More on transformers: LC resonator filters

- Transistors
- Transistor S-parameters and Y-parameters
- Transistors and transformers in solve.
LC (Helical) Resonator Filters

- Back to back electrostatically coupled transformers.
- Make excellent narrowband filters.
LC resonator filter design technique

➤ We design in the same way as Pi and T matching networks.
➤ A semi intuitive design procedure.
➤ Divide the circuit into two halves through the coupling capacitor $C_c$.
➤ Consider matching the source (or load) $Z_o$ to a *virtual resistance*, $R'$.
LC resonator filter design technique

➤ Absorb the coupling capacitor $C_c$ into the tuning capacitor $C_T$.

$$R = R' + \frac{1}{\omega^2 C_c^2 R'} C_c \rightarrow \frac{C_c}{1 + (\omega C_c R')^2}$$

➤ We can now simplify to the following,
LC resonator filters: Things to notice...

- We now have a matching network which matches $Z_o$ to $R$.
- The coupling capacitor $C_c$ has been eliminated from the design flow. This means that we will not be able to choose it mathematically. Instead, we choose it \textit{physically} using MATLAB.
- $C_c$ nonetheless plays an important role in the circuit, for without it, no signal can pass from one side of the circuit to the other.
- It should be clear that for a \textbf{narrowband} filter we are going to design a \textbf{high Q} L-matching network.
LC resonator filter design technique using autotransformer theory

- We have a matching network to match \( Z_o \) to \( R \).
- We do not know \( R \). Instead we choose \( Q \).
- Specifying \( Q \) is the **only** step in the process!
- First we must model the autotransformer.
- Represent the autotransformer as the T-circuit equivalent of inductors...
The Autotransformer

- An autotransformer is a special transformer in which there is only one turn. The “primary” is formed by a tap on the “secondary”.
- In an autotransformer $V_1 = \nu V_2$, where $V_1$ is the primary tap voltage and $V_2$ is the voltage across the whole transformer. (the secondary!).
- $\nu$ is the ratio of the tap turn number to the total number of turns on the transformer.
- The mutual inductance, $M = \nu L$, where $L$ is the total inductance of the transformer and $L_1 = \nu^2 L$. 
Proof of these relations for the Autotransformer

➤ The equations of a general transformer are given by,

\[ V_1 = j\omega L_1 i_1 + j\omega M i_2 \]

\[ V_2 = j\omega M i_1 + j\omega L_2 i_2 \]

➤ The autotransformer and its transformer equivalent:
Proof of these relations for the Autotransformer

- Multiply the second equation by $\nu$.

\[ V_1 = j\omega L_1 i_1 + j\omega M i_2 \]

\[ \nu V_2 = V_1 = j\omega \nu M i_1 + j\omega \nu L_2 i_2 \]

- If we use $V_1 = \nu V_2$ to replace $V_2$, then we have two identical equations in $V_1$.

- Equate the coefficients of $i_1$ and $i_2$ and we obtain, $L_1 = \nu M$ and $M = \nu L_2$.

- Finally substitute $L$ for $L_2$ (the total inductance) and then $M = \nu L$ and $L_1 = \nu M = \nu^2 L$. -Q.E.D.
Proof of these relations for the Autotransformer

➤ We can replace the transformer T-circuit as follows.
Back to the LC resonator

The LC resonator now has the following simple form,
Back to the LC resonator

- Can still simplify this.
- In order to get a high Q circuit, the input impedance of the input to the transformer must be much lower than \( Z_o \), hence

\[
Z_o \gg \omega \nu (1 - \nu) L
\]
Finally express the left leg parallel combination of $Z_o$ and $\nu L$ as a series combination,
The LC resonator

Now apply the matching technique for L-type matching networks.

The only difference is that because we do not know \( R \) we must choose \( Q \) instead. A very convenient situation!

\[
R = \frac{(\omega \nu L)^2}{Z_o} \left( 1 + Q^2 \right) \approx \frac{(\omega \nu LQ)^2}{Z_o}
\]

provided \( Q \gg 1 \). And

\[
Q = \omega C_T R = \frac{Z_o \omega L}{(\omega \nu L)^2} = \frac{Z_o}{\omega L \nu^2}
\]
The LC resonator

If we express the left leg as a parallel rather than a series combination we obtain,

\[ R = \frac{Z_o}{\nu^2} \quad \text{and} \quad \omega^2 LC_T = 1 \]
The LC resonator design procedure

➤ Guess $L$. Say $L = 1\mu H$.

➤ Compute $C_T = 1/\omega^2/L$

➤ Choose $Q$.

➤ Compute $\nu$

$$\nu = \sqrt{\frac{Z_0}{\omega Q L}}$$

➤ $\nu$ gives us the tap location, $L_1 = \nu^2 L$, the mutual inductance, $M = \nu L$.

➤ Also $L_2 = L$. 
Example

- Design a narrowband filter for 50 MHz.
- Let $Q = 100$ and $L=100$ nH.
- Choose a value of $C_c$
Choice of coupling capacitor

Loose coupling

Tight coupling
A Simpler Narrowband Circuit for low Q

Consider the following

\[ L_T \quad C_C \quad L_T \quad Z_o \quad C_T \quad Z_o \]
A Simpler Narrowband Circuit for low Q

➤ Divide in half as usual

\[ L_T \quad 2C_C \quad Z_o \quad C_T \quad R' \]

\[ L_T \quad Z_o \quad C_T \quad C \quad R \]
A Simpler Narrowband Circuit for low Q

We can show that

\[ C = \frac{2C_c}{1 + (2\omega C_c R)^2} \]

and

\[ R = Z_o \left( 1 + Q^2 \right) \]

\[ C + C_T = \frac{Q}{\omega R}, \quad \text{and} \quad L_T = \frac{QZ_o}{\omega} \]
A Simpler Narrowband Circuit for low Q

➤ Once again $C_c$ is not in the design flow so we have to guess it while using MATLAB to improve the performance.

➤ Note that if we change the centre frequency $\omega$ to $\omega/r$, then we can scale the circuit components as follows,

$$L_T = rL_T, \ C_T = rC_T \text{ and } C_c = rC_c.$$ 

➤ Thus to do a design for a different frequency just change $r$. 