AUSTRALIAN NATIONAL UNIVERSITY
Department of Engineering

ENGN 3227 Analogue Electronics
Problem Set #06 Active Filters

Q1

Consider the circuit shown in Figure 1.
Assume $R_1 = 1$ kΩ, $R_2 = 2$ kΩ, $C_1 = 1$ μF.
(a) Find the s-domain transfer function of the circuit in standard form.
(b) Write the set of MATLAB commands (4 lines expected) to obtain the Bode plot.

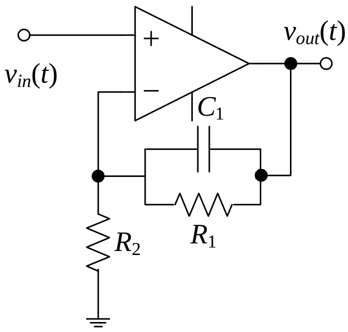

Figure 1: The circuit for Question 1.

Q2

Consider the circuit shown in Figure 2.
Assume $R_1 = 1$ kΩ, $C_1 = 1$ μF, $C_2 = 10$ μF.
(a) Find the s-domain transfer function of the circuit in standard form.
(b) Write the set of MATLAB commands (4 lines expected) to obtain the Bode plot.

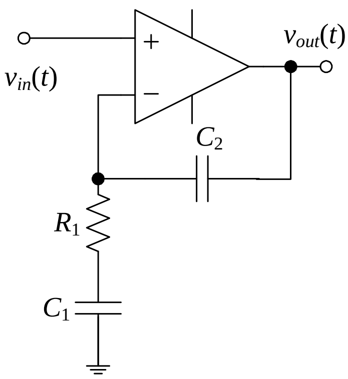

Figure 2: The circuit for Question 2.

Q3

Consider the Sallen-Key low-pass filter circuit shown in Figure 3.
Find the s-domain transfer function of the circuit in standard form.

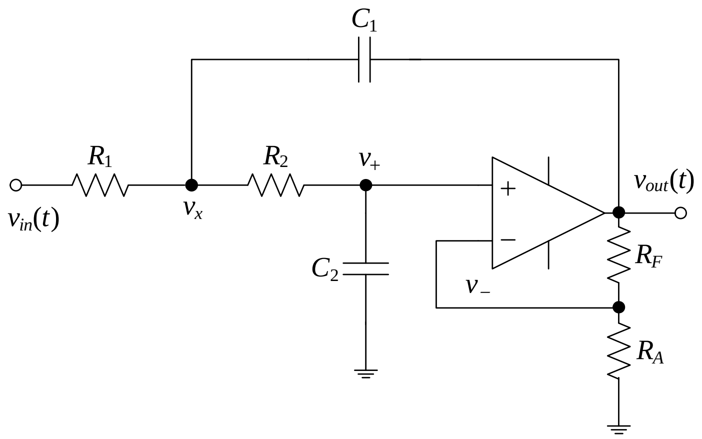

Figure 3: The circuit for Question 3.

Q4

Consider the Sallen-Key high-pass filter circuit shown in Figure 4.
Find the s-domain transfer function of the circuit in standard form.

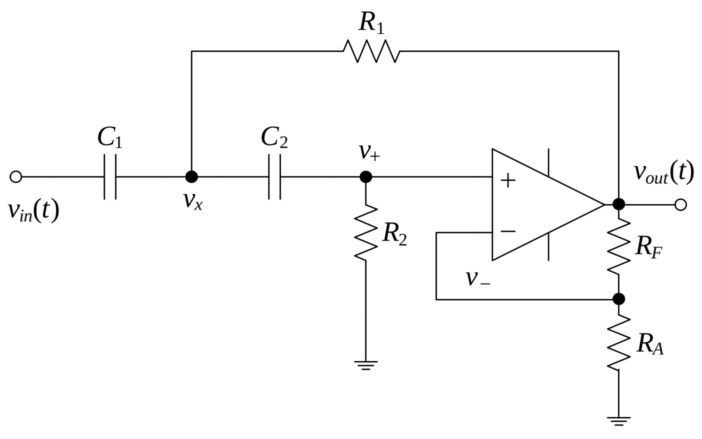

Figure 4: The circuit for Question 4.

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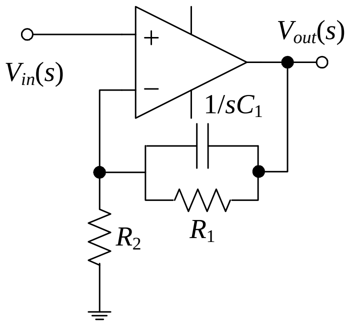
Problem Set #06 Solution

Q1

Complete Solution

The given circuit data is $R_1 = 1$ kΩ, $R_2 = 2$ kΩ, $C_1 = 1$ μF.
Re-drawing the circuit in s-domain, we have



(a)

Applying KCL at -ve pin,

$$\frac{V_-(s) - 0}{R_2} + \frac{V_-(s) - V_{out}(s)}{R_1} + \frac{V_-(s) - V_{out}(s)}{\frac{1}{sC_1}} = 0$$

Solving

$$\frac{V_{out}(s)}{V_-(s)} = \frac{sC_1 + \frac{1}{R_1} + \frac{1}{R_2}}{sC_1 + \frac{1}{R_1}}$$

From circuit, $V_{in}(s) = V_+(s)$. Applying op-amp assumption, $V_+(s) = V_-(s)$. Hence

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{sC_1 + \frac{1}{R_1} + \frac{1}{R_2}}{sC_1 + \frac{1}{R_1}}$$

Converting to standard form

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{s + \frac{1}{C_1}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}{s + \frac{1}{R_1 C_1}}$$

(a)

Substituting the values, the transfer function is

$$H(s) = \frac{s + 1500}{s + 1000}$$

The MATLAB commands are:

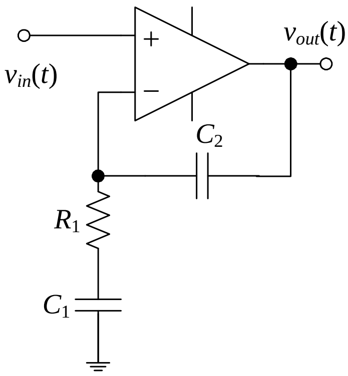
```
>> num=[1 1500];
>> den=[1 1000];
>> H=tf(num,den);
>> bode(H);
```

From the shape of the magnitude bode plot, the given circuit provides 3.5 dB gain to low frequencies and allows high frequencies to pass unchanged (0dB gain).

Q2

Partial Solution

The given circuit data is $R_1 = 1$ kΩ, $C_1 = 1$ μF, $C_2 = 10$ μF.
The given circuit is



(a)

In standard form

$$H(s) = \frac{\frac{1}{R_1C_2}}{s + \frac{1}{R_1C_1}}$$

(b)

$$H(s) = \frac{100}{s + 1000}$$

The MATLAB commands are:

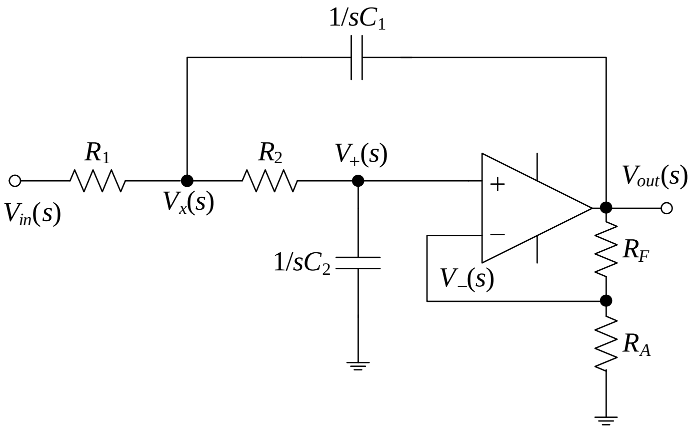
```
>> num=[100];
>> den=[1 1000];
>> H=tf(num,den);
>> bode(H);
```

From the shape of the magnitude bode plot, the given circuit is a low-pass filter with roll-off -20 dB/decade.

Q3

Partial Solution

Re-drawing the circuit in s-domain, we have



Let

$$G = 1+\frac{R_F}{R_A} = \frac{R_A+R_F}{R_A}$$

Step 1: Apply KCL at -ve pin

$$\frac{V_-(s)-V_{out}(s)}{R_F}+\frac{V_-(s)-0}{R_A} = 0$$

Solving, we have

$$V_-(s) = \frac{R_A}{R_A+R_F}V_{out}(s) = \frac{V_{out}(s)}{G}$$

Applying op-amp assumption, $V_+(s)=V_-(s)$. Hence

$$V_+(s) = \frac{V_{out}(s)}{G}$$

Step 2: Apply KCL at +ve pin

$$\frac{V_+(s)-V_x(s)}{R_2}+\frac{V_+(s)-0}{\frac{1}{sC_2}} = 0$$

Substituting the value $V_+(s)=\frac{V_{out}(s)}{G}$ and solving, we get

$$V_x(s) = (sC_2R_2+1)\frac{V_{out}(s)}{G}$$

Step 3: Apply KCL at node x

$$\frac{V_x(s)-V_+(s)}{R_2}+\frac{V_x(s)-V_{out}(s)}{\frac{1}{sC_1}}+\frac{V_x(s)-V_{in}(s)}{R_1} = 0$$

$$\frac{V_x(s)-\frac{V_{out}(s)}{G}}{R_2}+\frac{V_x(s)-V_{out}(s)}{\frac{1}{sC_1}}+\frac{V_x(s)-V_{in}(s)}{R_1} = 0$$

Simplifying, we have

$$\begin{aligned}
\left(sC_1+\frac{1}{R_1}+\frac{1}{R_2}\right)V_x(s) &= \frac{1}{R_1}V_{in}(s)+sC_1V_{out}(s)+\frac{1}{GR_2}V_{out}(s)\\
(sC_1R_1R_2+R_2+R_1)V_x(s) &= R_2V_{in}(s)+sC_1R_1R_2V_{out}(s)+\frac{R_1}{G}V_{out}(s)\\
(sC_1R_1R_2+R_2+R_1)(sC_2R_2+1)\frac{V_{out}(s)}{G} &= R_2V_{in}(s)+sC_1R_1R_2V_{out}(s)+\frac{R_1}{G}V_{out}(s)\\
(sC_1R_1R_2+R_2+R_1)(sC_2R_2+1)V_{out}(s) &= GR_2V_{in}(s)+sGC_1R_1R_2V_{out}(s)+R_1V_{out}(s)
\end{aligned}$$

$$\begin{aligned}
(sC_1R_1R_2+R_2+R_1)(sC_2R_2+1)V_{out}(s)-sGC_1R_1R_2V_{out}(s)-R_1V_{out}(s) &= GR_2V_{in}(s)\\
(sC_1C_2R_1R_2^2+sC_1R_1R_2+sC_2R_2^2+R_2+sC_2R_1R_2+R_1)V_{out}(s)-sGC_1R_1R_2V_{out}(s)-R_1V_{out}(s) &= GR_2V_{in}(s)\\
(s^2C_1C_2R_1R_2^2+sC_1R_1R_2(1-G)+sC_2R_1R_2+sC_2R_2^2+R_2)V_{out}(s) &= GR_2V_{in}(s)\\
(s^2C_1C_2R_1R_2+sC_1R_1(1-G)+sC_2R_1+sC_2R_2+1)V_{out}(s) &= GV_{in}(s)
\end{aligned}$$

Hence

$$\begin{aligned}
\frac{V_{out}(s)}{V_{in}(s)} &= \frac{G}{s^2C_1C_2R_1R_2+sC_1R_1(1-G)+sC_2R_1+sC_2R_2+1}\\
&= \frac{G}{s^2C_1C_2R_1R_2+[C_1R_1(1-G)+C_2R_1+C_2R_2]s+1}
\end{aligned}$$

Step 4: Convert transfer function to standard form.

Dividing numerator and denominator by $C_1C_2R_1R_2$

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{G\,\frac{1}{C_1C_2R_1R_2}}{s^2+\left[\frac{1-G}{C_2R_2}+\frac{1}{C_1R_2}+\frac{1}{C_1R_1}\right]s+\frac{1}{C_1C_2R_1R_2}}$$

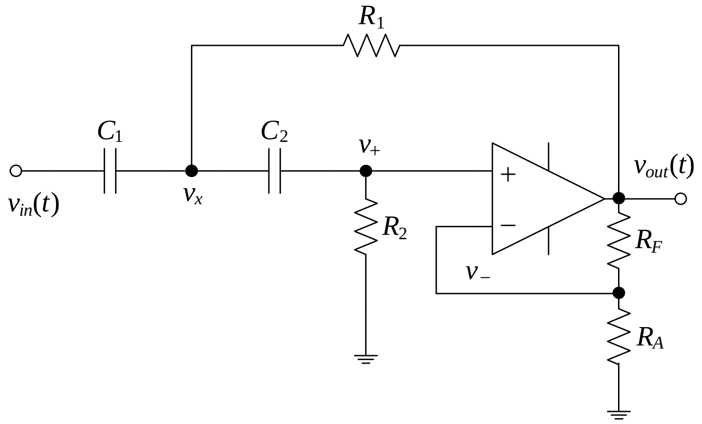
Hence

$$H(s) = \frac{G\,\frac{1}{C_1C_2R_1R_2}}{s^2+\left[\frac{1-G}{C_2R_2}+\frac{1}{C_1R_2}+\frac{1}{C_1R_1}\right]s+\frac{1}{C_1C_2R_1R_2}}$$

Q4

Solution

The given circuit is



$$H(s) = \frac{Gs^2}{s^2+\left[\frac{1-G}{C_1R_1}+\frac{1}{C_1R_2}+\frac{1}{C_2R_2}\right]s+\frac{1}{C_1C_2R_1R_2}}$$