Effect of Angular Energy Distribution of an Incident Signal on the Spatial Fading Correlation of a Uniform Linear Array

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Abstract

Array antennas have been widely used in wireless communication systems to improve the overall system coverage, capacity and link quality. The performance of different antenna array architectures such as Diversity and Beamforming arrays is critically dependant on the spatial correlation between antenna elements. In this paper we examine the effect of different angular distributions of an arriving signal on the spatial fading correlation between antenna elements of a Uniform Linear Array (ULA). Using Monte Carlo simulations, we investigate the effect of Angle of Arrival (AOA) distribution on the mean Bit Error Rate (BER) performance of a Code Division Multiple Access (CDMA) system employing a ULA at the Base Station (BS). The results show that the spatial correlation and BER are mainly dependent on the value of angular spread (i.e. the standard deviation of the underlying azimuth energy distribution) and not on the type of the assumed energy distribution.

I. INTRODUCTION

The use of array antennas at base stations of wireless communication systems has been proposed as a means of improving the overall system coverage, capacity and link quality. The two array architectures commonly employed to achieve these goals are Diversity arrays and Beamforming arrays [1]. The former employs widely spaced antenna elements while the latter employs antenna elements that are closely spaced, with typical inter-element spacing of half wavelength. The performance of both array antenna architectures is critically dependent on the fading correlation between the antenna elements. The cross-correlation between the signals received by antenna elements depends not only on the antenna array configuration but also on the operating environment, e.g. the direction of the incoming multipath waves and their angular spread.

Traditionally, beamforming and diversity have conflicting requirements for optimum performance. Diversity arrays work best when the spatial correlation between antenna elements is small. In comparison, conventional beamforming techniques based on AOA estimation work best with highly correlated fading across the array antenna elements. Recently, a hybrid scheme of diversity and beamforming called Hierarchical Beamforming architecture has been proposed to concurrently use beamforming and diversity gains [2], [3]. Thus in order to predict and evaluate the performance of such array antennas, the spatial correlation is a very important figure of merit.

In this work, we assume a Uniform Linear Array antenna that is employed at a BS of a wireless communication system. Other array configurations have been considered in [4], [5]. We assume that antenna elements feature an omnidirectional radiation pattern in azimuth. For an incident signal, we consider Uniform and Gaussian energy distributions for AOA in azimuth. We investigate the effect of specific energy distribution on the spatial fading correlation between the array elements. Also we study the Mean BER of a CDMA system employing this type of array antenna. We show that the dominating factor affecting the system performance is the standard deviation of the underlying azimuth angular distribution. Computer simulations confirm these theoretical results.

This paper is organised as follows. The channel model is presented in Section I. The theoretical expressions for Spatial Correlation coefficient are reviewed in Section II. The theoretical and simulation results are discussed in Section III. Finally conclusions are drawn in Section IV.

II. SPATIAL VECTOR CHANNEL MODEL

In order to perform the intended investigations, first it is necessary to develop the vector channel model for the array antenna. In this paper, we consider a parameterized model to characterize the wireless channel between a single antenna at the Mobile Station (MS) and a Uniform Linear Array (ULA) of N omnidirectional antenna elements at the BS [6]. We assume that each resolvable multipath is itself a superposition of S sub-paths originating from local scatterers in the vicinity of the MS. These local reflections from the mobile arrive at the base station from a random direction $\theta_s^{k,l} = \theta^{k,l} + \vartheta_s^{k,l}$, where $\theta^{k,l}$ is the mean Angle Of Arrival AOA and $\vartheta_s^{k,l}$ is a zero mean random angular deviation with standard deviation σ_{AOA}.

A. Channel Response Vector

The multiplicative distortion of the lth resolvable multipath of the kth user at the nth antenna can be given as

$$h^{k,l,n}(t) = \sqrt{\frac{\Omega^{k,l}}{S}} \sum_{s=1}^{S} e^{[j(\phi_s^{k,l}+2\pi f_D t \cos \Psi_s^{k,l})]} e^{[-j\mathcal{K}d(n-1)\sin(\theta^{k,l}+\vartheta_s^{k,l})]} = \sqrt{\frac{\Omega^{k,l}}{S}} \sum_{s=1}^{S} \alpha_s^{k,l} e^{[-j\mathcal{K}d(n-1)\sin(\theta_s^{k,l})]} \tag{1}$$

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where $n = 1, 2, \ldots, N$ is the antenna index, $k = 1, 2, \ldots, K$ is the user index, $l = 1, 2, \ldots, L$ is the multipath index, S is the number of sub-paths for each resolvable path, $\alpha_s^{k,l}$ is the complex sub-path amplitude, $\Omega^{k,l}$ is the mean path power of the lth multipath, $\mathcal{K} = 2\pi/\lambda$ is the wave number, d is the inter-element distance, f_D is the maximum Doppler frequency which is the ratio of the mobile velocity v and the wavelength, $\phi_s^{k,l}$ is random phase of each ray, assumed to be uniformly distributed over $[0, 2\pi]$ and $\Psi_s^{k,l}$ is the Angle of Departure (AOD) for each sub-path relative to the motion of the mobile, modelled by a uniform probability density function [7].

In vector notation, the channel vector can be written as $\mathbf{h}^{k,l}(t) = \sqrt{\frac{\Omega^{k,l}}{S}} \sum_{s=1}^{S} \alpha_s^{k,l} \mathbf{a}(\theta^{k,l} + \vartheta_s^{k,l})$ where $\mathbf{a}(\theta)$ is the $N \times 1$ steering vector, which models the spatial response of the array due to an incident plane wave from the θ direction and is given as [8]

$$\mathbf{a}(\theta) = [a_1(\theta)\ a_2(\theta)\ \cdots\ a_N(\theta)]^T = [1\ e^{-j\mathcal{K}d\sin\theta}\ \ldots\ e^{-j\mathcal{K}d(N-1)\sin\theta}]^T \quad (2)$$

B. Uniform and Gaussian PDF in Angle of Arrival

The Probability Density Function (PDF) in the AOA describes the angular distribution of the waves arriving at the Base Station (BS) in azimuth. For urban environments, a commonly used assumption for azimuth energy distribution is a Uniform distribution [9] over an azimuth angle range of 2Δ about the Mean AOA, given by $f_{AOA}(\theta) = \frac{1}{2\Delta}$ (for $\theta^{k,l} - \Delta \le \theta \le \theta^{k,l} + \Delta$), where Δ is called the scattering angle and is related to the standard deviation of the Uniform distribution as $\sigma_{AOA} = \frac{\Delta}{\sqrt{3}}$.

Another distribution that has been suggested is a Gaussian pdf, given by $f_{AOA}(\theta) = \frac{C_g}{\sqrt{2\pi}\,\sigma_{AOA}} \exp\left(\frac{(\theta-\theta^{k,l})^2}{2\sigma_{AOA}^2}\right)$ (for $-\pi+\theta^{k,l} \le \theta \le \pi+\theta^{k,l}$) where σ_{AOA} is the standard deviation and controls the spread of the PDF function (from probability theory, 99% of the paths are confined in the range $\pm 3\sigma_{AOA}$), $C_g = 1/\mathrm{erf}(\pi/\sqrt{2}\,\sigma_{AOA})$ is a constant that ensures that $f_{AOA}(\theta)$ fulfils the requirements of a probability density function and $\mathrm{erf}(x) = (2/(\sqrt{\pi}))\int_0^x e^{-t^2} dt$ is the error function. Note that when angular spread is small, C_g is almost unity.

In addition to the Uniform and Gaussian distributions, other distributions have also been proposed, e.g. $\cos^n(\theta)$ and Laplacian distributions [10]. However, according to [11], a Gaussian pdf best matches the azimuth pdf in typical urban environments. In this work we employ and compare both Uniform and Gaussian distributions to model the AOA's and consider the effect on array spatial correlation. To allow a consistent comparison, the angle spread (AS) is assumed equal to the standard deviation σ_{AOA} of the underlying PDF.

III. Spatial Correlation Coefficient

The array spatial correlation between the nth and pth antenna elements for the ULA is defined as

$$\mathbf{R}_s(n,p) = E\{a_n(\theta)a_p^*(\theta)\} = \int_\theta a_n(\theta)a_p^*(\theta)\,f_{AOA}(\theta) \equiv \Re\{\mathbf{R}_s(n,p)\} + j\Im\{\mathbf{R}_s(n,p)\} \quad (3)$$

where $(\cdot)^*$ denotes complex conjugate, $f_{AOA}(\theta)$ is the PDF of the AOA distribution and $a_n(\theta)$ and $a_p(\theta)$ are elements of the array steering vector from Eq. (2).

The Spatial envelope correlation coefficient ρ_s is then defined as

$$\rho_s = |\mathbf{R}_s(n,p)|^2 = |\Re\{\mathbf{R}_s(n,p)\} + j\Im\{\mathbf{R}_s(n,p)\}|^2 \quad (4)$$

where $\Re\{\mathbf{R}_s(n,p)\}$ and $\Im\{\mathbf{R}_s(n,p)\}$ are the real and imaginary parts of signal correlation respectively. The required antenna separation for $\rho_s = 0.7$ is also known as the correlation distance.

A. Case of Uniform PDF in AOA

Assuming a uniform distribution in AOA, the real and imaginary parts of signal correlation between the nth and pth antenna elements $\mathbf{R}_s(n,p)$ is given by [9], [12], [13]

$$\Re\{\mathbf{R}_s(n,p)\} = J_0(z_{np}) + 2\sum_{v=1}^{\infty} J_{2v}(z_{np})\,\cos(2v\theta)\,\mathrm{sinc}(2v\Delta) \quad (5)$$

$$\Im\{\mathbf{R}_s(n,p)\} = 2\sum_{v=0}^{\infty} J_{2v+1}(z_{np})\,\sin[(2v+1)\theta]\,\mathrm{sinc}[(2v+1)\Delta] \quad (6)$$

where $z_{np} = 2\pi|p-n|d/\lambda$, d is the inter-element distance, λ is the wavelength, θ is the mean AOA, Δ is the scattering angle, $J_n(x)$ is the nth order Bessel function of the first kind and $\mathrm{sinc}(x) = \frac{\sin(x)}{x}$ is the sinc function.

B. Case of Gaussian PDF in AOA

Assuming a gaussian distribution in AOA, it can be shown that the real and imaginary parts of $\mathbf{R}_s(n,p)$ are given by [14], [15]

$$\Re\{\mathbf{R}_s(n,p)\} = J_0(z_{np}) + 2C_g\sum_{v=1}^{\infty} J_{2v}(z_{np})\,\cos(2v\theta)\,e^{-2v^2\sigma_{AOA}^2}\,\Re\left\{\mathrm{erf}\left(\frac{\pi + j2v\sigma_{AOA}^2}{\sqrt{2}\,\sigma_{AOA}}\right)\right\} \quad (7)$$

$$\Im\{\mathbf{R}_s(n,p)\} = 2C_g\sum_{v=0}^{\infty} J_{2v+1}(z_{np})\sin\{(2v+1)\theta\}\,e^{\frac{-(2v+1)^2\sigma_{AOA}^2}{2}}\,\Re\left\{\mathrm{erf}\left(\frac{\pi + j(2v+1)\sigma_{AOA}^2}{\sqrt{2}\,\sigma_{AOA}}\right)\right\} \quad (8)$$

where $\mathrm{erf}(a + jb)$ is the complex input error function, which can be numerically evaluated using MATLAB.

IV. RESULTS

A. Spatial Behaviour of the Channel Model

We look at the behaviour of the channel model under spatial averaging. Specifically, we consider the same time instant and look at the spatial correlation. For simplicity, we consider the case of single path i.e. $L = 1$ and $\Omega^{k,l} = 1$. From Eq. (1), the spatial correlation $\mathbf{R}_s(n,p)$ can be estimated by taking the following expectation, which can be approximated by a simple average that is taken over the number of scatterers as

$$\mathbf{R}_s(n,p) = E\left\{\frac{1}{S}\sum_{s=1}^{S} a_n(\theta_s^{k,l})\, a_p^H(\theta_s^{k,l})\right\} \tag{9}$$

where S is the number of sub-paths and $E(\cdot)$ denotes expectation operator.

Figs. 1 and 2 show a comparison of the theoretical and estimated values of spatial correlation coefficient assuming Uniform and Gaussian AOAs respectively for different angle spread σ_{AOA} and Mean AOA θ. We consider AOAs in the range $[0°, 60°]$, since most outdoor wireless systems use sectorization (120° sectors). The theoretical values are calculated using Eqs. (5)-(8), with the summations carried out over 30 terms, which is sufficient for accuracy of six digit after the decimal point [15]. For the simulations, we use $S = 15$ and average over 10,000 independent Monte Carlo channel realizations. We see that the simulated values show good agreement with theory.

B. Effect of Mean AOA and Angle Spread on Spatial Correlation

Fig. 1 shows how the Spatial Correlation coefficient between adjacent antenna elements varies with the antenna spacing d, angle spread σ_{AOA} and Mean AOA θ, assuming Uniform AOA. We see that as expected, the fading correlation decreases as d increases. For small angle spreads, the decrease is uniform while ripples are observed for higher angle spreads. The figures show that in general, the spatial correlation decreases with increasing angular spread and decreasing angle of incidence, measured from array broadside. The correlation is higher for end-fire than for broadside incidence. The consequences of this correlation behaviour are two fold:

- If the azimuth angular spread is small, the output signals of the elements of the BS antenna array are strongly correlated. Therefore the antenna will provide beamforming gain only.
- If the azimuth angular spread is moderate or large, the output signals of the elements of the BS antenna array are not strongly correlated. Therefore in this situation, the antenna can provide diversity gain from spatial fading in addition to the beamforming gain. A beamforming algorithm to exploit this additional spatial diversity gain was recently proposed in [16].

C. Effect of Type of PDF in AOA on Spatial Correlation

Fig. 1 shows how the Spatial correlation coefficient between adjacent antenna elements varies with the antenna spacing d, angle spread σ_{AOA} and Mean AOA θ, assuming Gaussian AOA. Comparing Figs. 1 and 2 it can be observed that uniform PDF has slightly lower correlation than Gaussian PDF. This can be justified from the fact that the uniform PDF has more paths located further away from the mean AOA, while Gaussian PDF has more paths that are incident closer to the mean AOA. However the impact of different PDFs on the spatial correlation can be regarded as negligible.

D. Effect of Type of PDF in AOA on Mean BER for a CDMA System

To investigate this, we consider an uncoded IS-95 CDMA system with a combined beamformer and Rake receiver (2D Rake receiver) [12], [17]. We assume perfect beamforming i.e. the channel coefficients are known and used for beamforming weights. Fig. 3 illustrates how the choice of the PDF in AOA affects the Mean BER. In particular it shows the Mean BER vs. E_b/N_o for $N = 6$ antenna element ULA for single user with single path assuming different PDF's in AOA and different angle spreads respectively. We see that the BER performance for $\sigma_{AOA} = 5°$ is almost identical for Uniform and Gaussian distributions (in this case, for Uniform PDF all the sub-paths are confined in the range $\Delta = \sqrt{3}\sigma_{AOA} = \pm 8.66°$, while for Gaussian PDF 99% of the sub-paths are confined in the range $\pm 3\,\sigma_{AOA} = \pm 15°$ about the mean AOA). For comparison we also consider the situation with Uniform PDF for $\Delta = 15°(\sigma_{AOA} = 8.66°)$. We see that in this case, the performance is closer to $\sigma_{AOA} = 10°$ for Gaussian PDF. Fig. 4 shows the plot of Mean BER vs. E_b/N_o for $N = 6$ antennas, 15 users with $L = 2$ equal strength paths assuming Uniform and Gaussian PDFs in AOA and $\sigma_{AOA} = 10°$ respectively. The figure confirms that the key parameter for moderate angle spread values is σ_{AOA} and not the type of the PDF.

V. CONCLUSION

In this paper, we have reported on the impact of various angular distributions of an incident signal on the spatial fading correlation and Mean BER of a Uniform Linear Array antenna. It has been shown that the Uniform and Gaussian distributions give a similar fading correlation for the same value of the angle spread, characterised by the standard deviation σ_{AOA} of the assumed azimuth energy distribution (angle spread in the range $0° - 60°$ was considered in this paper). In the next step, we have performed simulations of a CDMA system with combined beamformer and Rake receiver (2D Rake receiver) assuming a linear array at a base station. The simulations have shown that the effect of type of PDF in AOA is negligible on the Mean BER of this system. The main parameter, which affects spatial correlation and thus the system performance is the angle spread σ_{AOA} and not the type of distribution.

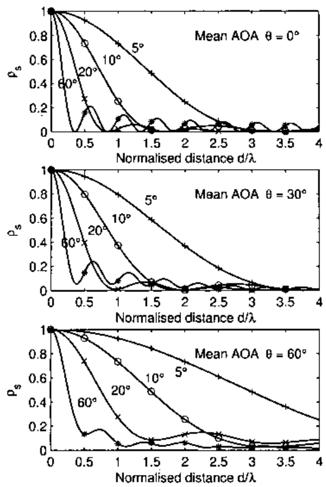

Fig. 1. Spatial envelope correlation coefficient p_s for different angle spreads ($\sigma_{AOA} = 5°, 10°, 20°, 60°$) and mean AOA $\theta = 0°, 30°, 60°$ assuming Uniform PDF in AOA (theory - lines, simulation - markers).

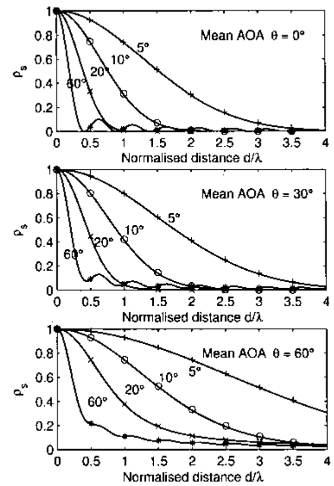

Fig. 2. Spatial envelope correlation coefficient p_s for different angle spreads ($\sigma_{AOA} = 5°, 10°, 20°, 60°$) and mean AOA $\theta = 0°, 30°, 60°$ assuming Gaussian PDF in AOA (theory - lines, simulation - markers).

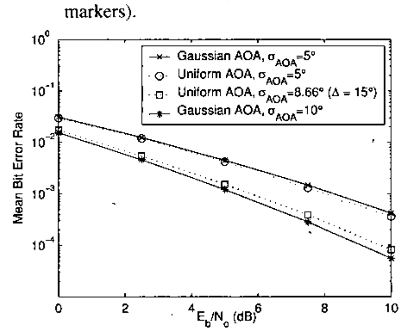

Fig. 3. Mean BER vs. E_b/N_o for $N = 6$ antennas, $K = 1$ user with single path assuming Uniform and Gaussian PDFs in AOA and different angle spreads.

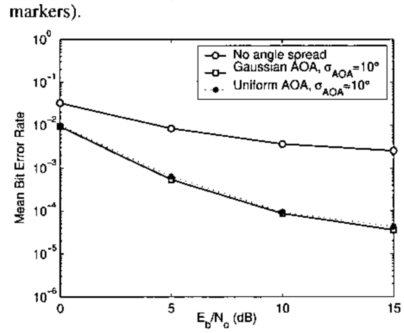

Fig. 4. Mean BER vs. E_b/N_o for $N = 6$ antennas, $K = 15$ users with $L = 2$ equal strength paths assuming Uniform and Gaussian PDFs in AOA and $\sigma_{AOA} = 0°$ and $10°$ respectively.

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