AUSTRALIAN NATIONAL UNIVERSITY
Department of Engineering

ENGN 3226 Digital Communications
Problem Set #8 Block Codes

Q1

Consider a (6,3) linear block code defined by the generator matrix

$$\vec{G} = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

(a) Determine if the code is a Hamming code. Find the parity check matrix $\vec{H}$ of the code in systematic form.
(b) Find the encoding table for the linear block code.
(c) What is the minimum distance d_{min} of the code. How many errors can the code detect. How many errors can the code correct.
(d) Draw the hardware encoder diagram.
(e) Find the decoding table for the linear block code.
(f) Draw the hardware syndrome generator diagram.
(g) Suppose $\vec{c} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix}$ is sent and $\vec{r} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$ is received. Show how the code can correct this error.

Q2

Consider a (7,4) linear block code defined by the generator matrix

$$\vec{G} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

(a) Determine if the code is a Hamming code. Find the parity check matrix $\vec{H}$ of the code in systematic form.
(b) Find the encoding table for the linear block code.
(c) What is the minimum distance d_{min} of the code. How many errors can the code detect. How many errors can the code correct.
(d) Draw the hardware encoder diagram.
(e) Find the decoding table for the linear block code.
(f) Draw the hardware syndrome generator diagram.
(g) Suppose $\vec{c} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$ is sent and $\vec{r} = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$ is received. Show how the code can correct this error.

Q3

Consider a (5,1) linear block code defined by the generator matrix

$$\vec{G} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

(a) Find the parity check matrix $\vec{H}$ of the code in systematic form.
(b) Find the encoding table for the linear block code.
(c) What is the minimum distance d_{min} of the code. How many errors can the code detect. How many errors can the code correct.
(d) Draw the hardware encoder diagram.
(e) Find the decoding table for the linear block code (consider single bit errors only).
(f) Draw the hardware syndrome generator diagram.
(g) Suppose $\vec{c} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix}$ is sent and $\vec{r} = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \end{bmatrix}$ is received. Show how the code can correct this error.

Q4

Consider the generator polynomial for a (7,3) cyclic code defined by

$$g(p) = p^4 + p^3 + p^2 + 1$$

(a) Find the encoding table for the cyclic code.
(b) What is the minimum distance d_{min} of the code.

Q5

Consider the generator polynomial for a (7,4) cyclic code defined by

$$g(p) = p^3 + p^2 + 1$$

(a) Find the encoding table for the cyclic code.
(b) What is the minimum distance d_{min} of the code.
(c) Find the systematic output codeword for input $\overrightarrow{c} = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}$.

AUSTRALIAN NATIONAL UNIVERSITY
Department of Engineering

ENGN 3226 Digital Communications
Problem Set #8 Solution

Q1: Complete Solution

(a)

Testing for hamming code, we have

$$\begin{aligned} m &= n-k = 6-3 = 3 \\ k &= 2^m - m - 1 = 2^3 - 3 - 1 = 4 \neq 3 \\ n &= 2^m - 1 = 2^3 - 1 = 7 \neq 6 \end{aligned}$$

Hence $(6,3)$ is not a Hamming code.

We have

$$\begin{aligned}
\overrightarrow{G} &= \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix} \\
\overrightarrow{P} &= \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \\
\overrightarrow{P}^T &= \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \\
\overrightarrow{I}_3 &= \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \\
\overrightarrow{H} &= [\overrightarrow{P}^T \vdots \overrightarrow{I}_{n-k}] \\
\overrightarrow{H} &= \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}$$

(b)

The encoding table for $(6,3)$ linear block code is

Message	Code word	Weight of code word
000	000000	0
001	001101	3
010	010011	3
011	011110	4
100	100110	3
101	101011	4
110	110101	4
111	111000	3

This is calculated as follows

$$\begin{aligned}
\vec{c}_0 = \vec{m}_0\vec{G} &= \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\
\vec{c}_1 = \vec{m}_1\vec{G} &= \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix} \quad \text{(3rd row of } \vec{G}) \\
\vec{c}_2 = \vec{m}_2\vec{G} &= \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 1 \end{bmatrix} \quad \text{(2nd row of } \vec{G}) \\
\vec{c}_3 = \vec{m}_3\vec{G} &= \begin{bmatrix} 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix} \quad \text{(2nd row of } \vec{G} \text{ + 3rd row of } \vec{G}\text{)} \\
\vec{c}_4 = \vec{m}_4\vec{G} &= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \end{bmatrix} \quad \text{(1st row of } \vec{G}) \\
\vec{c}_5 = \vec{m}_5\vec{G} &= \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} \quad \text{(1st row of } \vec{G} \text{ + 3rd row of } \vec{G}\text{)} \\
\vec{c}_6 = \vec{m}_6\vec{G} &= \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 1 \end{bmatrix} \quad \text{(1st row of } \vec{G} \text{ + 2nd row of } \vec{G}\text{)} \\
\vec{c}_7 = \vec{m}_7\vec{G} &= \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix} \quad \text{(1st row of } \vec{G} \text{ + 2nd row of } \vec{G} \text{ + 3rd row of } \vec{G})
\end{aligned}$$

(c)

From encoding table, we have

$$\begin{aligned}
d_{min} &= 3 \\
e &= d_{min} - 1 = 2 \\
t &\leq \left\lfloor \frac{1}{2}(d_{min} - 1) \right\rfloor \leq 1
\end{aligned}$$

Hence the $(6,3)$ linear block code can detect 2 bit errors and correct 1 bit error in 6 bit output codeword.

(d)

The output for general code word is

$$\vec{c} = \vec{m}\vec{G} = \begin{bmatrix} m_1 & m_2 & m_3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} m_1 & m_2 & m_3 & m_1+m_3 & m_1+m_2 & m_2+m_3 \end{bmatrix}$$

The hardware encoder implementation is

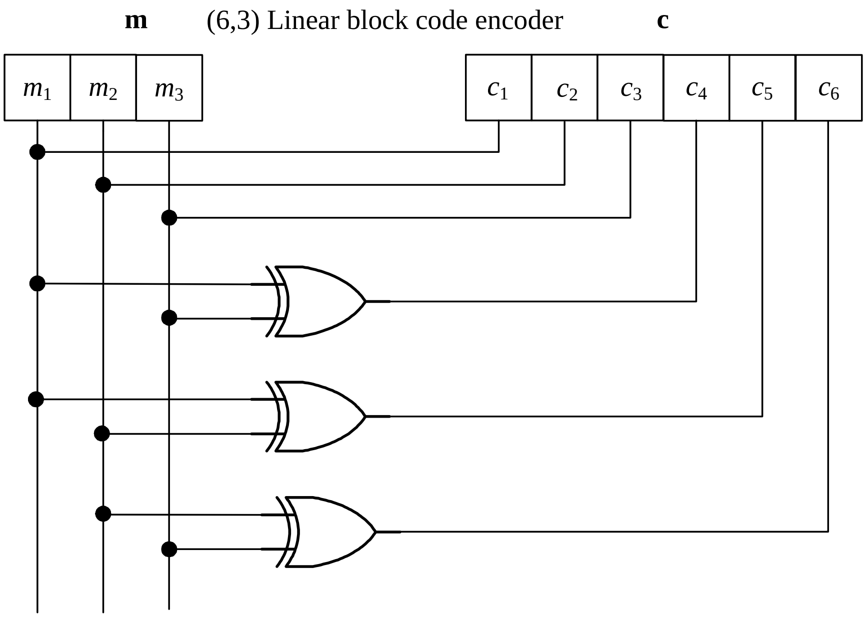

Figure 1: Figure for Question 1 (d).

(e)

We have

$$\vec{H} = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$\vec{H}^T = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The decoding table is

Error Pattern	Syndrome	Comment
000000	000	all 0's
100000	110	1st row of $\vec{H}^T$
010000	011	2nd row of $\vec{H}^T$
001000	101	3rd row of $\vec{H}^T$
000100	100	4th row of $\vec{H}^T$
000010	010	5th row of $\vec{H}^T$
000001	001	6th row of $\vec{H}^T$

(f)

The syndrome for general received word is

$$\begin{aligned}\vec{s} = \vec{r}\vec{H}^T &= \begin{bmatrix} r_1 & r_2 & r_3 & r_4 & r_5 & r_6 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} r_1+r_3+r_4 & r_1+r_2+r_5 & r_2+r_3+r_6 \end{bmatrix}\end{aligned}$$

The hardware syndrome generator implementation is

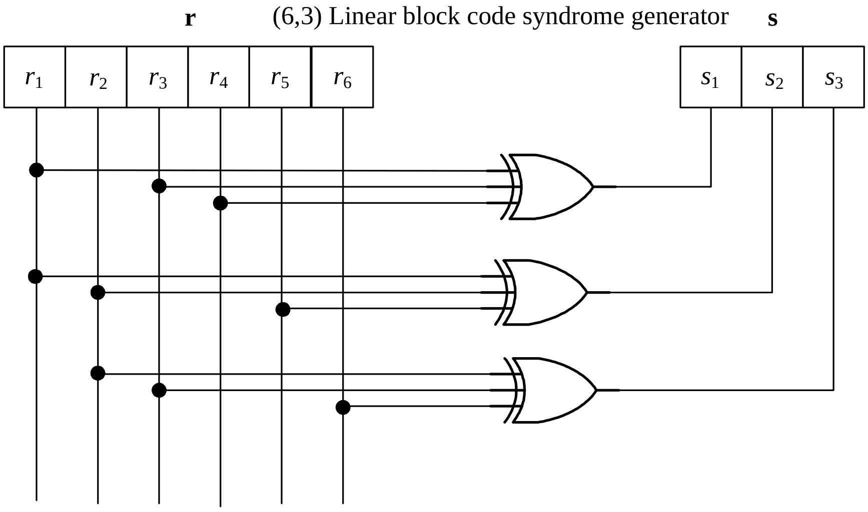

Figure 2: Figure for Question 1 (h).

(g)

Given that $\vec{c} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix}$ is sent and $\vec{r} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$ is received.

$$\begin{aligned}\vec{s} &= \vec{r}\vec{H}^T = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}\end{aligned}$$

From decoding table, this syndrome corresponds to error pattern $\vec{e} = [000001]$. Hence the corrected code word is

$$\begin{aligned}\vec{y} &= \vec{r} + \vec{e} \\ &= \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix}\end{aligned}$$

Q2: Partial Solution

(a)

Testing for hamming code, we have

$$\begin{aligned} m &= n-k = 7-4 = 3 \\ k &= 2^m - m - 1 = 2^3 - 3 - 1 = 4 \\ n &= 2^m - 1 = 2^3 - 1 = 7 \end{aligned}$$

Hence $(7,4)$ is a Hamming code.

We have

$$\begin{aligned} \overrightarrow{G} &= [\overrightarrow{I}_k \vdots \overrightarrow{P}] \\ \overrightarrow{G} &= \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix} \\ \overrightarrow{H} &= [\overrightarrow{P}^T \vdots \overrightarrow{I}_{n-k}] \\ \overrightarrow{H} &= \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

(b)

The encoding table for $(7,4)$ linear block code is

Message	Code word	Weight of code word
0000	0000000	0
0001	0001101	3
0010	0010111	4
0011	0011010	3
0100	0100011	3
0101	0101110	4
0110	0110100	3
0111	0111001	4
1000	1000110	3
1001	1001011	4
1010	1010001	3
1011	1011100	4
1100	1100101	4
1101	1101000	3
1110	1110010	4
1111	1111111	7

(c)

From encoding table, we have

$$\begin{aligned} d_{min} &= 3 \\ e &= d_{min} - 1 = 2 \\ t &\leq \left\lfloor \frac{1}{2}(d_{min} - 1) \right\rfloor \leq 1 \end{aligned}$$

Hence the $(7,4)$ linear block code can detect 2 bit errors and correct 1 bit error in 7 bit output codeword.

(d)

The output for general code word is

$$\vec{c} = \vec{m}\vec{G} = \begin{bmatrix} m_1 & m_2 & m_3 & m_4 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} m_1 & m_2 & m_3 & m_4 & m_1+m_3+m_4 & m_1+m_2+m_3 & m_2+m_3+m_4 \end{bmatrix}$$

The hardware encoder implementation is

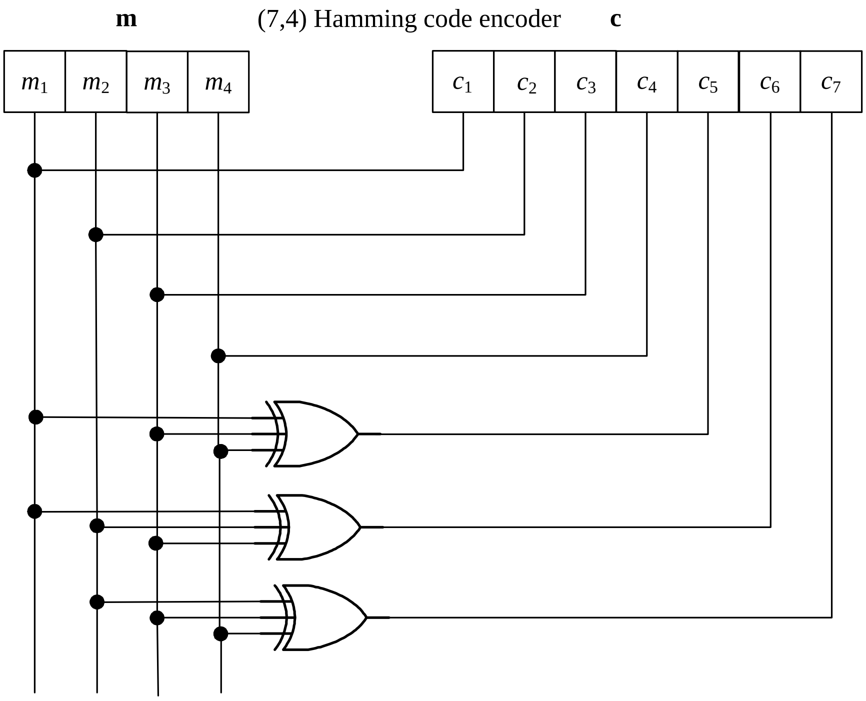

Figure 3: Figure for Question 2 (d).

(e)

We have

$$\vec{H}^T = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The decoding table is

Error Pattern	Syndrome
0000000	000
1000000	110
0100000	011
0010000	111
0001000	101
0000100	100
0000010	010
0000001	001

(f)

The syndrome for general received word is

$$\vec{s} = \vec{r}\vec{H}^T = \begin{bmatrix} r_1 & r_2 & r_3 & r_4 & r_5 & r_6 & r_7 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} r_1+r_3+r_4+r_5 & r_1+r_2+r_3+r_6 & r_2+r_3+r_4+r_7 \end{bmatrix}$$

The hardware syndrome generator implementation is

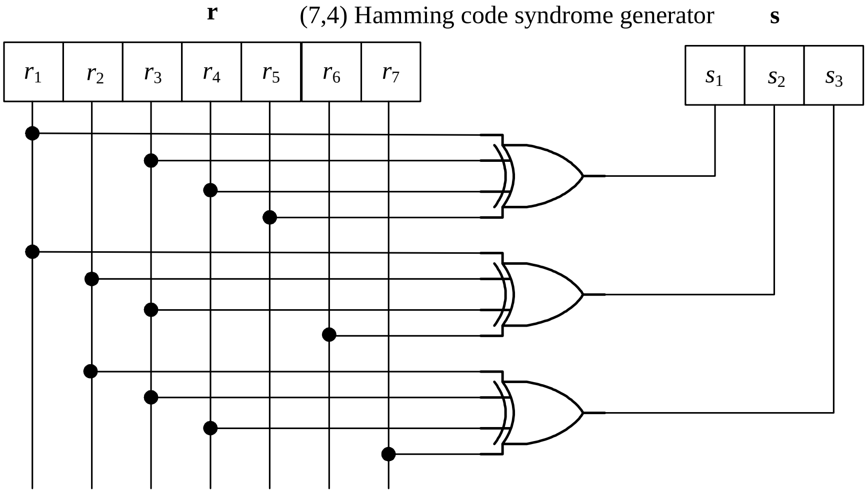

Figure 4: Figure for Question 2 (h).

(g)

Given that $\vec{c} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$ is sent and $\vec{r} = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$ is received.

$$\vec{s} = \vec{r}\vec{H}^T = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 1 \end{bmatrix}$$

From decoding table, this syndrome corresponds to error pattern $\vec{e} = [0100000]$. Hence the corrected code word is

$$\begin{aligned} \vec{y} &= \vec{r} + \vec{e} \\ &= \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} \end{aligned}$$

See also `Lecture 19, Example 1`.

Q3: Solution

(a)

$$\begin{aligned}
\overrightarrow{G} &= [\overrightarrow{I}_k \vdots \overrightarrow{P}] \\
\overrightarrow{G} &= \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix} \\
\overrightarrow{H} &= [\overrightarrow{P}^T \vdots \overrightarrow{I}_{n-k}] \\
\overrightarrow{H} &= \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}$$

(b)

The encoding table for $(5,1)$ linear block code is

Message	Code word	Weight of code word
0	00000	0
1	11111	5

(c)

From encoding table, we have

$$\begin{aligned}
d_{min} &= 5 \\
e &= d_{min} - 1 = 4 \\
t &\leq \left\lfloor \frac{1}{2}(d_{min} - 1) \right\rfloor \leq 2
\end{aligned}$$

Hence the $(5,1)$ linear block code can detect 4 bit errors and correct 2 bit errors in 5 bit output codeword.

(d)

The output for general code word is

$$\begin{aligned}
\overrightarrow{c} = \overrightarrow{m}\overrightarrow{G} &= \begin{bmatrix} m_1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix} \\
&= \begin{bmatrix} m_1 & m_1 & m_1 & m_1 & m_1 \end{bmatrix}
\end{aligned}$$

The hardware encoder implementation is

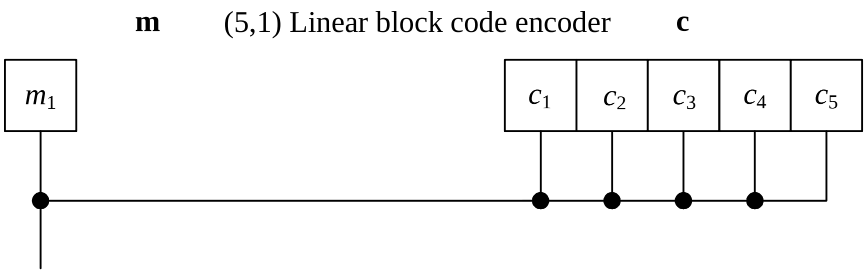

Figure 5: Figure for Question 3 (d).

(e)

We have

$$\vec{H}^T = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The decoding table is

Error Pattern	Syndrome
00000	0000
10000	1111
01000	1000
00100	0100
00010	0010
00001	0001

(f)

The syndrome for general received word is

$$\vec{s} = \vec{r}\vec{H}^T = \begin{bmatrix} r_1 & r_2 & r_3 & r_4 & r_5 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} r_1+r_2 & r_1+r_3 & r_1+r_4 & r_1+r_5 \end{bmatrix}$$

The hardware syndrome generator implementation is

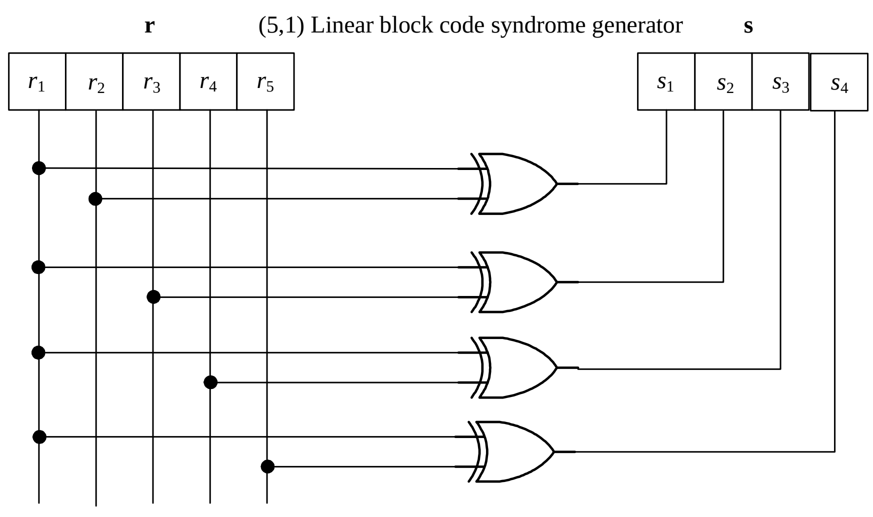

Figure 6: Figure for Question 3 (h).

(g)

Given that $\overrightarrow{c} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix}$ is sent and $\overrightarrow{r} = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \end{bmatrix}$ is received.

$$\begin{aligned} \overrightarrow{s} = \overrightarrow{r}\overrightarrow{H}^T &= \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \end{aligned}$$

From decoding table, this syndrome corresponds to error pattern $\overrightarrow{e} = [1000]$. Hence the corrected code word is

$$\begin{aligned} \overrightarrow{y} &= \overrightarrow{r} + \overrightarrow{e} \\ &= \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix} \end{aligned}$$

Q4: Complete Solution

(a)

Given that the generator polynomial for a (7,3) cyclic code is

$$g(p) = p^4 + p^3 + p^2 + 1$$

The output code words are given by

$$c(p) = M(p)g(p)$$

Tabulating the results

Input	$M(p)$	$c(p) = M(p)g(p)$	Code word	Weight of code word
000	0	0	0000000	0
001	1	$p^4+p^3+p^2+1$	0011101	4
010	p	$p^5+p^4+p^3+p$	0111010	4
011	$p+1$	p^5+p^2+p+1	0100111	4
100	p^2	$p^6+p^5+p^4+p^3$	1111000	4
101	p^2+1	$p^6+p^5+p^3+1$	1101001	4
110	p^2+p	$p^6+p^3+p^2+p$	1001110	4
111	p^2+p+1	p^6+p^4+p+1	1010011	4

Note: XOR addition is used here, e.g.

$$\begin{aligned} (p^2+1)(p^4+p^3+p^2+1) &= p^6+p^5+p^4+p^2+p^4+p^3+p^2+1 \\ &= p^6+p^5+(1+1)p^4+p^3+(1+1)p^2+1 \\ &= p^6+p^5+(0)p^4+p^3+(0)p^2+1 \\ &= p^6+p^5+p^3+1 \end{aligned}$$

(b)

$d_{min} = 4$.

Q5: Partial Solution

(a)

See Lecture 19, Slide 18 for solution.

(b)

$d_{min} = 3$.

(c)

Given that $\overrightarrow{c} = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}$.

Given that the generator polynomial for a (7,4) cyclic code is

$$g(p) = p^3 + p^2 + 1$$

The systematic output code word is

$$\begin{aligned}
p^{n-k} &= p^3 \\
M(p) &= p^3 + p^2 + p + 1 \\
p^{n-k}M(p) &= (p^3)(p^3 + p^2 + p + 1) = p^6 + p^5 + p^4 + p^3 \\
\frac{p^{n-k}M(p)}{g(p)} &= \frac{p^6 + p^5 + p^4 + p^3}{p^3 + p^2 + 1} \\
&= p^3 + p + 1 + \frac{p^2 + p + 1}{p^3 + p^2 + 1} \\
b(p) &= p^2 + p + 1 \\
c(p) &= p^{n-k}M(p) + b(p) \\
&= p^6 + p^5 + p^4 + p^3 + p^2 + p + 1 \\
\overrightarrow{c} &= \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}
\end{aligned}$$

See Lecture 19, Example 3 for detailed steps.