SER Analysis of Multi-Way Relay Networks with M-QAM Modulation in the Presence of Imperfect Channel Estimation

Shama N. Islam, Salman Durrani, and Parastoo Sadeghi

Abstract: **Multi-way relay networks (MWRNs) allow multiple users to exchange information with each other through a single relay terminal. MWRNs are often incorporated with capacity achieving lattice codes to enable the benefits of high-rate signal constellations to be extracted. In this paper, we analytically characterize the symbol error rate (SER) performance of a functional decode and forward (FDF) MWRN in the presence of channel estimation errors. Considering M-ary quadrature amplitude modulation (QAM) with square constellations as an important special case of lattice codes, we obtain asymptotic expressions for the average SER for a user in FDF MWRN. The accuracy of the analysis at high signal-to-noise ratio is validated by comparison with the simulation results. The analysis shows that when a user decodes other users with better channel conditions than itself, the decoding user experiences better error performance. The analytical results allow system designers to accurately assess the non-trivial impact of channel estimation errors and the users' channel conditions on the SER performance of a FDF MWRN with M-QAM modulation.**

Index Terms: **Channel estimation, functional decode and forward (FDF), lattice codes, multi-way relay network (MWRN), symbol error rate (SER).**

I. INTRODUCTION

RECENTLY, multi-way relay networks (MWRNs), where multiple users can exchange information with each other through a single relay terminal, have emerged as a significant research area in the field of relay networks for their enhanced capacity and spectral efficiency benefits [1], [2]. MWRNs have interesting potential applications such as exchanging data and information in wireless networks, sensor networks or satellite communication networks [1]. MWRNs are generalized version of two-way relay networks (TWRNs) [3]–[6] and enable the benefits of network coding for multiple users. Another way of realizing the benefits of network coding in a multi-user scenario is a multi-user TWRN, where users exchange messages with their pre-assigned partners and has been studied widely in the literature [7], [8]. Since, a MWRN is a general version of a multi-user TWRN, we focus on MWRNs in this paper.

The performance of MWRNs has been studied assuming amplify and forward (AF) [9]–[11], functional decode and forward (FDF) [2], [12] and compute and forward [13], [14] relaying protocols. It was shown in [12] that pairwise FDF (at the relay) for additive white Gaussian noise (AWGN) MWRN is theoretically the optimal strategy since it achieves the common-rate capacity. In this paper, we consider FDF relaying protocol, as it achieves higher rate, as well as, superior error performance compared to AF relaying. Apart from the relaying protocols in MWRNs, there have been some studies on optimal pairing schemes in MWRNs, that can maximize the sum-rate and the common rate under different channel conditions [15]–[19]. It was shown in [10] that a non-pairwise transmission strategy can offer larger spectral efficiency but incurs signal processing complexity at the relay. For this reason, we choose to use pairwise transmission strategy for simpler implementation at the relay.

In a pairwise transmission based MWRN, physical-layer network coding (PNC) protocols have been widely used in the literature. In PNC based FDF MWRNs, the relay utilizes the additive nature of physical electromagnetic waves and decodes and forwards a function of the users' messages. Recent studies on MWRNs have considered lattice code based transmission, which are known to achieve the Shannon capacity in an AWGN channel [14]. By utilizing the property that the sum of two lattice points is another point in the same lattice, lattice codes can achieve higher rates and better spectral efficiency compared to the uncoded transmissions. Moreover, lattice codes have been incorporated with PNC protocols in [20]. Hence, we consider lattice code based transmission protocols in our system.

There are three important practical issues that need to be considered for pairwise transmission based FDF MWRNs with lattice codes.

1) To perfectly recover the message of the other users by self-interference cancelation, the channels need to be perfectly estimated [15] at both the users and the relay, which is generally not possible in practice. For TWRNs, recent studies have quantified the impact of imperfect channel estimation in terms of error performance of AF [21] and FDF [22] TWRNs with relay selection, optimum power allocation for AF TWRNs [23] and presented suitable channel estimation algorithms [24], [25]. Despite this, the impact of imperfect channel estimation on MWRNs has not been addressed in the literature of MWRNs to date. In a MWRN, the message detection of each user influences decisions about other users in a complex manner which makes the investigation of the interdependencies between channel estimation error and the average SER of a MWRN quite non-trivial.
2) Lattice code based pairwise MWRNs have been investi-

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gated in [12] in terms of the capacity and the achievable rates. However, a complete characterization of the pairwise MWRNs also requires error performance analysis. Though the error performance of a pairwise MWRN has been analyzed in [9], [26] for simple binary phase shift keying (BPSK) modulation, the error performance of pairwise MWRNs with more general lattice codes has not been considered in the literature to date.

3) If a user wrongly decodes another user's message in MWRNs with pairwise data exchange, then this error propagates through the subsequent decoding operations. This occurrence, termed as error propagation, significantly affects the average bit error rate (BER) [26]. In the presence of imperfect channel estimation in a pairwise transmission based MWRN, the estimation error adds to the performance degradation resulting from error propagation and the error performance gets even worse.

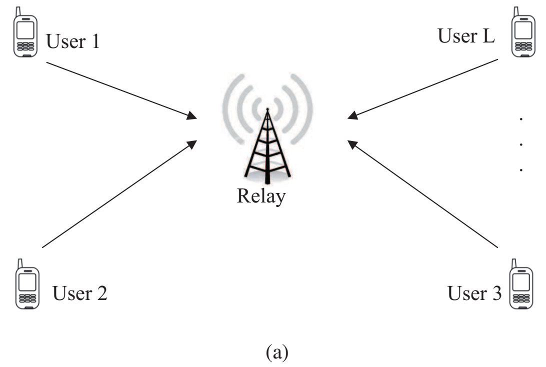
To the best of our knowledge, the joint impact of channel estimation error and error propagation on lattice code based MWRNs has not yet been addressed.

Addressing the above practical issues, we make the following novel contributions in this paper:

- Considering an L-user FDF MWRN employing M-ary quadrature amplitude modulation (QAM), we derive the expressions for the average symbol error rate (SER) with imperfect channel estimation and unequal average channel gains for the users. M-QAM is an important special case of lattice codes and is often used in practical wireless communication systems. The derived expressions for M-QAM can more accurately predict the system behavior at high SNR.
- We show that the average SER of FDF MWRN with M-QAM modulation is an increasing function of both the estimation error and the number of users. This behavior is a result of the fact that small and large number of errors are equiprobable for FDF MWRN due to error propagation and thus, the chance of larger number of error increases with the increasing number of users.
- We show that when a user decodes other users with better channel conditions, the decoding user's error performance improves, compared to the case when other users have worse channel conditions. This is because the relay correctly decodes the network coded message of the transmitting users with larger probability when the channels between these users and the relay experience good conditions.

The rest of the paper is organized in the following manner. The system model assumptions are presented in Section II and the proposed signal model for channel estimation in a lattice code based MWRN is presented in Section III. The SNR analysis is provided in Section III-C. The average SER for a user in FDF MWRN is derived in Section IV. The simulation results for verification of the analytical solutions are provided in Section V. Finally, conclusions are provided in Section VI.

Throughout this paper, we have used the following notations: $\bigoplus$ denotes XOR operation, $\hat{(\cdot)}$ denotes the estimate of a variable, $\hat{\hat{(\cdot)}}$ denotes that the variable is estimated for the second time, $\tilde{(\cdot)}$ denotes the estimation error, $|\cdot|$ denotes absolute value of a complex variable, $\arg(\cdot)$ denotes the argument, $\min(\cdot)$ denotes the minimum value, $(\cdot)^{\text{pe}}$ in the superscript means perfect estimation and $Q(x) = \frac{1}{\sqrt{2\pi}} \int_0^x e^{-t^2/2} dt$ is the Gaussian Q-function.



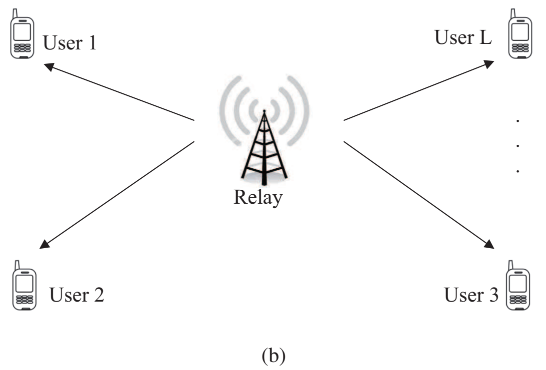


Fig. 1. Schematic diagram of a multi-way relay network: (a) MAC phase and (b) BC phase.

II. SYSTEM MODEL

We consider an L-user FDF MWRN with pairwise transmission, where the users exchange their information through a single relay. We assume that there is no direct link between the users and they transmit in a half-duplex manner. In an L-user MWRN with pairwise transmission, at the ℓ^{th} time slot, where, $\ell \in [1, L-1]$, the user pair formed by the ℓ^{th} and the $(\ell+1)^{\text{th}}$ user transmit simultaneously. Thus, in this scheme, the first and the last user transmit only once while the remaining $L-2$ users transmit twice. This transmission scheme is adopted widely in the literature of MWRNs as this scheme is found to achieve the capacity for MWRNs in AWGN channels [2], [9], [10], [15], [17], [27]. The information exchange among users is completed in two phases$-$multiple access and broadcast phase. A schematic diagram for MWRNs is illustrated in Fig. 1.

The channel from the i^{th} user (relay) to the relay (i^{th} user) is denoted by $h_{i,r}$ ($h_{r,i}$). We make the following assumptions regarding the channels:

- The channels are assumed to be block Rayleigh fading channels, which remain constant during one message packet transmission in a certain time slot. The channels in different time slots are considered to be independent. Also, the channels between users and the relay are reciprocal.
- The fading channel coefficients are zero mean complex-valued Gaussian random variables with variances $\sigma^2_{h_{i,r}} = \sigma^2_{h_{r,i}}$.

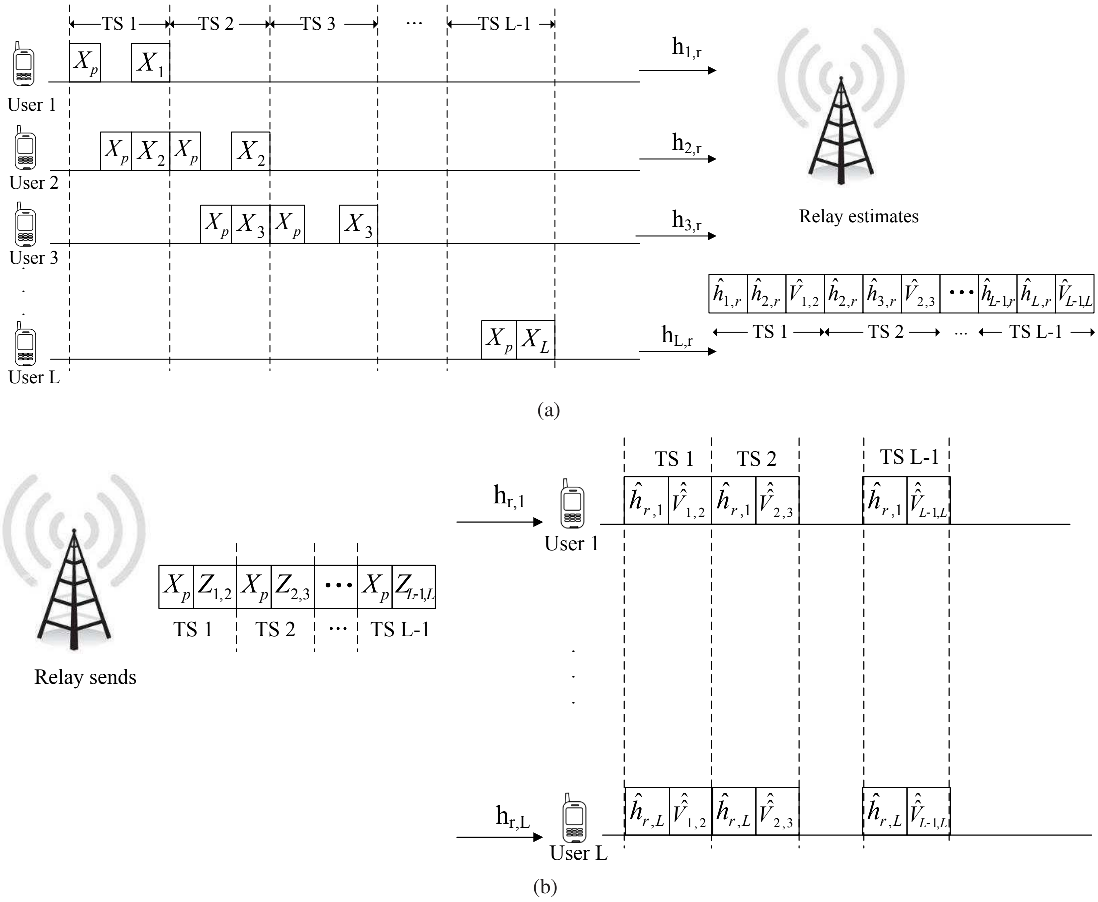


Fig. 2. Pilot and data transmission for an L-user FDF MWRN with imperfect channel estimation. The mathematical symbols are explained in Section II and Section III: (a) MAC phase and (b) BC phase.

- The channel coefficients are not known a priori at any of the users or the relay but the statistical parameters of the corresponding channels, for example, channel variances are known beforehand [23], [28] at the users and the relay.
- Perfect timing and phase synchronization is assumed at the users and the relay to obtain benchmark performance [22], [29]. This assumption is widely adopted in the literature of cooperative communications [30], [31] and MWRNs [2], [9], [15] for investigating different performance metrics.

III. LATTICE CODE BASED PROPOSED SIGNAL MODEL WITH CHANNEL ESTIMATION

In this set up, each phase is composed of a pilot transmission and a data transmission step. The pilot transmission is required for minimum mean square error (MMSE) based channel estimation and the data transmission is based on lattice codes. Thus, we model the channel $h_{i,r}$ as

$$h_{i,r} = \hat{h}_{i,r} + \tilde{h}_{i,r}, \tag{1}$$

where $\hat{h}_{i,r}$ is the estimated channel and $\tilde{h}_{i,r}$ represents the estimation error [23]. We denote the power of the pilot signal at the users and at the relay as P_s^{p} and P_r^{p}, respectively. Similarly, we denote the power of the data signal at the users and the relay by P_d and P_r, respectively. The above system model has been illustrated in Figs. 2(a) and 2(b).

In the following two subsections, we discuss the pilot and the data transmission protocols in the multiple access and the broadcast phases.

A. Pilot Transmission

A.1 Multiple Access Phase

In this phase, the users in a pair transmit their pilot symbols individually in different time slots and the relay estimates the corresponding channels through MMSE estimation. Such pilot symbol based MMSE is a frequently used technique for cellular channels [23], [32]. Since the channel is constant during one message packet transmission, only one pilot bit per message packet is required. So, in a single time slot, each user transmits a single pilot symbol and a single data packet. Thus, the overall signal transmitted at each time slot during the multiple access phase consists of two pilot symbols and one superimposed data packet from two simultaneously transmitting users. Similarly, the overall signal transmitted by the relay at each time slot during the broadcast phase consists of one pilot symbol and one network coded packet. That is, in total, $2(L-1)$ and $(L-1)$ pilot symbols are transmitted during the multiple access and broadcast phases, respectively. Note that when the data and the pilot powers are optimized, one pilot symbol transmission is optimal to achieve the highest information rate [23], [33]. However, assuming equal power for data and pilot symbols, as done in this paper, enables a simpler interpretation of the problem.

At the i^{th} time slot, first the i^{th} user transmits pilot symbol X_p and the relay receives the signal

$$Y_p = \sqrt{P_s^{\text{p}}} h_{i,r} X_p + n_p. \tag{2}$$

We assume that $X_p = 1$ and n_p is a zero mean complex valued AWGN with variance $\sigma_n^2 = N_0/2$ per dimension. The relay then obtains the estimate [34]

$$\hat{h}_{i,r} = \frac{\sqrt{P_s^{\text{p}}}\sigma^2_{h_{i,r}}}{P_s^{\text{p}}\sigma^2_{h_{i,r}} + \sigma_n^2} Y_p, \tag{3}$$

and the estimation error variance at the relay is [34]

$$\sigma^2_{\tilde{h}_{i,r}} = \frac{\sigma^2_{h_{i,r}}\sigma_n^2}{P_s^{\text{p}}\sigma^2_{h_{i,r}} + \sigma_n^2}. \tag{4}$$

Note that, the estimation error is independent of the channel estimate $\hat{h}_{i,r}$ because $\hat{h}_{i,r}$ is the MMSE estimate of $h_{i,r}$ [21]. Similarly, the relay can estimate the channel coefficient of the $(i+1)^{\text{th}}$ user.

A.2 Broadcast Phase

In this phase, the relay broadcasts its own pilot, as well as, the estimated channel coefficients in the multiple access phase. Then the m^{th} ($m \in [1, L]$) user performs MMSE estimation to obtain the estimate $\hat{h}_{r,m}$ as in (3). The channel estimation error is $\tilde{h}_{r,m} = h_{r,m} - \hat{h}_{r,m}$, with variance

$$\sigma^2_{\tilde{h}_{r,m}} = \frac{\sigma^2_{h_{r,m}}\sigma_n^2}{P_r^{\text{p}}\sigma^2_{h_{r,m}} + \sigma_n^2}. \tag{5}$$

B. Data Transmission

Here, we discuss the general lattice code based data transmissions in a FDF MWRN with pairwise transmission. Our notations for lattice codes follow those of [12], [35], [36]. Further details on lattice codes are available in [14], [37]–[39].

B.1 Multiple Access Phase

In this phase, the users simultaneously transmit their data in a pairwise manner using FDF based on lattice[1] codes and the relay receives the sum of the signals. That is, at the i^{th} time slot, users i and $i+1$ simultaneously transmit messages W_i and W_{i+1} using lattice codes [1], [12]:

$$X_i = (\psi(W_i) + d_i) \mod \Lambda, \tag{6a}$$
$$X_{i+1} = (\psi(W_{i+1}) + d_{i+1}) \mod \Lambda, \tag{6b}$$

where W_i and W_{i+1} are generated independently and uniformly over a finite field, $\psi(\cdot)$ denotes the mapping of messages from a finite dimensional field to lattice points and d_i and d_{i+1} are the dither vectors[2] for the i^{th} and the $(i+1)^{\text{th}}$ user. The dither vectors, generated at the corresponding users, are transmitted to the relay prior to message transmission in the multiple access phase [35].

The relay receives the signal

$$r_{i,i+1} = \sqrt{P_d} h_{i,r} X_i + \sqrt{P_d} h_{i+1,r} X_{i+1} + n_1, \tag{7}$$

where n_1 is the zero mean complex AWGN at the relay with noise variance $\sigma^2_{n_1} = N_0/2$ per dimension.

B.2 Broadcast Phase

In this phase, the relay broadcasts the decoded network coded message to all the users. When all the users have the network coded messages corresponding to each user pair, they utilize self information to extract the messages of the other users.

First, the relay decodes the received signal with the estimated channel coefficients $\hat{h}_{i,r}$ and $\hat{h}_{i+1,r}$ and obtains an estimate of the corresponding network coded message (which is a function of the transmitting users' messages). The relay then broadcasts the estimated network coded signal after pilot transmission.

That is, the relay scales the received signal with a scalar coefficient α and removes the dithers d_i, d_{i+1} scaled by $\sqrt{P_d}\hat{h}_{i,r}$ and $\sqrt{P_d}\hat{h}_{i+1,r}$, respectively. The resulting signal is given by[3]

$$\begin{aligned} X_r &= [\alpha r_{i,i+1} - \sqrt{P_d}\hat{h}_{i,r} d_i - \sqrt{P_d}\hat{h}_{i+1,r} d_{i+1}] \mod \Lambda \\ &= [\sqrt{P_d}\hat{h}_{i,r} X_i + \sqrt{P_d}\hat{h}_{i+1,r} X_{i+1} + (\alpha - 1)\sqrt{P_d}(\hat{h}_{i,r} X_i \\ &\quad + \hat{h}_{i+1,r} X_{i+1}) + \alpha n_1 + \alpha\sqrt{P_d}(\tilde{h}_{i,r} X_i + \tilde{h}_{i+1,r} X_{i+1}) \\ &\quad - \sqrt{P_d}\hat{h}_{i,r} d_i - \sqrt{P_d}\hat{h}_{i+1,r} d_{i+1}] \mod \Lambda \\ &= [\sqrt{P_d}\hat{h}_{i,r}\psi(W_i) + \sqrt{P_d}\hat{h}_{i+1,r}\psi(W_{i+1}) + n] \mod \Lambda, \end{aligned} \tag{8}$$

where, $n = (\alpha - 1)\sqrt{P_d}(\hat{h}_{i,r} X_i + \hat{h}_{i+1,r} X_{i+1}) + \alpha n_1 + \alpha\sqrt{P_d}(\tilde{h}_{i,r} X_i + \tilde{h}_{i+1,r} X_{i+1})$ and α is chosen to minimize the noise variance and computed using the estimated channel coefficients.

The relay decodes the signal in (8) with a lattice quantizer[4] to obtain an estimate $\hat{V}_{i,i+1}$ approaching $(\psi(W_i) + \psi(W_{i+1}))$ $\mod \Lambda$. For lattice code based transmissions, a message is correctly decoded when the received signal is within the voronoi region[5] $\mathcal{V}$ of the transmitted signal, i.e., in effect, when the noise in the received signal is within the voronoi region. Since, for sufficiently large N, the voronoi region has a larger volume which leads to $Pr(n \notin \mathcal{V}) \to 0$, $\hat{V}_{i,i+1}$ approaches $(\psi(W_i) + \psi(W_{i+1})) \mod \Lambda$. The relay then adds a dither d_r with the network coded message which is generated at the relay and broadcast to the users prior to message transmission in the broadcast phase. Then it broadcasts the resulting message using lattice codes, given as $Z_{i,i+1} = (\hat{V}_{i,i+1} + d_r) \mod \Lambda$.

[1] An N-dimensional lattice is a discrete subgroup of the N-dimensional complex field under the vector addition and reflection operations.

[2] Dithering is a well known randomization technique which is necessary for achieving statistical independence between the input vector and the error vector [37].

[3] Note that we do not need different coefficients to multiply with the i^{th} and the $(i+1)^{\text{th}}$ users' lattice coded signals as in [14]. This is because we need only one combination of the signals and during each time slot, one of the signals in (8) will be already known to the decoding user.

[4] Lattice quantizers are multi-dimensional generalization of uniform quantizers, which map a point from the complex field to the nearest lattice point.

[5] The fundamental Voronoi region denotes the set of all points in the N-dimensional complex field which are closest to the zero vector.

Now, we consider the decoding at the m^{th} user and denote it as the decoding user. The m^{th} user receives

$$Y_{i,i+1} = \sqrt{P_r} h_{r,m} Z_{i,i+1} + n_2, \tag{9}$$

where n_2 is the zero mean complex AWGN at the user with noise variance $\sigma^2_{n_2} = N_0/2$ per dimension.

At the end of the broadcast phase, the m^{th} user scales the received signal with a scalar coefficient β_m and removes the dithers d_r multiplied by $\sqrt{P_r}\hat{h}_{r,m}$. The resulting signal is

$$\begin{aligned} &[\beta_m Y_{i,i+1} - \sqrt{P_r}\hat{h}_{r,m} d_r] \mod \Lambda \\ &= [\sqrt{P_r}\hat{h}_{r,m}\hat{V}_{i,i+1} + n'] \mod \Lambda, \end{aligned} \tag{10}$$

where, $n' = (\beta_m - 1)\sqrt{P_r}\hat{h}_{r,m}\hat{V}_{i,i+1} + \beta_m n_2 + \beta_m \sqrt{P_r}\tilde{h}_{r,m}\hat{V}_{i,i+1}$ and β_m is chosen to minimize the noise variance. The users then detect the received signal with a lattice quantizer and obtain the estimate $\hat{\hat{V}}_{i,i+1}$ approaching $(\psi(W_i) + \psi(W_{i+1})) \mod \Lambda$, assuming that the lattice dimension is large enough such that $\Pr(n' \notin \mathcal{V})$ approaches zero. After decoding all the network coded messages, each user performs message extraction of every other user by canceling self information.

B.3 Message Extraction

At first, the i^{th} user subtracts the scaled lattice point corresponding to its own message, i.e., $\psi(W_i)$ from the network coded message received at the $(i+1)^{\text{th}}$ time slot (i.e., $\hat{\hat{V}}_{i,i+1}$) and extracts the message of the $(i+1)^{\text{th}}$ user as $\psi(\hat{W}_{i+1})$. After that, it utilizes the extracted message of the $(i+1)^{\text{th}}$ user to obtain the messages of the $(i+2)^{\text{th}}$ user to the L^{th} user in the downward extraction process in a similar manner. The downward message extraction process can be shown as

$$\begin{aligned} \psi(\hat{W}_{i+1}) &= (\hat{\hat{V}}_{i,i+1} - \psi(W_i)) \mod \Lambda, \\ \psi(\hat{W}_{i+2}) &= (\hat{\hat{V}}_{i+1,i+2} - \psi(\hat{W}_{i+1})) \mod \Lambda, \cdots, \\ \psi(\hat{W}_L) &= (\hat{\hat{V}}_{L-1,L} - \psi(\hat{W}_{L-1})) \mod \Lambda. \end{aligned} \tag{11}$$

This process is also continued upward to recover the messages of the $(i-1)^{\text{th}}$ user to the 1^{st} user. It is clear from (11) that if $\hat{W}_{i+1} \neq W_{i+1}$, then W_{i+2} will be incorrectly decoded. Thus, decisions about any user are dependent on the previous decisions about other users and there is a chance of error propagation.

Remark 1: (8) and (10) show that the error performance of FDF MWRN depends on the channel estimation error. The expressions of the channel estimates (see (3)) and estimation error (see (4) and (5)) show that these are functions of the noise variance and the channel variance. Thus, we expect the channel variance and the noise variance to play a key role in determining the error performance of FDF MWRNs.

C. SNR Analysis

In a FDF MWRN, the decoding operation is performed after both the multiple access phase and the broadcast phase. Thus, we need to consider the SNR at the relay and the SNR at the users, separately.

C.1 SNR at the Relay

The SNR of the received signal with imperfect channel estimation at the relay is obtained from (8) as in (12) at the top of the next page, where the numerator represents the power of the signal part in (8) and the denominator represents the power of the noise terms n in (8). For the general case of the user pair formed by the m^{th} and the $(m \pm 1)^{\text{th}}$ user, the SNR expression in (12) can be written as in (13) at the top of the next page.

The optimum value of α can be obtained by setting $\frac{dn}{d\alpha} = 0$ as $\alpha = \frac{P_d|\hat{h}_{m,r}|^2 + P_d|\hat{h}_{m\pm1,r}|^2}{P_d|h_{m,r}|^2 + P_d|h_{m\pm1,r}|^2 + N_0}$. Now, substituting α in (13) and after some algebraic manipulations, the SNR at the relay can be expressed as in (14) at the top of the next page.

C.2 SNR at the Users

The signal transmission from the relay to the m^{th} ($m \in [1, L]$) user is the same as that in a point-to-point fading channel. Thus, the SNR of the j^{th} ($j \in [1, L]$) user's signal received at the m^{th} user is given by:

$$\gamma_m = \frac{P_r \mid \hat{h}_{r,m} \mid^2}{\mid \beta_m \mid^2 N_0 + P_r \mid \beta_m - 1 \mid^2 \mid \hat{h}_{r,m} \mid^2 + |\beta_m|^2 P_r \sigma^2_{\tilde{h}_{r,m}}}, \tag{15}$$

where the numerator and the denominator represent the power of the signal part and the noise term n', respectively in (10). The optimum value of β_m is obtained by setting $\frac{dn'}{d\beta_m} = 0$ as $\beta_m = \frac{P_r|\hat{h}_{r,m}|^2}{P_r|h_{r,m}|^2 + N_0}$. Then substituting β_m in (15) and after some algebraic manipulations, the SNR at the m^{th} user is obtained as:

$$\gamma_m = \frac{P_r \mid \hat{h}_{r,m} \mid^2}{P_r \sigma^2_{\tilde{h}_{r,m}} + N_0}. \tag{16}$$

C.3 Special Case: Perfect Channel Estimation

When the channel estimation is perfect (i.e., $\sigma^2_{\tilde{h}_{i,r}} = \sigma^2_{\tilde{h}_{i+1,r}} = \sigma^2_{\tilde{h}_{r,m}} = 0$), the SNR at the relay is:

$$\gamma_r^{pe}(i) = \frac{\min(|h_{i,r}|^2, |h_{i+1,r}|^2)}{|h_{i,r}|^2 + |h_{i+1,r}|^2} + \frac{P_d \min(\mid h_{i,r} \mid^2, \mid h_{i+1,r} \mid^2)}{N_0}. \tag{17}$$

Also, the SNR at the m^{th} user is given by:

$$\gamma_m^{pe} = \frac{P_r \mid h_{r,m} \mid^2}{N_0}. \tag{18}$$

The expressions (17) and (18) coincide with the results in [15]. Thus, the results in [15] can be considered as a special case of the formulations in (15) and (16).

IV. ERROR PERFORMANCE ANALYSIS

In this section, we characterize the error performance of FDF MWRN through average SER analysis. We apply the analysis technique in [26]. It must be noted that in [26], the average BER analysis has been performed for BPSK modulation in AWGN and fading channels with perfect estimation and equal channel variances. Here we provide the analytical derivations for square M-QAM modulation, which is a 2 dimensional lattice code and

$$\gamma_r(i) = \frac{P_d \min(|\ \hat{h}_{i,r}\ |^2, |\ \hat{h}_{i+1,r}\ |^2)}{|\ \alpha\ |^2\ N_0 + P_d\ |\ \alpha - 1\ |^2\ (|\ \hat{h}_{i,r}\ |^2 + |\ \hat{h}_{i+1,r}\ |^2) + P_d\ |\ \alpha\ |^2\ (\sigma^2_{\tilde{h}_{i,r}} + \sigma^2_{\tilde{h}_{i+1,r}})}, \tag{12}$$

$$\gamma_r(m) = \frac{P_d \min(|\ \hat{h}_{m,r}\ |^2, |\ \hat{h}_{m\pm1,r}\ |^2)}{|\ \alpha\ |^2\ N_0 + P_d\ |\ \alpha - 1\ |^2\ (|\ \hat{h}_{m,r}\ |^2 + |\ \hat{h}_{m\pm1,r}\ |^2) + P_d\ |\ \alpha\ |^2\ (\sigma^2_{\tilde{h}_{m,r}} + \sigma^2_{\tilde{h}_{m\pm1,r}})}, \tag{13}$$

$$\gamma_r(m) = \frac{\min(|\hat{h}_{m,r}|^2, |\hat{h}_{m\pm1,r}|^2)}{|\hat{h}_{m,r}|^2 + |\hat{h}_{m\pm1,r}|^2} + \frac{P_d \min(|\ \hat{h}_{m,r}\ |^2, |\ \hat{h}_{m\pm1,r}\ |^2)}{P_d\sigma^2_{\tilde{h}_{m,r}} + P_d\sigma^2_{\tilde{h}_{m\pm1,r}} + N_0}, \tag{14}$$

incorporate channel estimation error and unequal channel variances in the average SER analysis.

A. Data Transmission with Square M-QAM Modulation

In the M-QAM modulated FDF MWRN system, at the i^{th} time slot, the i^{th} user and the $(i+1)^{\text{th}}$ user transmit messages W_i and W_{i+1} which are M-QAM modulated to X_i and X_{i+1}, respectively, where $X_i, X_{i+1} = a + jb$ and $a, b \in \{\pm1, \pm3, \cdots, \pm(\sqrt{M}-1)\}$. The relay receives the signal $r_{i,i+1}$ (see (7)) and decodes it to obtain $\hat{V}_{i,i+1}$ of the network coded symbol $V_{i,i+1} = (W_i + W_{i+1}) \mod M$ as in [3], [40]. The relay then broadcasts the estimated network coded signal after M-QAM modulation as $Z_{i,i+1}$. The j^{th} ($j \in [1, L]$) user receives $Y_{i,i+1}$ (see (9)) and detects the received signal to obtain the estimate $\hat{\hat{V}}_{i,i+1}$. After decoding the network coded messages, each user performs message extraction upward and downward. In the downward extraction process, the i^{th} user subtracts its own message W_i from the network coded message $\hat{\hat{V}}_{i,i+1}$ and then performs the modulo-M operation. The process can be shown as

$$\begin{aligned}\hat{W}_{i+1} &= (\hat{\hat{V}}_{i,i+1} - W_i + M) \mod M,\\ \hat{W}_{i+2} &= (\hat{\hat{V}}_{i+1,i+2} - W_{i+1} + M) \mod M, \cdots,\\ \hat{W}_L &= (\hat{\hat{V}}_{L-1,L} - \hat{W}_{L-1} + M) \mod M.\end{aligned} \tag{19}$$

The upward message extraction process can be shown similarly.

B. Steps for Error Performance Analysis

In this subsection, we outline the general steps for obtaining the average SER of a MWRN. These steps summarize how the analysis technique in [26] can be applied to the more general problem involving imperfect channel estimation considered in this paper.

- *Step 1:* Obtain the probability of incorrectly decoding a $\sqrt{M}$-PAM network coded message, $P_{\sqrt{M}-PAM,NC}(i,k)$. This is important because any M-QAM signal with square constellation (i.e., $\sqrt{M} \in \mathbb{Z}$) can be decomposed into two $\sqrt{M}$-PAM signals [41]. Thus, the network coded signal resulting from M-QAM signals is correctly decoded when both the component $\sqrt{M}$-PAM signals are correctly decoded.
- *Step 2:* Obtain the probability of incorrectly decoding a network coded message, $P_{FDF}(i,k)$.
- *Step 3:* Obtain the probability of the k^{th} error event, $P_i(k)$, in terms of $P_{FDF}(i,k)$, where the k^{th} error event occurs when exactly k number of users' messages are incorrectly decoded.
- *Step 4:* Since, there are $L-1$ possible error events in an L-user MWRN, find the expected probability of all these error events to obtain the average SER, $P_{i,avg}$.

The next section summarizes the main results from steps 1–4 through Lemmas 1–3 and Theorem 1.

C. SER Analysis

In this section, we obtain the average SER for FDF MWRN with imperfect CSI following the steps outlined in Section IV-B. First, we obtain the probability of incorrectly decoding a network coded message resulting from $\sqrt{M}$-PAM network coded signals.

Lemma 1: The probability of incorrectly decoding a network coded message resulting from the sum of two $\sqrt{M}$-PAM signals from the k^{th} and the $(k\pm1)^{\text{th}}$ user at the i^{th} user with imperfect CSI, is

$$P_{\sqrt{M}-PAM,NC}(i,k) = \frac{1}{\sqrt{M}}\left(\sum_{p,q=0}^{\sqrt{M}-1} c_{p,q} \sum_{p',q'=0, p'\neq p, q'\neq q}^{\sqrt{M}-1} d_{p',q'}\right), \tag{20}$$

where $c_{p,q}$ can be expressed as

$$c_{p,q} = \begin{cases} \sum\limits_{u=1,u=\text{odd}}^{2(2\sqrt{M}-2)-1} a_{p,q,u} Q(u\sqrt{\gamma_r(k)}), & p \neq q;\\ 1 + \sum\limits_{u=1,u=\text{odd}}^{2(2\sqrt{M}-2)-1} a_{p,q,u} Q(u\sqrt{\gamma_r(k)}), & p = q,\end{cases} \tag{21}$$

and $\gamma_r(k)$ represents the SNR of the k^{th} and the $(k\pm1)^{\text{th}}$ users' signal at the relay with imperfect CSI for $\sqrt{M}$-PAM modulation and can be obtained as $\gamma_r(k) = \frac{P_d \min(|\hat{h}_{k,r}|^2, |\hat{h}_{k\pm1,r}|^2)}{E_{av}(\sigma^2_{\tilde{h}_{k,r}} + \sigma^2_{\tilde{h}_{k\pm1,r}} + N_0)}$, E_{av} is the average energy of symbols for $\sqrt{M}$-PAM modulation (e.g., $E_{av} = 5$ for $M = 16$) and $d_{p,q}$ can be expressed as

$$d_{p',q'} = \begin{cases} \sum\limits_{v=1,v=\text{odd}}^{2(\sqrt{M}-1)-1} b_{p',q',v} Q(v\sqrt{\gamma_i}), & p' \neq q';\\ 1 + \sum\limits_{v=1,v=\text{odd}}^{2(\sqrt{M}-1)-1} b_{p',q',v} Q(v\sqrt{\gamma_i}), & p' = q',\end{cases} \tag{22}$$

where $\gamma_i = \frac{P_r|\hat{h}_{r,i}|^2}{E_{av}(\sigma^2_{\hat{h}_{r,i}}+N_0)}$ represents the SNR at the i^{th} user.

Proof: The proof is given in [36] and uses the results from [40]. It is not repeated here for brevity. The coefficients $a_{p,q,u}$ and $b_{p',q',v}$ for $M = 16$ (or $\sqrt{M} = 4$), are tabulated in Table 1 at the top of this page. □

Lemma 2: The probability that the i^{th} user incorrectly decodes the M-QAM network coded message of the k^{th} and the $(k \pm 1)^{\text{th}}$ user in a FDF MWRN is given as:

$$P_{FDF}(i,k) = 1 - \left(1 - P_{\sqrt{M}-PAM,NC}(i,k)\right)^2, \qquad (23)$$

where $P_{\sqrt{M}-PAM,NC}(i,k)$ is expressed in (20).

Proof: (23) follows from the fact that any M-QAM signal with square constellation can be decomposed to two $\sqrt{M}$-PAM signals [41]. Thus, the network coded signal resulting from M-QAM signals is correctly decoded when both the component $\sqrt{M}$-PAM signals are correctly decoded. □

Using (23) and (20) and following the steps in IV-B, the probability of k error events with imperfect CSI for FDF relaying can be obtained as in the following lemma.

Lemma 3: At high SNR, the expression for the probability of the k^{th} error event is given by:

$$P_i(k) = \begin{cases} P_{FDF}(i, L-k), & i = 1, 2; \\ P_{FDF}(i,k), & i = L, L-1; \\ P_{FDF}(i,k) + P_{FDF}(i, L-k), & i \notin \{1, 2, L-1, L\}. \end{cases} \qquad (24)$$

Proof: See Appendix A. □

Using Lemmas 1-3 (corresponding to the main results from steps 1-3 in Section IV-B), we obtain the main result in this paper, which is stated below.

Theorem 1: At high SNR, the average SER of a FDF MWRN with imperfect CSI is given as:

$$P_{i,avg} = \frac{1}{L-1} \begin{cases} \sum_{k=1}^{L-1} kP_{FDF}(i, L-k), & i = 1, 2 \\ \sum_{k=1}^{L-1} kP_{FDF}(i,k), & i = L, L-1 \\ \sum_{k=1}^{L-1} kP_{FDF}(i, L-k), & \\ +kP_{FDF}(i,k), & i \notin \{1, 2, L-1, L\}. \end{cases}$$

Proof: Averaging the probability of the k^{th} error event over the $L-1$ possible error events, the average SER at the i^{th} user can be obtained as:

$$P_{i,avg} = \frac{1}{L-1} \sum_{k=1}^{L-1} kP_i(k), \qquad (25)$$

Then, substituting (24) into (25) gives the average SER of a FDF MWRN, which completes the proof. □

Remark 2: Note that the expression in (25) represents the instantaneous error performance of a MWRN. However, in Section V, we have numerically averaged (26) over large number of channel realizations to obtain the average SER of a MWRN.

V. NUMERICAL RESULTS

In this section, we verify the error performance analysis results with Monte Carlo simulations and discuss insights from the analysis. We consider a FDF MWRN with imperfect channel estimation, where each user transmits a message packet of $T = 2,000$ bits and uses 16-QAM modulation. For each of the message packets, one pilot bit is transmitted. The pilot and data power at the users and at the relay are assumed to be equal and normalized to unity. In all the figures in this section, we plot the analytical results for average SER by numerically averaging the expression in (25) over a large number of channel realizations. Following [10], the average channel gain for the j^{th} user is modeled by $\sigma^2_{h_{j,r}} = (1/(d_j/d_0))^\nu$, where d_0 is the reference distance, d_j is the distance between the j^{th} user and the relay which is assumed to be uniformly randomly distributed between 0 and d_0, and ν is the path loss exponent, which is assumed to be 3. Such a distance based channel model takes into account large scale path loss and has been widely considered in the literature [3], [7], [23], [28], [34], [42]. Note that, in this model, the estimation error variance (see (4)) is a function of distances. The SNR (x-axis variable in Figs. 3 and 7) is assumed to be SNR per message per user. We denote the decoding user as the i^{th} user, where i is assumed to be 1 and other users as the m^{th} user, where $m \in [1, L], m \neq i$. The simulation results are averaged over $1,000$ Monte Carlo trials per SNR point.

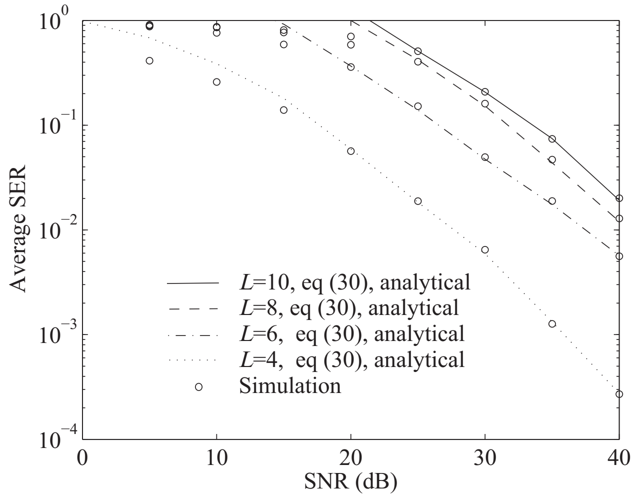


Fig. 3. Average SER for $L = 4$, $L = 6$, $L = 8$ and $L = 10$ users in a FDF MWRN with imperfect channel estimation, where the estimation error is given in (4).

A. Verification of the Analytical Results

Fig. 3 shows the average SER for FDF MWRN with $L = 4, 6, 8, 10$ users in the presence of imperfect channel estimation. Here, the analytical results are plotted using (25) and compared with the simulation results. The analytical results match very well with the simulations at high SNR. The discrepancies between the analytical and simulation results at low SNR are expected due to the high SNR approximations in the analysis (see Lemma 3). Larger number of users results into larger average SER, as expected from (25).

B. Impact of Estimation Error

Fig. 4(a) plots average SER for $L = 4, 6, 8, 10$ user FDF MWRNs for different levels of the estimation error and different channel conditions. In this analysis, the estimation errors are set

Table 1. Illustration of the coefficients $a_{p,q,u}$ and $b_{p',q',v}$ for $M = 16$ corresponding to (21) and (22), respectively [36].

p, p'	u \ q	$a_{p,q,u}$: $q=0$	$q=1$	$q=2$	$q=3$	v \ q'	$b_{p',q',v}$: $q'=0$	$q'=1$	$q'=2$	$q'=3$
$p=0$	$u=1$	$-7/4$	1	0	$3/4$	$v=1$	$1/4$	$1/4$	0	0
	$u=3$	0	-1	$7/4$	$-3/4$					
	$u=5$	0	$3/4$	-1	$1/4$	$v=3$	0	$-1/4$	$1/4$	0
	$u=7$	1	$-3/4$	0	$-1/4$					
	$u=9$	$-1/4$	$1/4$	0	0	$v=5$	0	0	$-1/4$	$1/4$
	$u=11$	0	$-1/4$	$1/4$	0					
$p=1$	$u=1$	1	1	0	0	$v=1$	$1/4$	$-1/4$	$1/4$	0
	$u=3$	-1/2	0	$-1/2$	1					
	$u=5$	1/2	0	$1/2$	-1	$v=3$	$-1/4$	$1/4$	$-1/4$	$1/4$
	$u=7$	$-1/2$	1	$-1/2$	0					
	$u=9$	$1/2$	-1	$1/2$	0	$v=5$	0	$1/4$	0	$-1/4$
	$u=11$	0	0	0	0					
$p=2$	$u=1$	1	1	$-7/4$	$3/4$	$v=1$	0	$1/4$	$-1/4$	$1/4$
	$u=3$	7/4	-1	0	$-3/4$					
	$u=5$	-1	$3/4$	0	$1/4$	$v=3$	$1/4$	$-1/4$	$1/4$	$1/4$
	$u=7$	0	$-3/4$	1	$-1/4$					
	$u=9$	0	$1/4$	$-1/4$	0	$v=5$	$-1/4$	0	$1/4$	0
	$u=11$	$1/4$	$-1/4$	0	0					
$p=3$	$u=1$	1	0	1	-2	$v=1$	0	0	$1/4$	$-1/4$
	$u=3$	-1	2	-1	0					
	$u=5$	1	-2	1	0	$v=3$	$1/4$	$1/4$	$-1/4$	0
	$u=7$	0	0	0	0					
	$u=9$	0	0	0	0	$v=5$	0	$-1/4$	0	0
	$u=11$	0	0	0	0					

using the technique explained below, which for the sake of clarity, is illustrated for two estimation errors of 0.1% and 0.01% of the combined variance of the fading channel and the complex AWGN noise. These values of channel estimation errors have been introduced by setting σ_n^2 equal to 0.001 and 0.0001, respectively in the expression for estimation error in (4) and noting that $\sigma^2_{h_{i,r}} \approx \sigma^2_{\hat{h}_{i,r}} + \sigma_n^2$ at high SNR and $\nu = 3$. It can be noted from this figure that the average SER is an increasing function of the estimation error. Also, for larger number of users, the average SER is higher compared to the case for smaller number of users.

Fig. 4(b) plots average SER for increasing number of users and different levels of estimation error. From this figure, it can be seen that average SER of a FDF MWRN is an increasing function of the number of users for different levels of estimation error. This is because, with the increasing number of users, the probability of having an incorrect network coded message increases (see (25)).

Fig. 5 shows the impact of channel estimation error and the users' distances on the maximum number of users that can be present in a MWRN for a certain level of average SER. To generate this figure, we conduct a number of simulation trials. In each trial, for a certain average SER requirement and a certain channel estimation error, we increment the number of users and average the SER over a large number of channel realizations and check that the average SER constraint is met. The trials are continued to determine the maximum number of users for which the average SER constraint is met. From this figure, we can see that as the estimation error increases, smaller number of users can be accommodated in the network to maintain a certain level of average SER. In this figure, we plot the results for two cases (i) $d_i < d_m$ (i^{th} user has better channel conditions than the m^{th} users) and (ii) $d_i > d_m$ (i^{th} user has poorer channel conditions than the m^{th} users). When only the decoding user (i^{th} user) has better channel conditions, smaller number of users can be accommodated to maintain the required average SER. However, when more users have better channel conditions, larger number of users can take part in the transmission while maintaining the same average SER.

C. Impact of the User's Channel Conditions

Fig. 6 plots the average SER for $L = 10$ user FDF MWRN with increasing distance of the decoding user from the relay for different levels of estimation error. In this figure, the results are plotted for two cases: (i) $d_i < d_m$ and (ii) $d_i > d_m$. These represent the cases that the i^{th} user has (i) better and (ii) poorer channel conditions, respectively, compared to the m^{th} users. The results show that the average SER increases with the level of channel estimation error. For different levels of channel estimation error, the average SER is smaller for $d_i > d_m$, compared to the case $d_i < d_m$. This can be intuitively explained as follows. For FDF relaying, both the relay and the decoding user are decoding messages in two phases. Thus, when the decoding user has better channel conditions, compared to the transmitting users, it can decode the message from the relay correctly with higher probability but an incorrect detection at the relay is highly probable (as the transmitting users' channel conditions are not good). However, when the decoding user has poorer channel conditions compared to the transmitting users, the channel conditions for the channels between the transmitting users and the relay are good. So, the relay can decode the network coded messages correctly with higher probability and the decoding user makes decision based on the transmitted signal from the relay. That is why, FDF MWRN achieves better error performance when the decoding user has poorer average channel gain compared to the other users.

Fig. 7 shows average SER for $L = 10$ user FDF MWRN for the cases when (i) 10% of the users have distances below $0.1d_0$ (corresponds to the case when most of the users have poor chan-

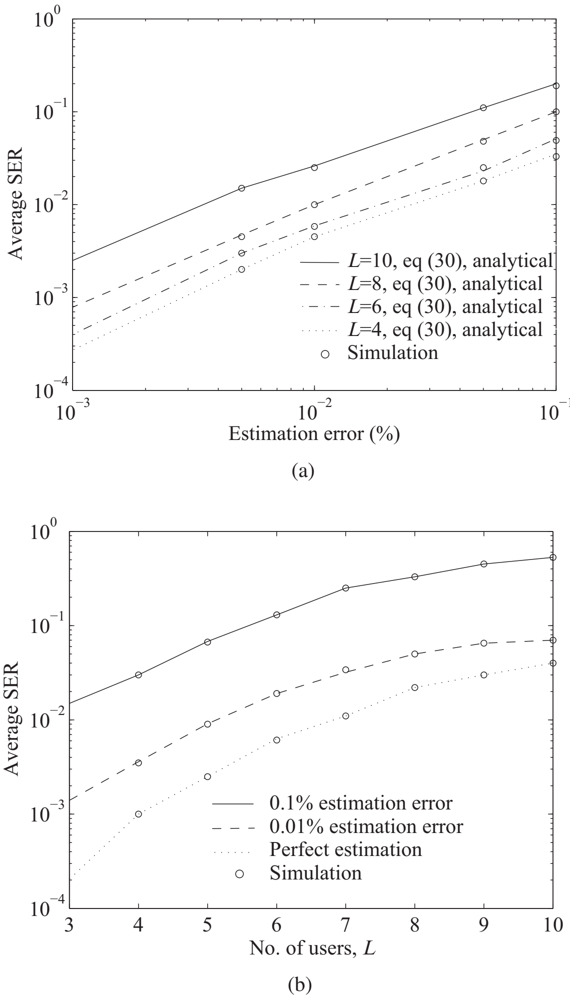


Fig. 4. Impact of estimation error on the average SER in a FDF MWRN with different number of users: (a) Average SER vs. estimation error and (b) Average SER vs. number of users

nel conditions) and (ii) 90% of the users have distances below $0.1d_0$ (corresponds to the case when most of the users have good channel conditions). For both the imperfect and perfect estimation, when most of the users experience good channel conditions, the average SER of FDF MWRN improves compared to the other case. This is because, when most of the users' channel conditions are good, the chance of error propagation in the decoding process is less. Moreover, when most of the users have good channel conditions, FDF MWRN with imperfect CSI performs closer to the perfect CSI performance. Thus, the overall channel conditions of the users have a greater impact on the average SER when perfect CSI is not available.

VI. CONCLUSIONS

In this paper, we have presented a method for analyzing the average SER for a user in a FDF MWRN with imperfect CSI and M-QAM modulation. We obtained the analytical expressions for the SNR at the relay and the users and also the average SER in the presence of channel estimation error. We have shown that the average SER of a FDF MWRN is an increasing function of both the estimation error from MMSE channel estimation and the number of users. We also observed that when the users other than the decoding user have better channel conditions, the error performance improves than the case when the decoding user

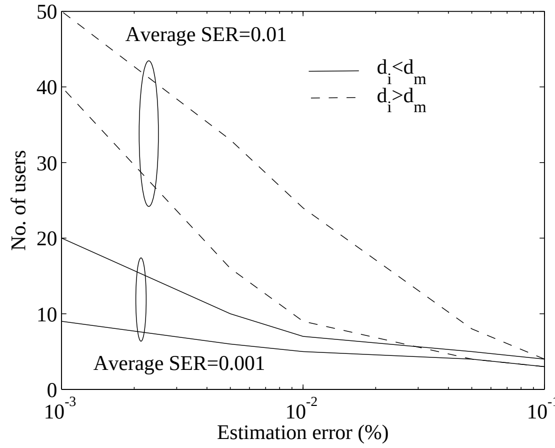


Fig. 5. The maximum number of users that can be accommodated in a MWRN to achieve a certain level of average SER in the presence of channel estimation error.

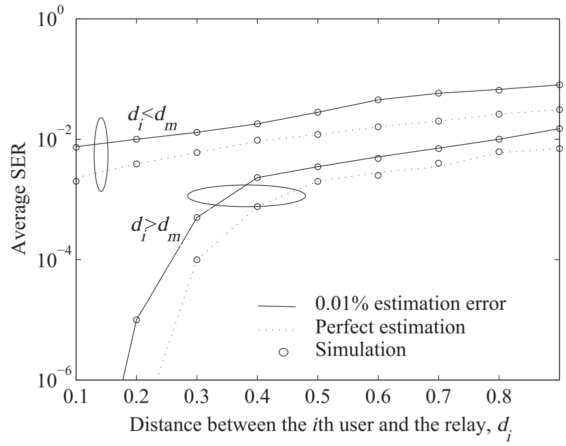


Fig. 6. Impact of the decoding user's channel conditions in $L = 10$ users FDF MWRN.

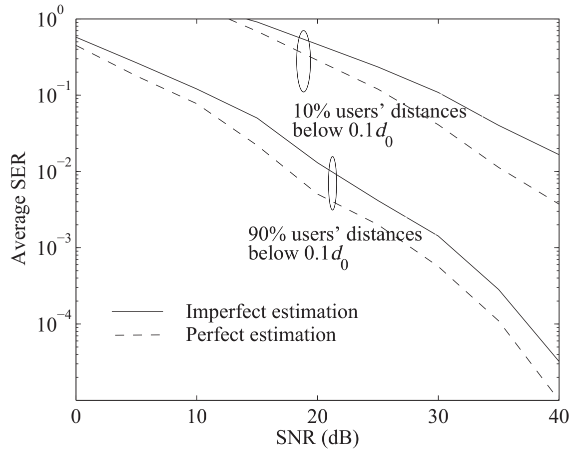


Fig. 7. Impact of the overall channel conditions in $L = 10$ users FDF MWRN.

has better channel conditions. Moreover, we observed that when most of the users experience good channel conditions, the error performance gap between imperfect and perfect CSI decreases because of reduced error propagation.

APPENDIX A: PROOF OF LEMMA 3

In this appendix, we derive the probability of the general case of k error events in a FDF MWRN. First, we need to investigate different error cases for the k^{th} error event in a FDF MWRN [26]. For $k = 1$, the possible error cases are

- when two consecutive erroneous network coded messages occur or,
- when an error in the network coded messages involving one of the end users occurs.

For larger values of k, there will be many more error cases and considering all the possible error cases would make the analysis complicated. For a tractable analysis, we consider only the dominating error cases that influence the k^{th} error event at high SNR. That is, we consider the higher order error terms (e.g., P_{FDF}^2) and the corresponding error cases are negligible.

At high SNR, the dominating case for the k^{th} error event occurs when the network coded message involving the k^{th} and the $(k+1)^{\text{th}}$ (or $(L-k+1)^{\text{th}}$ and $(L-k)^{\text{th}}$) users is incorrectly decoded, resulting in error about k users' messages. For example, if $k = 2$, 2 error events result from an error in the network coded message $V_{2,3}$ or $V_{L-2,L-1}$. Thus, the dominating error cases for the k^{th} error event is expressed as:

$$P_D = P_{FDF}(i,k) \prod_{m=1, m\neq k}^{L-1} (1 - P_{FDF}(i,m)), \tag{26a}$$

$$P_{D'} = P_{FDF}(i,L-k) \prod_{m=1, m\neq L-k}^{L-1} (1 - P_{FDF}(i,m)). \tag{26b}$$

Here, the subscripts D and D' indicate the case of k consecutive errors involving the first user and the $k-1$ following users and the case of k consecutive errors involving the last user and the $k-1$ preceding users, respectively. At high SNR, the terms $\prod_{m=1,m\neq k}^{L-1}(1 - P_{FDF}(i,m))$ in (26a) and $\prod_{m=1,m\neq L-k}^{L-1}(1 - P_{FDF}(i,m))$ in (26b) can be approximated to 1. Thus, (26) can be rewritten as:

$$P_D = P_{FDF}(i,k), P_{D'} = P_{FDF}(i,L-k). \tag{27}$$

Now for $i = 1, 2, L-1, L$, there is only one possible user combination in which the messages of the first user and the $k-1$ following users (or the last user and the $k-1$ preceding users) can be incorrectly decoded. Thus the expression for the probability of the k^{th} error event can be given by:

$$P_i(k) = \begin{cases} P_{D'} & i = 1, 2 \\ P_D & i = L, L-1 \\ P_D + P_{D'} & i \notin \{1, 2, L-1, L\}. \end{cases} \tag{28}$$

Then substituting (27) in (28) completes the proof.

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