NF are known, then the required input signal level for a given output SNR can be calculated,

\[ S_i = NF + N_o + SNR_o \]
Specifying Phase Noise

➤ Common to specify phase noise as,

\[ S_c(f) = \frac{S_N(\Delta f)/2}{\text{Carrier Power}} \]

where \( S_N(f) = V_o^2 S_{\Delta \theta}(f) \) and the carrier power = \( V_o^2 \).

➤ The factor of 2 dividing the P.S.D. arises because we only consider one sideband in the definition of \( S_c(f) \).

➤ \( S_c(f) \) has the units of dBc/Hz.

\[ S_c(f) = S_{\Delta \theta}(f)/2 : \quad S_c(f)(\text{dB}) = 10 \log_{10} (S_c(f)) \]
Specifying Phase Noise

Figure 4. Single-sideband phase noise representation
Spectrum Analyser Revision

➤ LO Sweep generator is mixed with incoming signal
➤ IF signal is passed through two filters.
➤ *IF filter*: Resolution Bandwidth.
➤ *DC filter*: Video Bandwidth.
➤ Thus be wary when measuring the phase noise with a spectrum analyser.
Effective Aperture of a Dish Antenna

- Imagine a planar light beam illuminating a round hole on a black screen at normal incidence.

- The Rayleigh condition for a diffraction limited aperture describes the angle of expansion of the beam on exit from the hole.

\[ \Delta \theta_B = \frac{4 \lambda}{\pi d} \]

where \( \lambda \) is the wavelength, \( \Delta \theta_B \) is the opening angle of the beam and \( d \) is the diameter of the aperture.
Antenna Aperture: Useful to compute received power.

- The effective aperture of any antenna is given by:

\[ A_e = \frac{G\lambda^2}{4\pi} \]

where \( \lambda \) is the wavelength, \( G \) is the antenna gain.

- Effective aperture only depends on antenna gain and the wavelength of operation.

- E.G. A low gain monopole tuned to 3 MHz has an aperture

\[ A_e = \frac{G\lambda^2}{4\pi} \approx \frac{100^2}{4\pi} = 800m^2 \]
Antenna Aperture

If an antenna is oriented for maximum signal and correctly tuned to the load, it will intercept a maximum signal power equal to:

\[ P = S_i A_e \]

where \( S_i \) is the incident power flux density (Watts per \( m^2 \)) and \( A_e \) is the antenna effective aperture.

An antenna absorbs half this power into a matched load and reradiates (scatters) the other half.
The Friis Transmission Formula

- We know how to calculate the power radiated by an antenna, the maximum flux density of an antenna from its gain and the power intercepted by an antenna from $A_e$

- If we assume that the antennas are aligned for maximum transmission and reception, then in free space,

$$P_r = \frac{G_t A_r P_t}{4\pi r^2}$$

where $A_r$ is the receiving aperture of the receiving antenna.

- Since $A_r = G_r \lambda^2 / (4\pi)$

$$P_r = G_t G_r P_t \left[ \frac{\lambda}{4\pi r} \right]^2$$
Antennas (Cont.): Antenna Noise

♣ Random noise comes from the sky: E.G. The cosmic radiation background at $3^\circ K$.

♣ Black body radiation $\Rightarrow$ it must be there at finite temperature even in a vacuum!

♣ This noise can be picked up by antennas. In a receiver it adds to the noise of the receiver electronics.

♣ PSD = $N_o = KT$ where $K = 1.38 \times 10^{-23} J/^{\circ}K$ and $T$ is the absolute temperature. Thus the noise power is

$$P_n = kTB$$

♣ Such noise picked up by the antenna leads to the definition of antenna temperature.
Link Budget: Friis transmission

The Friis transmission formula describes e.m. propagation between line of sight antennas:

\[ P_r = P_t \frac{G_1 G_2 \lambda^2}{(4\pi r)^2} \]

where \( P_t \) and \( P_r \) are the transmit and received powers, \( G(= G_1, G_2) \) is the gains of the antennas at each end of the link, \( r \) is the distance between the antennas and \( \lambda \) the wavelength.

Note in particular the dB with respect to 1 mW.. dBm

\[ P(dBm) = 10 \log_{10} \frac{P(Watts)}{.001} \]
Link Budgets

Consider communications between a dish and an arbitrary antenna each matched to a pair of signal generators.