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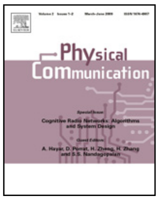

Full length article

A novel user pairing scheme for functional decode-and-forward multi-way relay network

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In this paper, we consider a functional decode and forward (FDF) multi-way relay network (MWRN) where a common user facilitates each user in the network to obtain messages from all other users. We propose a novel user pairing scheme, which is based on the principle of selecting a common user with the best average channel gain. This allows the user with the best channel conditions to contribute to the overall system performance. Assuming lattice code based transmissions, we derive upper bounds on the average common rate and the average sum rate with the proposed pairing scheme. Considering M-ary quadrature amplitude modulation with square constellation as a special case of lattice code transmission, we derive asymptotic average symbol error rate (SER) of the MWRN. We show that in terms of the achievable rates, the proposed pairing scheme outperforms the existing pairing schemes under a wide range of channel scenarios. The proposed pairing scheme also has lower average SER compared to existing schemes. We show that overall, the MWRN performance with the proposed pairing scheme is more robust, compared to existing pairing schemes, especially under worst case channel conditions when majority of users have poor average channel gains.

1. Introduction

Multi-way relay networks (MWRNs), where a single relay facilitates all users in the network to exchange information with every other user, have important potential applications in teleconferencing, data exchange in a sensor network or file sharing in a social network [1–12]. A MWRN is a generalization of two-way relay networks (TWRNs), which enable bidirectional information exchange between two users and are widely recognized in the literature for their improved spectral efficiency, compared to conventional relaying [13–19]. Note that multi-user TWRNs [20–25], where each user exchanges information with a pre-assigned user only, can be considered as a special case of MWRNs.

The users in a MWRN can adopt either pairwise transmission [1,5,9] or non-pairwise transmission [4,6,8,26] strategy for message exchange. Though non-pairwise transmission can offer larger spectral efficiency, its benefits come at the expense of additional signal processing complexity at the relay [6]. Hence, in this paper, we focus on pairwise transmission strategy. Recently, pairwise transmission based MWRNs have been studied for different relaying protocols, e.g., functional decode and forward (FDF) [1], decode and forward [4], amplify and forward [5] and compute and forward [7] protocols. It was shown in [1] that pairwise FDF with binary linear codes for MWRN, where the relay decodes a function of the users' messages rather than the individual messages from a user pair, is theoretically the optimal strategy since it achieves the common rate. Also it was shown in [2] that for a MWRN with lattice codes in an Additive White Gaussian Noise (AWGN) channel, the pairwise FDF achieves the common rate. Hence, in this paper, we consider FDF MWRN.

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In a pairwise transmission based FDF MWRN, user pair formation is a critical issue. In this regard, two different pairing schemes have been proposed in the literature. In the pairing scheme in [1], two consecutive users are paired with each other (i.e., user 1 with user 2, user 2 with user 3, user $L-1$ with user L, etc. where L is the number of users in the MWRN). Thus, in general, the ℓth and the $(\ell+1)$th users form a pair at the ℓth time slot, where $\ell \in [1, L-1]$. In the pairing scheme in [9], instead of consecutive users as in the pairing scheme in [1], two users in a pair are chosen from the two ends of a sequence such that the second user in one pair becomes the first user in the next pair (i.e., the pairs would be $(1, L)$, $(L, 2)$, $(2, L-1)$, etc.). Thus, in general, the ℓth and the $(L-\ell+1)$th user form a pair at the ℓth time slot when $1 \leq \ell \leq \lfloor L/2 \rfloor$ and the $(\ell+1)$th and $(L-\ell+1)$th user form a pair at the ℓth time slot when $\lfloor L/2 \rfloor < \ell \leq L-1$, where $\lfloor\cdot\rfloor$ denotes the floor operation. The achievable rates for these two existing pairing schemes were analyzed in [1,2,9], while the average bit error rate (BER) for the first pairing scheme was analyzed in [27].

A major drawback of the pairing scheme in [1] is that they do not take the users' channel information into account when pairing the users. In [9], the authors have considered only one case of asymmetric channel conditions, which is $|h_{1,r}|^2 < |h_{2,r}|^2 < \cdots < |h_{L,r}|^2$. However, they have not utilized the channel gain information for intelligent choice of the user pairs. Rather, both in [1] and [9], the users with good and bad channel conditions transmit the same amount of time. The only difference between the pairing schemes in [1] and [9] is the ordering of the user pair. Thus, if the performance metrics of [1] and [9] are being averaged over a number of settings for average channel gain, [1] and [9] would give the same results. Though in [9], the authors have shown that their pairing scheme is optimal in terms of the common rate for DF protocol and the considered channel conditions, the pairing schemes in [1] and [9], are not optimal in terms of the sum rate and error performance. This is because both in [1] and [9], the users with good and bad channel conditions transmit the same amount of time and the overall throughput will be less than that if the user with good channel conditions are allowed to transmit more times. Moreover, in a MWRN, the decision about each user depends on the decisions about all other users transmitting before it. Thus, in the above pairing schemes, if any user experiences poor channel conditions, it can lead to incorrect detection of another user's message, which can adversely impact the system performance due to error propagation. We also note that a recent paper on opportunistic pairing [11] also suffers from the error propagation problem similar to [1].

In this paper, we propose a novel pairing scheme for user pair formation in a FDF MWRN. In this scheme, each user is paired with a common user, which is chosen by the relay as the user with the best average channel gain. This allows the user with the best channel conditions to contribute to improving the overall system performance by reducing the error propagation in the network. In our prior work in [28], we considered a pairing scheme to reduce error propagation in an amplify and forward (AF) MWRN. However, we considered simple binary phase shift keying (BPSK) modulation in [28], which is not suitable for practical high data rate systems. Also, our prior work in [28] reduces error propagation for a specific channel gain scenario but may not be optimal in terms of the common rate and sum rate. These major limitations of our prior work have motivated us to generalize and extend the prior work. The major contributions of this paper are as follows:

- Considering an L-user FDF MWRN employing sufficiently large dimension lattice codes, we derive upper bounds for the common rate and sum rate with the proposed pairing scheme (cf. Theorems 1–2).
- Considering an L-user FDF MWRN with M-ary quadrature amplitude modulation (QAM) based transmission, which is a special case of lattice code based transmission, we derive the asymptotic average SER with the proposed pairing scheme (cf. Theorem 3).
- We present important insights, obtained from a careful analysis of the results in Theorems 1–3, in the form of Propositions 1–9. Analyzing the results in Theorems 1–3, we compare the performance of the proposed pairing scheme with the existing pairing schemes and show that:
  - For the equal average channel gain scenario, the average common rate and the average sum rate are the same for the proposed and existing pairing schemes, but the average SER improves with the proposed pairing scheme (cf. Propositions 1, 4 and 7).
  - For the unequal average channel gain scenario, the average common rate, the average sum rate and the average SER all improve for the proposed pairing scheme (cf. Propositions 2, 5 and 8).
  - For the variable average channel gain scenario, the average common rate for the proposed pairing scheme is practically the same as the existing schemes, whereas, the average sum rate and the average SER improve for the proposed pairing scheme (cf. Propositions 3, 6 and 9).

The rest of the paper is organized as follows. The generalized system model is presented in Section 2. The proposed pairing scheme is discussed in Section 3 and the general lattice code based transmissions with the proposed pairing scheme are presented in Section 4. The common rate and the sum rate for a FDF MWRN with the proposed scheme is derived in Section 5. The average SER is derived in Section 6. The numerical and simulation results for verification of the analytical solutions are provided in Section 7. Finally, conclusions are provided in Section 8.

Throughout this paper, we use the following notations: $\hat{(\cdot)}$ denotes the estimate of a message, $\hat{\hat{(\cdot)}}$ denotes that the message is estimated for the second time, $|\cdot|$ denotes absolute value of a complex variable, $\|\cdot\|$ denotes Euclidean norm, $\arg(\cdot)$ denotes the argument, $\max(\cdot)$ denotes the maximum value, $\min(\cdot)$ denotes the minimum value, $E_H[\cdot]$ denotes the expected value with respect to random channel coefficients, $\lfloor\cdot\rfloor$ denotes the floor operation, $\log(\cdot)$ denotes logarithm to the base two and $Q(\cdot)$ is the Gaussian Q-function.

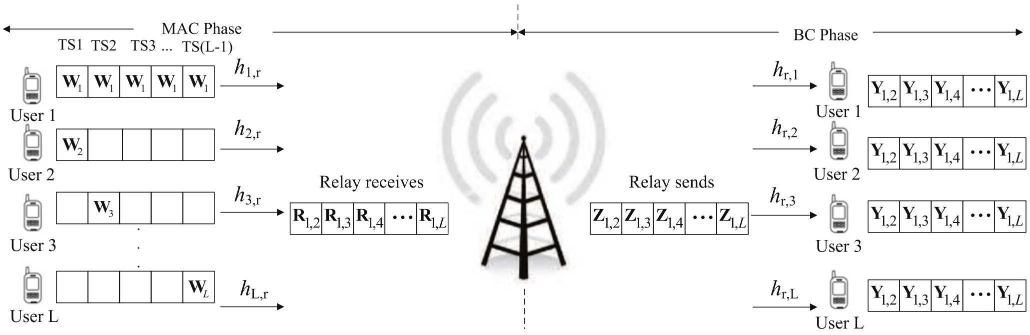

Fig. 1. System model for an L-user multi-way relay network (MWRN), where the users exchange information with each other via the relay R. Here, 'TS' means time slot and user 1 is considered to be the common user (for illustration purpose).

2. Generalized system model

We consider a generalized L-user MWRN, where all the users exchange their information with each other through a single relay. In this setup, a pair of users communicates with each other at a time, while, the remaining users are silent. We assume that the users transmit in a half-duplex manner and they do not have any direct link in between them. The information exchange takes place in two phases – multiple access and broadcast phase – each comprising $L-1$ time slots for an L-user MWRN [1]. In the *multiple access phase*, the users transmit their data in a pairwise manner. That is, in each time slot, a pair of users transmits simultaneously. The user pair formation can be performed using any one of the three different pairing schemes—the existing scheme proposed by Ong et al. [1], the existing scheme proposed by Noori et al. [9] and the proposed pairing scheme.

- In [1], at the ℓth time slot, the ℓth and the $(\ell+1)$th users form a pair. That is, the end users transmit once and the middle users transmit twice. Thus, the total number of user pairs is $\frac{1+2(L-2)+1}{2}=L-1$. For example, in an $L=6$ user MWRN with pairing scheme [1], the user pairs are $(1,2)$, $(2,3)$, $(3,4)$, $(4,5)$ and $(5,6)$. Since, the first (last) user needs to form a pair only with the immediately following (immediately preceding) user and the middle users need to form pairs with both the immediately following and the immediately preceding users, $L-1$ user pairs are sufficient to allow each user to extract all the other users' messages. Further details of this scheme are provided in Section 3.1.
- In [9], at the ℓth time slot, the ℓth and the $(L-\ell+1)$th users form a pair. That is, the first and the $(\lfloor\frac{L}{2}\rfloor+1)$th users transmit once and the remaining $L-2$ users transmit twice, leading to $L-1$ user pairs. For example, in an $L=6$ user MWRN with pairing scheme [9], the user pairs are $(1,6)$, $(6,2)$, $(2,5)$, $(5,3)$ and $(3,4)$. Further details of this scheme are provided in Section 3.1.
- In the proposed pairing scheme, at the ℓth time slot, the common user and the ℓth user form a pair. That is, the common user transmits $L-1$ times and the other users transmit once, leading to $\frac{L-1+L-1}{2}=L-1$ user pairs. For example, in an $L=6$ user MWRN with the proposed pairing scheme and user 1 as the common user, the user pairs are $(1,2)$, $(1,3)$, $(1,4)$, $(1,5)$ and $(1,6)$. Further details of this scheme are provided in Section 3.2.

Thus, in an L user MWRN, the possible number of user pairs is $L-1$ and so, the multiple access phase is comprised of $L-1$ time slots.[1] In the *broadcast phase*, the relay broadcasts the decoded network coded messages to all users. Since each network coded message is broadcast at separate time slots, this phase is also comprised of $L-1$ time slots. After $2(L-1)$ time slots, all users have the network coded messages corresponding to each user pair and then they utilize self information to extract the messages of all the other users. We refer to these $2(L-1)$ time slots in the two phases as one *time frame*. We denote the total amount of information that a user exchanges at each time slot as one *message packet* comprised of T message blocks. That is, in each time frame, each user transmits a message packet and the relay transmits $(L-1)$ message packets, each of length T. Thus, a total of $(2L-1)$ message packets are communicated in an entire time frame. We choose the index for time slot and time frame as t_s and t_f, respectively, and the message block index as t where, $t_s\in[1,L-1]$, $t\in[1,T]$ and $t_f\in[1,F]$, where, F is the total number of time frames. The transmission power of each user is P, whereas, the transmission power of the relay is P_r. At the t_fth time frame and the t_sth time slot, the channel from the jth user to the relay is denoted by $h_{j,r}^{t_s,t_f}$ and the channel from the relay to the jth user by $h_{r,j}^{t_s,t_f}$, where $j\in[1,L]$. The above system model is illustrated for the proposed pairing scheme in Fig. 1, where at 'TS' 1, users 1 and 2 simultaneously transmit W_1 and W_2 and the relay receives the sum of the signals as $R_{1,2}$. Then the relay decodes a function of W_1 and W_2 as $Z_{1,2}$ and broadcasts to

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[1] Note that if the common user in the proposed scheme or the end user in schemes [1] and [9] are allowed to vary, the possible number of user pairs become $C(L,2)$ (i.e., each user forms pair with every other user), where $C(L,2)$ denotes the number of combinations formed by selecting 2 users from L users. However, once the common user or the end user is fixed, the possible number of user pairs is $L-1$ and this is in fact independent of the specific pairing scheme. Hence, the system model in Section 2 is applicable to any pairwise transmission based MWRN.

all the users who receive $Y_{1,2}$ as the network coded message. Similarly, at 'TS' 2, users 1 and 3 transmit, at 'TS' $L-1$ users 1 and L transmit, etc. and the relay performs similar operations as before.

We make the following assumptions regarding the channels:

- The channels are assumed to be block Rayleigh fading channels, which remain constant during one message packet transmission in a certain time slot in a certain multiple access or broadcast phase. The channels in different time slots (e.g., $h_{1,r}^{1,1}$ and $h_{1,r}^{2,1}$) and different time frames (e.g., $h_{1,r}^{1,1}$ and $h_{1,r}^{1,2}$) are considered to be independent. Also, the channels from users to the relay (e.g., $h_{j,r}^{t_s,t_f}$) and the channels from the relay to users (e.g., $h_{r,j}^{t_s,t_f}$) are reciprocal.
- The fading channel coefficients are zero mean complex-valued Gaussian random variables with variances $\sigma^2_{h_{j,r}} = \sigma^2_{h_{r,j}}$ which can be equal for all $j \in [1, L]$ or can be unequal depending upon the channel scenario. This type of channel model is widely adopted in the literature of relay networks [6,29,30].
- The path loss exponent is assumed to be ν. We do not consider shadowing as the effect of shadowing can be disregarded in the long term performance averages. This assumption is adopted in many relevant research works on relay networks [6,29,30].
- The perfect instantaneous channel state information (CSI) of all users is available to the relay. The users have access to the self CSI only, which has been assumed in many research works [29,31,32].
- Perfect channel phase synchronization is assumed because physical layer network coding requires that the signals arrive at the relay with the same phase and this allows benchmark performance to be determined [13,17].

We consider the following three different channel scenarios in this work:

1. *Equal average channel gain scenario:* All the channels from the relay to the users and the users to the relay have equal average channel gain, which remain fixed for all time frames. That is, $E_H[|h_{1,r}^{t_s,t_f}|^2] = E_H[|h_{2,r}^{t_s,t_f}|^2] = \cdots = E_H[|h_{L,r}^{t_s,t_f}|^2]$.
2. *Unequal average channel gain scenario:* All the channels from the relay to the users and the users to the relay have unequal average channel gains which remain fixed for all the time frames. That is, $E_H[|h_{1,r}^{t_s,t_f}|^2] \neq E_H[|h_{2,r}^{t_s,t_f}|^2] \neq \cdots \neq E_H[|h_{L,r}^{t_s,t_f}|^2]$ and $E_H[|h_{j,r}^{t_s,1}|^2] = E_H[|h_{j,r}^{t_s,2}|^2] = \cdots = E_H[|h_{j,r}^{t_s,F}|^2]$.
3. *Variable average channel gain scenario:* All the channels from the relay to the users and the users to the relay have unequal average channel gains and the channel conditions change after a block of T_f' ($T_f' < F$) time frames. That is, $E_H[|h_{1,r}^{t_s,t_f}|^2] \neq E_H[|h_{2,r}^{t_s,t_f}|^2] \neq \cdots \neq E_H[|h_{L,r}^{t_s,t_f}|^2]$ and $E_H[|h_{j,r}^{t_s,aT_f'+1}|^2] = E_H[|h_{j,r}^{t_s,aT_f'+2}|^2] = \cdots = E_H[|h_{j,r}^{t_s,(a+1)T_f'}|^2]$ for $j \in [1, L]$ and $0 \leq a \leq \frac{F}{T_f'} - 1$, where T_f' is the number of time frames after which the unequal average channel gains change.

The above scenarios can model a wide variety of practical channel scenarios. For example, the equal average channel gain scenario is applicable to satellite communications, where the users are equidistant from the relay. The unequal average channel gain scenario is applicable to fixed users (e.g., located at home or workplace) in a network, where the users' distances from the relay are unequal but remain fixed. The variable average channel gain scenario is applicable to mobile users in a network, where the users' distances from the relay are unequal and vary due to user mobility.

3. Proposed pairing scheme for MWRN

In this section, we propose a new pairing scheme for user pair formation in the multiple access phase. Before that we briefly discuss the pairing processes of the two existing pairing schemes in the following subsection.

3.1. Existing pairing schemes

Here we discuss the two reference pairing schemes with which we are going to compare the proposed pairing scheme in the following sections. The first of these two pairing schemes is proposed by Ong et al. in [1]. The pairing scheme forms user pairs with two consecutive users at each time slot. That is, user 1 is paired with user 2, user 2 with user 3, user $L-1$ with user L, etc. Thus, in general, at the ℓth time slot, users ℓ and $(\ell+1)$ are paired together. For extracting messages in [1], the ℓth user first subtracts it own message from the received network coded message from the relay and obtains an estimate of the $(\ell+1)$th user's message. In the following step, the ℓth user uses the extracted message to obtain the estimate of the following user. The second pairing scheme is proposed by Noori et al. in [9]. This scheme forms user pairs by choosing two users from two ends of a sequence in such a way that the second user in each user pair becomes the first user in the following user pair. That is, user 1 is paired with user L, user L with user 2, user 2 with user $L-1$, etc. Thus, in general, the ℓth and the $(L-\ell+1)$th user form a pair at the ℓth time slot when $1 \leq \ell \leq \lfloor L/2 \rfloor$ and the $(\ell+1)$th and $(L-\ell+1)$th user form a pair at the ℓth time slot when $\lfloor L/2 \rfloor < \ell \leq L-1$, where $\lfloor \cdot \rfloor$ denotes the floor operation. The decoding operation is similar to that of [1], as explained before with $\ell+1$ replaced by $L-\ell+1$.

Now, we define the set of principles for the proposed pairing scheme in the following subsection.

3.2. *Principles for the proposed pairing scheme*

P1 Each user in the system is paired with a common user that has the best average channel gain in the system. This common user transmits in all the time slots in the multiple access phase and the other users take turns to form a pair with this common user.
P2 Prior to each multiple access phase, the relay searches for the maximum channel gain user and obtains the common user's index. The index is broadcast by the relay only if it is different from that obtained in the previous time frame.
P3 The common user is kept fixed for all the time slots within a certain time frame. After some time frames, the common user might change depending upon the changing channel conditions.

The proposed pairing scheme allows the best channel in the system to contribute towards the error-free detection of each user's message, which would not be possible if the common user is chosen without considering the channel conditions, as in [1,9]. Note that taking channel state information into account is a well established design principle in wireless communication systems [33].

In the proposed scheme, since the common user is involved in all the transmissions in the multiple access phase, an issue of transmission fairness arises. In the context of the proposed scheme, on average, each user should transmit the same number of times (equivalently consume the same amount of power overall). We propose to achieve transmission fairness for the three channel scenarios, considered in this work, in the following manner:

1. *Equal average channel gain scenario:* In this scenario, since all the users have the same average channel gain, all of them can be regarded as the best average channel gain user. In order to maintain transmission fairness among the users, we select a different common user in each time frame and thus, on average, every user gets the opportunity to become the common user.
2. *Unequal average channel gain scenario:* In this scenario, the common user's transmission power must be scaled by $(L-1)$, since it transmits $(L-1)$ times, whereas, other users transmit only once.
3. *Variable average channel gain scenario:* In this scenario, during each time frame, the user with the best average channel gain is chosen as the common user and this process is repeated for every time frame so that, on average, every user with changing channel conditions, gets the opportunity to become the common user.

Remark 1. Note that, the proposed pairing scheme requires a simple numerical search at the relay and the rest of the transmission mechanism is similar to that of a standard pairwise MWRN. Thus, the proposed pairing scheme does not add much computational complexity at the relay compared to the reference schemes in [1] and [9]. Moreover, the CSI overhead required for the proposed scheme is also the same as that in [1] and [9]. The additional transmission overhead at the relay for the proposed scheme is only due to the information regarding the common user's index, which does not increase the overhead significantly.

4. Signal transmissions with the proposed pairing scheme

In this section, we discuss the general lattice code based transmissions with the proposed pairing scheme in a MWRN. The signal transmission protocols presented in the following section are directly applicable to equal and variable average channel gain scenarios. For the unequal average channel gain scenario, they are applicable with P replaced by $\frac{P}{L-1}$. We denote the ith user as the common user and the ℓth user as the other users, where, $i, \ell \in [1, L]$ and $\ell \neq i$. For the rest of this paper, we consider message exchange within a certain time frame and choose to omit the superscript t_f from the symbols for simplifying the notations.

4.1. *Preliminaries on lattice codes*

As our proposed pairing scheme is based on lattice codes, we first present the definitions of some primary operations on lattice codes, which we have used in the later subsections. Our notations for lattice codes follow those of [2,8]. Further details on lattice codes are available in [26,34–37].

An N-dimensional lattice is a discrete subgroup of the N-dimensional complex field $\mathbb{C}^N$ under the normal vector addition operation and can be expressed as [8,37]:

$$\Lambda = \{\lambda = \mathbf{G}_\Lambda c : c \in \mathbb{Z}^N\} \tag{1}$$

where, $\mathbf{G}_\Lambda \in \mathbb{C}^{N\times N}$ is the generator matrix corresponding to the lattice Λ and $\mathbb{Z}$ is the set of integers.

- The nearest neighbor lattice quantizer maps a point $\mathbf{x} \in \mathbb{C}^N$ to a nearest lattice point $\boldsymbol{\lambda} \in \Lambda$ in Euclidean distance [8]. That is,

$$Q_\Lambda(\mathbf{x}) = \arg\min_{\boldsymbol{\lambda}} \|\mathbf{x} - \boldsymbol{\lambda}\|^2. \tag{2}$$

- The modulo-Λ operation is defined by $\mathbf{x} \bmod \Lambda = \mathbf{x} - Q_\Lambda(\mathbf{x})$ [2,34,35,37].

- The Voronoi region $\mathcal{V}(\Lambda)$ denotes the set of all points in the N-dimensional complex field $\mathbb{C}^N$, which are closest to the zero vector [8], i.e.,

$$\mathcal{V}(\Lambda) = \{\mathbf{x} \in \mathbb{C}^N : Q_\Lambda(\mathbf{x}) = \mathbf{0}\}. \tag{3}$$

- $\psi(\cdot)$ denotes the mapping of messages from a finite dimensional field to lattice points, i.e., $\psi(\mathbf{w}) \in \Lambda$, where $\mathbf{w}$ is a message from a finite dimensional field.
- A coarse lattice Λ is nested in a fine lattice Λ_f, i.e., $\Lambda \subseteq \Lambda_f$, so that the messages mapped into fine lattice points remain in the voronoi region of the coarse lattice.
- The dither vectors $\mathbf{d}$ are generated independently from a uniform distribution over the fundamental Voronoi region $\mathcal{V}(\Lambda)$.

4.2. *Multiple access phase*

In this phase, the common user and one other user transmit simultaneously using FDF based on lattice codes and the relay receives the sum of the signals, i.e., at the $(\ell - 1)$th (ℓth) time slot, users i and ℓ for $i < \ell$ ($i > \ell$) transmit simultaneously. Throughout the rest of this paper, we choose to explain the communication protocols for the case $i < \ell$. The explanations for $i > \ell$ would be the same as before with the $(\ell - 1)$th time slot replaced by the ℓth time slot.

4.2.1. *Communication protocol at the users*

In a certain time frame, the message packet of the ℓth user is denoted by

$$\mathbf{W}_\ell^{t_s} = \begin{cases} \{W_\ell^{t_s,1}, W_\ell^{t_s,2}, \ldots, W_\ell^{t_s,T}\} & t_s = \ell - 1 \\ 0 & t_s \neq \ell - 1, \end{cases} \tag{4}$$

where, the elements $W_\ell^{t_s,t}$ are generated independently and uniformly over a finite field. Similarly, the message packet of the ith user is given by $\mathbf{W}_i = \{W_i^{t_s,1}, W_i^{t_s,2}, \ldots, W_i^{t_s,T}\}$ for $t_s \in [1, L-1]$. Here we omit the index t_s because the ith user transmits the same message packet over all the time slots.

During a certain time frame, in the $t_s = (\ell - 1)$th time slot, the ith user and the ℓth user transmit their messages using lattice codes $\mathbf{X}_i = \{X_i^{t_s,1}, X_i^{t_s,2}, \ldots, X_i^{t_s,T}\}$ and $\mathbf{X}_\ell^{t_s} = \{X_\ell^{t_s,1}, X_\ell^{t_s,2}, \ldots, X_\ell^{t_s,T}\}$, respectively, which can be given by [2,4]:

$$X_i^{t_s,t} = (\psi(W_i^{t_s,t}) + d_i) \bmod \Lambda, \tag{5a}$$

$$X_\ell^{t_s,t} = (\psi(W_\ell^{t_s,t}) + d_\ell) \bmod \Lambda, \tag{5b}$$

where, d_i and d_ℓ are the dither vectors for the ith and the ℓth user. The dither vectors are generated at the users and transmitted to the relay prior to message transmission in the multiple access phase [8].

4.2.2. *Communication protocol at the relay*

The relay receives the signal $\mathbf{R}_{i,\ell}^{t_s} = \{r_{i,\ell}^{t_s,1}, r_{i,\ell}^{t_s,2}, \ldots, r_{i,\ell}^{t_s,T}\}$, where

$$r_{i,\ell}^{t_s,t} = \sqrt{P} h_{i,r}^{t_s} X_i^{t_s,t} + \sqrt{P} h_{\ell,r}^{t_s} X_\ell^{t_s,t} + n_1, \tag{6}$$

where n_1 is the zero mean complex AWGN at the relay with noise variance $\sigma_{n_1}^2 = \frac{N_0}{2}$ per dimension and N_0 is the noise power.

4.3. *Broadcast phase*

In this phase, the relay broadcasts the decoded network coded message and each user receives it.

4.3.1. *Communication protocol at the relay*

The relay scales the received signal with a scalar coefficient α [26] and removes the dithers d_i, d_ℓ scaled by $\sqrt{P} h_{i,r}^{t_s}$ and $\sqrt{P} h_{\ell,r}^{t_s}$, respectively. The resulting signal is given by

$$\begin{aligned} X_r^{t_s,t} &= [\alpha r_{i,\ell}^{t_s,t} - \sqrt{P} h_{i,r}^{t_s} d_i - \sqrt{P} h_{\ell,r}^{t_s} d_\ell] \bmod \Lambda \\ &= [\sqrt{P} h_{i,r}^{t_s} X_i^{t_s,t} + \sqrt{P} h_{\ell,r}^{t_s} X_\ell^{t_s,t} + (\alpha - 1)\sqrt{P}(h_{i,r}^{t_s} X_i^{t_s,t} + h_{\ell,r}^{t_s} X_\ell^{t_s,t}) + \alpha n_1 - \sqrt{P} h_{i,r}^{t_s} d_i - \sqrt{P} h_{\ell,r}^{t_s} d_\ell] \bmod \Lambda \\ &= [\sqrt{P} h_{i,r}^{t_s} \psi(W_i^t) + \sqrt{P} h_{\ell,r}^{t_s} \psi(W_\ell^{t_s,t}) + n] \bmod \Lambda, \end{aligned} \tag{7}$$

where, $n = (\alpha - 1)\sqrt{P}(h_{i,r}^{t_s} X_i^t + h_{\ell,r}^{t_s} X_\ell^{t_s,t}) + \alpha n_1$ and α is chosen to minimize the noise variance [34,35].

The relay decodes the signal in (7) with a lattice quantizer [26,34] to obtain an estimate $\hat{\mathbf{V}}_{i,\ell}^{t_s} = \{\hat{V}_{i,\ell}^{t_s,1}, \hat{V}_{i,\ell}^{t_s,2}, \ldots, \hat{V}_{i,\ell}^{t_s,T}\}$ which is a function of the messages $\mathbf{W}_i$ and $\mathbf{W}_\ell^{t_s}$. For lattice code based transmissions, a message is incorrectly decoded when the received signal is not within the voronoi region $\mathcal{V}$ of the transmitted signal, i.e., in effect, when the noise in the received signal is not within the voronoi region. Since, for sufficiently large N, the voronoi region has a larger volume which leads to $Pr(n \notin \mathcal{V}) \to 0$, $\hat{\mathbf{V}}_{i,\ell}^{t_s}$ approaches $(\psi(\mathbf{W}_i) + \psi(\mathbf{W}_\ell^{t_s})) \bmod \Lambda$. The relay then adds a dither d_r with the network coded message which is generated at the relay and broadcast to the users prior to message transmission in the broadcast phase [8]. Then it broadcasts the resulting message using lattice codes, which is given as $\mathbf{Z}_{i,\ell}^{t_s} = \{Z_{i,\ell}^{t_s,1}, Z_{i,\ell}^{t_s,2}, \ldots, Z_{i,\ell}^{t_s,T}\}$, where $Z_{i,\ell}^{t_s,t} = (\hat{V}_{i,\ell}^{t_s,t} + d_r) \bmod \Lambda$.

4.3.2. *Communication protocol at the users*

The jth user receives $\mathbf{Y}_{i,\ell}^{t_s} = \{Y_{i,\ell}^{t_s,1}, Y_{i,\ell}^{t_s,2}, \ldots, Y_{i,\ell}^{t_s,T}\}$, where

$$Y_{i,\ell}^{t_s,t} = \sqrt{P_r} h_{r,j}^{t_s} Z_{i,\ell}^{t_s,t} + n_2, \tag{8}$$

and n_2 is the zero mean complex AWGN at the user with noise variance $\sigma_{n_2}^2 = \frac{N_0}{2}$ per dimension. At the end of the broadcast phase, the jth user scales the received signal with a scalar coefficient β_j and removes the dithers d_r multiplied by $\sqrt{P_r} h_{r,j}$. The resulting signal is

$$\begin{aligned}[\beta_j Y_{i,\ell}^{t_s,t} - \sqrt{P_r} h_{r,j}^{t_s} d_r] \bmod \Lambda &= [\sqrt{P_r} h_{r,j}^{t_s} \hat{V}_{i,\ell}^{t_s,t} + (\beta_j - 1)\sqrt{P_r} h_{r,j}^{t_s} \hat{V}_{i,\ell}^{t_s,t} + \beta_j n_2] \bmod \Lambda \\ &= [\sqrt{P_r} h_{r,j}^{t_s} \hat{V}_{i,\ell}^{t_s,t} + n'] \bmod \Lambda,\end{aligned} \tag{9}$$

where, $n' = \sqrt{P_r} h_{r,j}^{t_s} (\beta_j - 1) \hat{V}_{i,\ell}^{t_s,t} + \beta_j n_2$ and β_j is chosen to minimize the noise variance [8]. The users then detect the received signal with a lattice quantizer [8] and obtain the estimate $\hat{\hat{\mathbf{V}}}_{i,\ell}^{t_s} = (\psi(\mathbf{W}_i) + \psi(\mathbf{W}_\ell^{t_s})) \bmod \Lambda$, assuming that the lattice dimension is large enough such that $\Pr(n' \notin \mathcal{V})$ approaches zero. After decoding all the network coded messages, each user performs message extraction of every other user by canceling self information.

4.3.3. *Message extraction at the common user*

For the common user (ith user), this message extraction involves simply subtracting the lattice point corresponding to its own message from the lattice network coded messages $\hat{\hat{\mathbf{V}}}_{i,\ell}^{t_s}$. The process can be shown as

$$\psi(\hat{\mathbf{W}}_\ell^{t_s}) = (\hat{\hat{\mathbf{V}}}_{i,\ell}^{t_s} - \psi(\mathbf{W}_i)) \bmod \Lambda, \quad \ell \in [1, L],\ \ell \neq i. \tag{10}$$

4.3.4. *Message extraction at the other users*

For other users, the process is different from the common user. At first, the ℓth user subtracts the scaled lattice point corresponding to its own message, i.e., $\psi(\mathbf{W}_\ell^{t_s})$ from the network coded message received in the $(\ell - 1)$th time slot (i.e., $\hat{\hat{\mathbf{V}}}_{i,\ell}^{t_s}$) and extracts the message of the ith user as $\psi(\hat{\mathbf{W}}_i)$. After that, it utilizes the extracted message of the ith user to obtain the messages of other users in a similar manner. The message extraction process in this case can be shown as

$$\psi(\hat{\mathbf{W}}_i) = (\hat{\hat{\mathbf{V}}}_{i,\ell}^{t_s} - \psi(\mathbf{W}_\ell^{t_s})) \bmod \Lambda, \qquad \psi(\hat{\mathbf{W}}_m^{t_s}) = (\hat{\hat{\mathbf{V}}}_{i,m}^{t_s} - \psi(\hat{\mathbf{W}}_i)) \bmod \Lambda, \quad m \in [1, L],\ m \neq i, \ell. \tag{11}$$

5. Common rate and sum rate analysis

In this section, we investigate common rate and sum rate of the MWRN with the proposed pairing scheme. We first analyze the SNR of each user pair in a MWRN and use these results to obtain expressions for the achievable rates. For the rest of this paper, we simplify the notations by omitting the time slot superscript t_s.

5.1. *SNR analysis*

In a FDF MWRN, the decoding operation is performed after both the multiple access phase and the broadcast phase. Thus, we need to consider the SNR at the users and the SNR at the relay, separately.

5.1.1. *SNR at the users*

The SNR at the users have the same expressions for all the three pairing schemes. The signal transmission from the relay to any user $j \in [1, L]$ is the same as that in a point-to-point fading channel. Thus, the SNR of the mth ($m \in [1, L]$) user's signal received at the jth user is given by:

$$\gamma_j = \frac{P_r |h_{r,j}|^2}{|\beta_j|^2 N_0 + P_r |\beta_j - 1|^2 |h_{r,j}|^2} \tag{12}$$

where, the numerator represents power of the signal part in (9) and the denominator represents the power of the noise term n' in (9).

5.1.2. SNR at the relay

In a FDF MWRN based on lattice coding with the proposed pairing scheme, the SNR of the received signal at the relay can be obtained from (7) as

$$\gamma_r(i,\ell) = \frac{P\min(|h_{i,r}|^2, |h_{\ell,r}|^2)}{|\alpha|^2 N_0 + P|\alpha-1|^2(|h_{i,r}|^2 + |h_{\ell,r}|^2)}, \tag{13}$$

where, the numerator represents the power of the signal part (i.e., $\sqrt{P}h_{i,r}^{t_s}\psi(W_i^t) + \sqrt{P}h_{\ell,r}^{t_s}\psi(W_\ell^{t_s,t})$ in (7)) and the denominator represents the power of the noise terms n in (7).

For the pairing scheme in [1], the SNR received at the relay can be expressed as

$$\gamma_r(i) = \frac{P\min(|h_{\ell,r}|^2, |h_{\ell+1,r}|^2)}{|\alpha|^2 N_0 + P|\alpha-1|^2(|h_{\ell,r}|^2 + |h_{\ell+1,r}|^2)}. \tag{14}$$

Similarly, for the pairing scheme in [9], the SNR at the relay is given by

$$\gamma_r(i) = \frac{P\min(|h_{\ell,r}|^2, |h_{L-\ell+2,r}|^2)}{|\alpha|^2 N_0 + P|\alpha-1|^2(|h_{\ell,r}|^2 + |h_{L-\ell+2,r}|^2)}. \tag{15}$$

Note that (13)–(15) have the same form and differ in the indices of the channel coefficients, which is determined by the pairing scheme.

5.2. Common rate

Common rate indicates the maximum possible information rate of the system that can be exchanged with negligible error. It can be a useful metric for the systems where all the users have the same amount of information to exchange [2]. Assuming lattice codes with sufficiently large dimensions are employed, the common rate for an L-user FDF MWRN is given by [1,9]

$$R_c = \frac{1}{L-1}\min_{\ell-1\in[1,L-1]}\{R_{c,\ell-1}\}, \tag{16}$$

where the factor $\frac{1}{L-1}$ is due to the fact that the message exchange in each of the multiple access and broadcast phases requires $L-1$ time slots and $R_{c,\ell-1}$ is the achievable rate in the $(\ell-1)$th time slot, given by

$$R_{c,\ell-1} = \min\{R_{M,\ell-1}, R_{B,\ell-1}\}, \tag{17}$$

where, $R_{M,\ell-1}$ and $R_{B,\ell-1}$ are the maximum achievable rates at the $(\ell-1)$th time slot during the multiple access phase and the broadcast phase, respectively. Next, we derive the upper bounds on the maximum achievable rates in the multiple access and broadcast phases.

Theorem 1. *For the proposed pairing scheme in a FDF MWRN, the maximum achievable rate during the $(\ell-1)$th time slot in the multiple access phase is upper bounded by*

$$R_{M,\ell-1} \le \frac{1}{2}\log\left(\min\left(\frac{|h_{i,r}|^2}{|h_{i,r}|^2+|h_{\ell,r}|^2} + \frac{P|h_{i,r}|^2}{N_0}, \frac{|h_{\ell,r}|^2}{|h_{i,r}|^2+|h_{\ell,r}|^2} + \frac{P|h_{\ell,r}|^2}{N_0}\right)\right), \tag{18}$$

and the maximum achievable rate during the $(\ell-1)$th time slot in the broadcast phase is upper bounded by

$$R_{B,\ell-1} \le \frac{1}{2}\log\left(1 + \frac{\min_{j\in[1,L]}|h_{j,r}|^2 P_r}{N_0}\right). \tag{19}$$

Proof. See Appendix A.

Note that the factor 1/2 in (18) and (19) comes from the fact that they are the standard expressions for the achievable rate of a multiple access channel and point-to-point channel, respectively, as in [38]. The common rate for the pairing scheme in [1] and in [9] can be obtained by replacing the subscript i with $\ell-1$ and $L-\ell+2$, respectively in (18) and using (19), (17) and (16).

Using Theorem 1 and substituting in (17) and (16), the average common rate for the proposed pairing scheme can be given as in (20d), where the inequality in (20b) holds from Jensen's inequality, the inequality (20c) comes from the fact that

$E_H[\min(A_1, A_2)] \le E_H[A_1], E_H[A_2]$, where A_1, A_2 are independent random variables and in (20d) $E_H[|h_{j,r}|^2] = \sigma^2_{h_{j,r}}$, which is obtained using the property that for a random variable X, $\sigma_X^2 = E[X^2] - (E[X])^2$ and exploiting the fact that the channel coefficients have zero mean.

$$E_H[R_c] \le \frac{1}{2(L-1)} E_H\left[\log\left(\min\left(\frac{1}{1+\frac{|h_{\ell,r}|^2}{|h_{i,r}|^2}} + \frac{P|h_{i,r}|^2}{N_0}, \frac{1}{1+\frac{|h_{i,r}|^2}{|h_{\ell,r}|^2}} + \frac{P|h_{\ell,r}|^2}{N_0}\right)\right)\right] \tag{20a}$$

$$\le \frac{1}{2(L-1)} \log\left(E_H\left[\min\left(\frac{1}{1+\frac{|h_{\ell,r}|^2}{|h_{i,r}|^2}} + \frac{P|h_{i,r}|^2}{N_0}, \frac{1}{1+\frac{|h_{i,r}|^2}{|h_{\ell,r}|^2}} + \frac{P|h_{\ell,r}|^2}{N_0}\right)\right]\right) \tag{20b}$$

$$\le \frac{1}{2(L-1)} \log\left(\min\left(E_H\left[\frac{1}{1+\frac{|h_{\ell,r}|^2}{|h_{i,r}|^2}} + \frac{P|h_{i,r}|^2}{N_0}\right], E_H\left[\frac{1}{1+\frac{|h_{i,r}|^2}{|h_{\ell,r}|^2}} + \frac{P|h_{\ell,r}|^2}{N_0}\right]\right)\right) \tag{20c}$$

$$= \frac{1}{2(L-1)} \log\left(\min\left(\frac{1}{1+\frac{\sigma^2_{h_{\ell,r}}}{\sigma^2_{h_{i,r}}}} + \frac{P\sigma^2_{h_{i,r}}}{N_0}, \frac{1}{1+\frac{\sigma^2_{h_{i,r}}}{\sigma^2_{h_{\ell,r}}}} + \frac{P\sigma^2_{h_{\ell,r}}}{N_0}\right)\right). \tag{20d}$$

Similarly, the average common rate for the pairing scheme in [1] can be expressed as

$$E_H[R_c] \le \frac{1}{2(L-1)} \log\left(\min\left(\frac{1}{1+\frac{\sigma^2_{h_{\ell,r}}}{\sigma^2_{h_{\ell-1,r}}}} + \frac{P\sigma^2_{h_{\ell-1,r}}}{N_0}, \frac{1}{1+\frac{\sigma^2_{h_{\ell-1,r}}}{\sigma^2_{h_{\ell,r}}}} + \frac{P\sigma^2_{h_{\ell,r}}}{N_0}\right)\right), \tag{21}$$

and the average common rate for the pairing scheme in [9] can be given as

$$\begin{aligned} E_H[R_c] \le \frac{1}{2(L-1)} \log\Bigg(\min\Bigg(&\frac{1}{1+\frac{\sigma^2_{h_{\ell-1,r}}}{\sigma^2_{h_{L-\ell+2,r}}}} + \frac{P\sigma^2_{h_{L-\ell+2,r}}}{N_0}, \frac{1}{1+\frac{\sigma^2_{h_{L-\ell+2,r}}}{\sigma^2_{h_{\ell-1,r}}}} + \frac{P\sigma^2_{h_{\ell-1,r}}}{N_0}, \\ &\frac{1}{1+\frac{\sigma^2_{h_{\ell,r}}}{\sigma^2_{h_{L-\ell+2,r}}}} + \frac{P\sigma^2_{h_{L-\ell+2,r}}}{N_0}, \frac{1}{1+\frac{\sigma^2_{h_{L-\ell+2,r}}}{\sigma^2_{h_{\ell,r}}}} + \frac{P\sigma^2_{h_{\ell,r}}}{N_0}\Bigg)\Bigg). \end{aligned} \tag{22}$$

While (20d)–(22) do not provide tight upper bounds on the average common rate, they allow an analytical comparison of the proposed and existing pairing schemes. The main results from the analytical comparison are summarized in Propositions 1–3. Note that in Section 7, the actual expressions of the instantaneous rates are averaged over a large number of channel realizations to corroborate the insights presented in Propositions 1–3.

Proposition 1. *The average common rate for the proposed pairing scheme and the pairing schemes in* [1] *and* [9] *are the same for the equal average channel gain scenario.*

Proof. See Appendix B.

Proposition 2. *The average common rate for the proposed pairing scheme is larger than that of the pairing schemes in* [1] *and* [9] *for the unequal average channel gain scenario.*

Proof. See Appendix B.

Proposition 3. *The average common rate for the proposed pairing scheme is practically the same as that of the pairing schemes in* [1] *and* [9], *for the variable average channel gain scenario.*

Proof. See Appendix B.

5.3. Sum rate

The sum rate indicates the maximum throughput of the system. For a FDF MWRN, the sum rate can be defined as the sum of the achievable rates of all users for a complete round of information exchange.

Theorem 2. *For the proposed pairing scheme in a FDF MWRN, the sum rate is given by:*

$$R_s = \frac{1}{2(L-1)} \sum_{\ell=1,\ell\neq i}^{L} \left(\log\left(\frac{|h_{i,r}|^2}{|h_{i,r}|^2 + |h_{\ell,r}|^2} + \frac{P|h_{i,r}|^2}{N_0} \right) + \log\left(\frac{|h_{\ell,r}|^2}{|h_{i,r}|^2 + |h_{\ell,r}|^2} + \frac{P|h_{\ell,r}|^2}{N_0} \right) \right). \tag{23}$$

Proof. see Appendix C.

Note that the sum rate for the pairing scheme in [1] and the pairing scheme in [9] can be obtained by replacing the subscript i with $\ell - 1$ and $L - \ell + 2$, respectively in (23).

Using Theorem 2, the average sum rate (averaged over all channel realizations) for the proposed pairing scheme can be given as in (24), using similar steps as in (20a), (20b) and (20d).

$$E_H[R_s] \leq \frac{1}{2(L-1)} \sum_{\ell=1,\ell\neq i}^{L} \left(\log\left(\frac{1}{1+\frac{\sigma^2_{h_{\ell,r}}}{\sigma^2_{h_{i,r}}}} + \frac{P\sigma^2_{h_{i,r}}}{N_0} \right) + \log\left(\frac{1}{1+\frac{\sigma^2_{h_{i,r}}}{\sigma^2_{h_{\ell,r}}}} + \frac{P\sigma^2_{h_{\ell,r}}}{N_0} \right) \right). \tag{24}$$

Similarly, the average sum rate for the pairing scheme in [1] can be written as

$$E_H[R_s] \leq \frac{1}{2(L-1)} \sum_{\ell=2}^{L} \left(\log\left(\frac{1}{1+\frac{\sigma^2_{h_{\ell,r}}}{\sigma^2_{h_{\ell-1,r}}}} + \frac{P\sigma^2_{h_{\ell-1,r}}}{N_0} \right) + \log\left(\frac{1}{1+\frac{\sigma^2_{h_{\ell-1,r}}}{\sigma^2_{h_{\ell,r}}}} + \frac{P\sigma^2_{h_{\ell,r}}}{N_0} \right) \right), \tag{25}$$

and the average sum rate for the pairing scheme in [9] can be written as

$$\begin{aligned} E_H[R_s] \leq \frac{1}{2(L-1)} & \sum_{\ell=2}^{\lfloor L/2\rfloor+1} \left(\log\left(\frac{1}{1+\frac{\sigma^2_{h_{\ell-1,r}}}{\sigma^2_{h_{L-\ell+2,r}}}} + \frac{P\sigma^2_{h_{L-\ell+2,r}}}{N_0} \right) + \log\left(\frac{1}{1+\frac{\sigma^2_{h_{L-\ell+2,r}}}{\sigma^2_{h_{\ell-1,r}}}} + \frac{P\sigma^2_{h_{\ell-1,r}}}{N_0} \right) \right) \\ & + \sum_{\ell=\lfloor L/2\rfloor+2}^{L} \left(\log\left(\frac{1}{1+\frac{\sigma^2_{h_{\ell,r}}}{\sigma^2_{h_{L-\ell+2,r}}}} + \frac{P\sigma^2_{h_{L-\ell+2,r}}}{N_0} \right) + \log\left(\frac{1}{1+\frac{\sigma^2_{h_{L-\ell+2,r}}}{\sigma^2_{h_{\ell,r}}}} + \frac{P\sigma^2_{h_{\ell,r}}}{N_0} \right) \right). \end{aligned} \tag{26}$$

(24)–(26) provide upper bounds on the actual average sum rate and they allow an analytical comparison of the proposed and existing pairing schemes. The main results are summarized in Propositions 4–6. Note that similar to the case of common rate, in Section 7, the actual expression for the instantaneous sum rate in (23) is averaged over a large number of channel realizations to validate the insights presented in Propositions 4–6.

Proposition 4. *The average sum rate of the proposed pairing scheme and the pairing schemes in* [1] *and* [9] *are the same for the equal average channel gain scenario.*

Proof. See Appendix D.

Proposition 5. *The average sum rate of the proposed pairing scheme is larger than that of the pairing schemes in* [1] *and* [9] *for the unequal average channel gain scenario.*

Proof. See Appendix D.

Proposition 6. *The average sum rate of the proposed pairing scheme is larger than that of the pairing schemes in* [1] *and* [9] *for the variable average channel gain scenario.*

Proof. See Appendix D.

6. Error performance analysis

In this section, we characterize the error performance of a FDF MWRN with the new pairing scheme. We provide the analytical derivations for M-QAM modulation, which is a 2 dimensional lattice code and is widely used in practical wireless communication systems.

6.1. System model

In the M-QAM modulated FDF MWRN system, during a certain time frame, in the $t_s = (\ell - 1)$th time slot, the ith user and the ℓth user transmit their messages $\mathbf{W}_i$ and $\mathbf{W}_\ell$ which are M-QAM modulated to $\mathbf{X}_i = \{X_i^1, X_i^2, \ldots, X_i^T\}$ and $\mathbf{X}_\ell = \{X_\ell^1, X_\ell^2, \ldots, X_\ell^T\}$, respectively, where $X_i^t, X_\ell^t = a + jb$ and $a, b \in \{\pm 1, \pm 3, \ldots, \pm(\sqrt{M} - 1)\}$. The relay receives the signal $\mathbf{R}_{i,\ell}$ (see (6)) and decodes it using ML criterion [31] and obtains an estimate $\hat{\mathbf{V}}_{i,\ell}$ of the network coded symbol $\mathbf{V}_{i,\ell} = (\mathbf{W}_i + \mathbf{W}_\ell) \mod M$ as in [13,39]. The relay then broadcasts the estimated network coded signal after M-QAM modulation, which is given as $\mathbf{Z}_{i,\ell}$. The jth ($j \in [1, L]$) user receives $\mathbf{Y}_{i,\ell}$ (see (8)) and detects the received signal through ML criterion [31] to obtain the estimate $\hat{\hat{\mathbf{V}}}_{i,\ell}$. After decoding all the network coded messages, each user performs message extraction. For the common user (ith user), this message extraction involves subtracting its own message $\mathbf{W}_i$ from the network coded messages $\hat{\hat{\mathbf{V}}}_{i,\ell}$ and then performing the modulo-M operation. The process can be shown as

$$\hat{\mathbf{W}}_\ell = (\hat{\hat{\mathbf{V}}}_{i,\ell} - \mathbf{W}_i + M) \bmod M, \qquad \hat{\mathbf{W}}_{\ell+1} = (\hat{\hat{\mathbf{V}}}_{i,\ell+1} - \mathbf{W}_i + M) \bmod M, \ldots, \hat{\mathbf{W}}_L = (\hat{\hat{\mathbf{V}}}_{i,L} - \mathbf{W}_i + M) \bmod M. \quad (27)$$

For other users, the message extraction process can be shown as

$$\hat{\mathbf{W}}_i = (\hat{\hat{\mathbf{V}}}_{i,\ell} - \mathbf{W}_\ell + M) \bmod M, \qquad \hat{\mathbf{W}}_{\ell+1} = (\hat{\hat{\mathbf{V}}}_{i,\ell+1} - \hat{\mathbf{W}}_i + M) \bmod M, \ldots, \hat{\mathbf{W}}_L = (\hat{\hat{\mathbf{V}}}_{i,L} - \hat{\mathbf{W}}_i + M) \bmod M. \quad (28)$$

6.2. SER analysis for the proposed pairing scheme

In this subsection, we investigate the error performance of a FDF MWRN with the proposed pairing scheme. Unlike the pairing schemes in [1] and [9], the error performance of all the users is not the same for the proposed pairing scheme. Hence, we need to obtain separate expressions for the error probabilities at the common user (ith user) and other users (ℓth user).

First, we obtain the probability of incorrectly decoding a network coded message at the common user and the other users. Since, any M-QAM signal with square constellation (i.e., $\sqrt{M} \in \mathbb{Z}$) can be decomposed to two $\sqrt{M}$-PAM signals [33], the network coded signal from a linear combination of two M-QAM signals can be decomposed to a network coded signal from two $\sqrt{M}$-PAM signals. Thus, we can obtain the probability that the ith (common) user incorrectly decodes the network coded message involving its own message and the mth user's message, as:

$$P_{FDF}(i, m) = 1 - \left(1 - P_{\sqrt{M}-PAM,NC}(i, m)\right)^2, \quad (29)$$

where $P_{\sqrt{M}-PAM,NC}(i, m)$ is the probability of incorrectly decoding a network coded message resulting from the sum of two $\sqrt{M}$-PAM signals from the ith and the mth user and is derived in Appendix E.

Similarly, The probability that the ℓth (other) user incorrectly decodes the network coded message involving the ith user's message and its own message or other user's messages is given as:

$$P_{FDF}(\ell, m) = \begin{cases} 1 - \left(1 - P_{\sqrt{M}-PAM,NC}(\ell, m)\right)^2 & m = i \\ 1 - \left(1 - P_{\sqrt{M}-PAM,NC}(i, m)\right)^2 & m \in [1, L], m \neq i, \ell \end{cases} \quad (30)$$

where $P_{\sqrt{M}-PAM,NC}(\ell, m)$ is the probability of incorrectly decoding a network coded message, i.e., the sum of two $\sqrt{M}$-PAM signals of the ℓth and the mth user and can be obtained from Appendix E.

Using (29) and (30), the average SER at the common user and the other users can be derived using the technique proposed in [27]. The main steps for obtaining the average SER results are outlined below. The details of these steps can be found in Appendix F.

1. Determine the probabilities that the ith user and the ℓth user incorrectly decode a network coded message, respectively.
2. Define the possible error cases for the kth ($k \in [1, L - 1]$) error event at the ith and the ℓth user, where the kth error event means that exactly k number of users' messages are incorrectly decoded.
3. Express the probabilities of the aforementioned error cases in terms of the probabilities of incorrectly decoding a network coded message.
4. Combine the probabilities of different error cases to determine the probability of the kth error event at the ith and the ℓth user.
5. Obtain the expected probability of all the error events to determine the exact average SER expression.
6. Apply the high SNR approximation to obtain approximate but accurate average SER expressions.

The average SER result, obtained through the above steps, is summarized in the following Theorem.

Theorem 3. *For the proposed pairing scheme in a FDF MWRN, the average SER at the ith (common) user, at high SNR, is given by:*

$$P_{i,avg} = \frac{1}{L-1}\sum_{m=1, m\neq i}^{L} P_{FDF}(i, m), \tag{31}$$

and the average SER at the ℓth (other) users, at high SNR, is given by:

$$P_{\ell,avg} = \frac{1}{L-1}\left(\sum_{m=1, m\neq i,\ell}^{L} P_{FDF}(\ell, m) + (L-1)P_{FDF}(\ell, i)\right). \tag{32}$$

Proof. See Appendix F.

Remark 2. From Theorem 3, it can be identified that the average SER at the other (ℓth) users is at least twice compared to the average SER at the common (ith) user. This can be intuitively explained from the fact that the ith user needs to correctly decode only one network coded message ($V_{i,m}$) to correctly decode the mth user's message. However, the ℓth user needs to correctly decode two network coded messages ($V_{i,m}$ and $V_{i,\ell}$) to correctly decode the mth user's message. Thus, the average SER at the other users would at least be twice compared to that at the common user.

Using Theorem 3 and the average SER result for the pairing scheme in [1], we can compare the performance of the proposed and the existing pairing schemes. Note that the error performance of the pairing scheme in [9] would be the same as the pairing scheme in [1], as the basic pairing process is the same for both these schemes and only the pairing orders are different. The main results are summarized in Propositions 7–9.

Proposition 7. *The average SER of an L-user FDF MWRN with the proposed pairing scheme is lower than the pairing scheme in* [1] *by a factor of* $\frac{L}{2}$ *for the common user and a factor of approximately* $\frac{L}{4}$ *for other users under the equal average channel gain scenario.*

Proof. See Appendix G.

Proposition 8. *The average SER of an L-user FDF MWRN with the proposed pairing scheme is always lower than the pairing scheme in* [1] *for all users under the unequal average channel gain scenario.*

Proof. See Appendix G.

Proposition 9. *The average SER of an L-user FDF MWRN with the proposed pairing scheme is always lower than the pairing scheme in* [1] *for all users under the variable average channel gain scenario.*

Proof. See Appendix G.

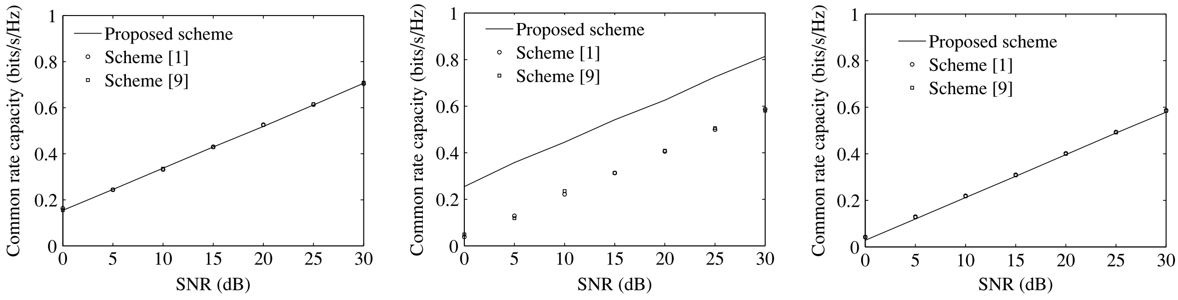
From Propositions 7–9, it is clear that choosing the user with the best average channel gain as the common user reduces the average SER of the FDF MWRN.

7. Results

In this section, we provide numerical results to verify the insights provided in Propositions 1–6. We also provide simulation results to verify Propositions 7–9. We consider an $L = 10$ user FDF MWRN where each user transmits a packet of $T = 2000$ bits and uses 16-QAM modulation. The power at the users, P and the power at the relay, P_r are assumed to be equal and normalized to unity. The SNR per bit per user is defined as $\frac{1}{N_0}$. Following [6], the channel variance for the jth user is modeled by $\sigma^2_{h_{j,r}} = (1/(d_j/d_0))^{\nu}$, where d_0 is the reference distance, d_j is the distance between the jth user and the relay which is assumed to be uniformly randomly distributed between 0 and d_0, and ν is the path loss exponent, which is assumed to be 3. All distances, once chosen, remain constant for unequal channel gain scenario and are randomly chosen every time frame (i.e., worst case, $T_f' = 1$) for variable channel gain scenario. Note that all the distances are the same for the equal average channel gain scenario. Though the pdf of the channel gain is often used for channel modeling as in [40–42], this distance based channel modeling is another approach which is widely adopted in the literature of cooperative relay networks [13,21,29,30,43] and it allows to consider the impact of long term path loss. All results are averaged over $F = 100$ time frames.

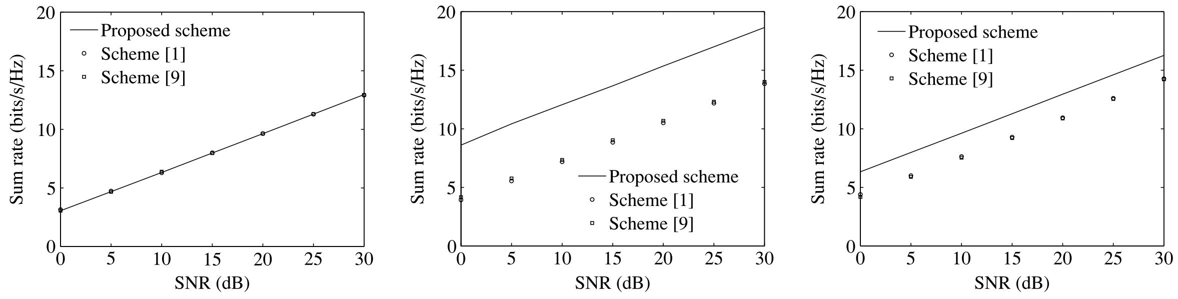
7.1. Common rate

Fig. 2 shows the common rate for the proposed and the existing pairing schemes in an $L = 10$ user FDF MWRN. All the numerical results are obtained by averaging the instantaneous common rates for the pairing schemes over a large number of



(a) Equal channel gain scenario. (b) Unequal channel gain scenario. (c) Variable channel gain scenario.

Fig. 2. Common rate for a $L = 10$ user FDF MWRN with different pairing schemes and different channel scenarios.



(a) Equal channel gain scenario. (b) Unequal channel gain scenario. (c) Variable channel gain scenario.

Fig. 3. Sum rate for a $L = 10$ user FDF MWRN with different pairing schemes and different channel scenarios.

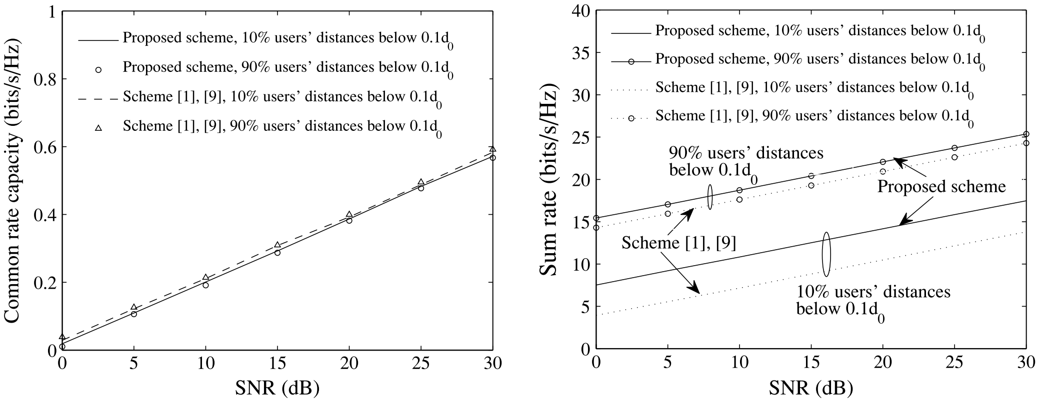
channel realizations. In this figure (and also in Figs. 3–4), we have considered only the upper bounds on the achievable rates because obtaining the actual achievable rates is a complicated problem as it depends upon the coding and the actual choice of the lattice. Thus, a detailed analysis of the actual achievable rates is outside the scope of this work. Now, Fig. 2(a) shows that all the pairing schemes have the same average common rate in equal average channel gain scenario, which verifies Proposition 1. The common rate of the proposed pairing scheme is larger than the existing pairing schemes for the unequal average channel gain scenario in Fig. 2(b). This is because, scaling the common user's power to ensure transmission fairness decreases the ratio of the maximum and the minimum average channel gains in (20d), resulting in a larger common rate. For variable average channel gain scenario, we can see that the common rate for the proposed scheme is practically the same as that of the existing pairing schemes. This verifies Propositions 2 and 3, respectively. Note that the pairing schemes in [1] and [9] give the same results in all the figures since the pairing process is the same for these schemes. Thus, when the achievable rates of these two schemes are averaged over a number of channel gain settings, the performances become the same.

7.2. Sum rate

Fig. 3 shows the sum rate for the proposed and the existing pairing schemes in an $L = 10$ user FDF MWRN for the three channel scenarios. All the numerical results are obtained by averaging the instantaneous sum rates for the pairing schemes over a large number of channel realizations. Fig. 3(a) shows that all the pairing schemes have the same average sum rate for equal average channel gain scenario, which verifies Proposition 4. Similarly, Figs. 3(b) and 3(c) show that the average sum rate for the proposed pairing scheme is larger than the existing pairing schemes, which is in line with Propositions 5 and 6. Intuitively, this can be explained as follows. In the proposed pairing scheme, the common user with the maximum average channel gain transmits more times than the other users. Unless all the average channel gains are equal, this results in a larger sum rate compared to the existing pairing schemes.

7.3. Robustness of the proposed pairing scheme

To illustrate robustness of the proposed pairing scheme, we consider two special cases of the variable average channel gain scenario, where (i) 10% of the users have distances below $0.1d_0$ (i.e., only a small proportion of the users are close to the relay and so, they have good channel conditions) and (ii) 90% of the users have distances below $0.1d_0$ (i.e., a large proportion of users have good channel conditions). Fig. 4(a) plots the average common rate and Fig. 4(b) plots the average sum rate for the proposed and existing pairing schemes. We can see from Fig. 4(a) that the common rate does not change much when either 10% or 90% of users have good channel conditions as it depends upon the minimum average channel gain in the



(a) Common rate.

(b) Sum rate.

Fig. 4. Common rate and sum rate of an $L = 10$ user FDF MWRN when 10% and 90% users have distances below $0.1d_0$.

system. However, we can see from Fig. 4(b) that when the number of users with good channel conditions falls from 90% to 10%, the sum rate of the proposed scheme degrades to a much lesser extent, compared to the existing pairing schemes. This is because the average sum rate of the proposed pairing scheme depends to a greater extent on the common user's average channel gain compared to the other users' average channel gain (as evident from (24)). However, for the existing pairing schemes, the sum rate depends on all the channel gains equally (as evident from (25) and (26)) and degrades to a greater extent. This illustrates the robustness of the proposed pairing scheme.

7.4. Average SER

Figs. 5(a), (b) and 6(a) plot the average SER of the proposed and the existing pairing schemes in an $L = 10$ user FDF MWRN for equal channel gain scenario (Fig. 5(a)), unequal channel gain scenario (Fig. 5(b)) and variable channel gain scenario (Fig. 6(a)). We can see from all the figures that the simulation results match perfectly with the analytical results at mid to high SNRs. This verifies the accuracy of Theorem 3. Note that the existing pairing schemes in [1] and [9] have the same average SER. So, only the results for pairing scheme in [1] have been shown in the above figures. Figs. 5(a), (b) and 6(a) show that the proposed pairing scheme outperforms the existing pairing schemes, in terms of average SER, which verifies Propositions 7–9. In addition, Fig. 5(a) shows that the average SER at the common user and other users are 5 times and nearly 2.5 times less than that of the existing pairing schemes. This verifies the insight presented by Remark 1 and Proposition 7.

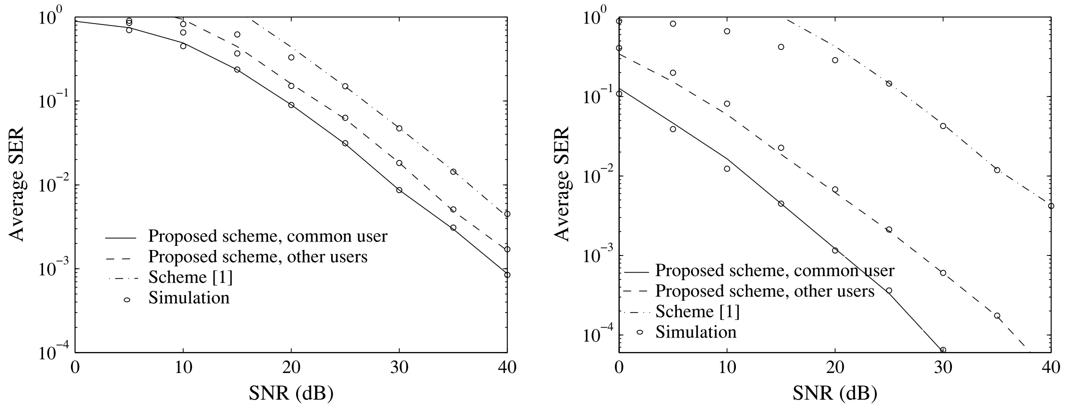
Fig. 6(b) plots the average SER of the proposed and the existing pairing schemes for the special cases of the variable average channel gain scenario when (i) 10% of the users have distances below $0.1d_0$ and (ii) 90% of the users have distances below $0.1d_0$. The figure shows that the average SER for the existing pairing schemes worsens by a larger extent compared to that of the proposed scheme with the degradation in the users' channel conditions. For the proposed pairing scheme, when the number of users with good channel conditions increases from 10% to 90%, the average SER at other users improve significantly and approaches the average SER at the common user. This is because the average SER at the ℓth user depends not only on its own channel conditions, but also the channel conditions of the common (ith) user and the mth user (see (32)). This improvement in the overall channel conditions results in improvement in the average SER, which illustrates the superiority of the proposed pairing scheme.

8. Conclusions

In this paper, we have proposed a novel user pairing scheme in a FDF MWRN. We have derived the upper bound on the average common rate (Theorem 1) and the average sum rate (Theorem 2) and the asymptotic average SER (Theorem 3) for the proposed pairing scheme. We have analyzed the results in Theorems 1–3 to compare the performance of the proposed scheme with existing pairing schemes under different channel scenarios. The main insights are summarized in Propositions 1–9. Our analysis shows that the proposed pairing scheme improves the aforementioned performance metrics compared to that of the existing pairing schemes for different channel conditions.

Appendix A. Proof of Theorem 1

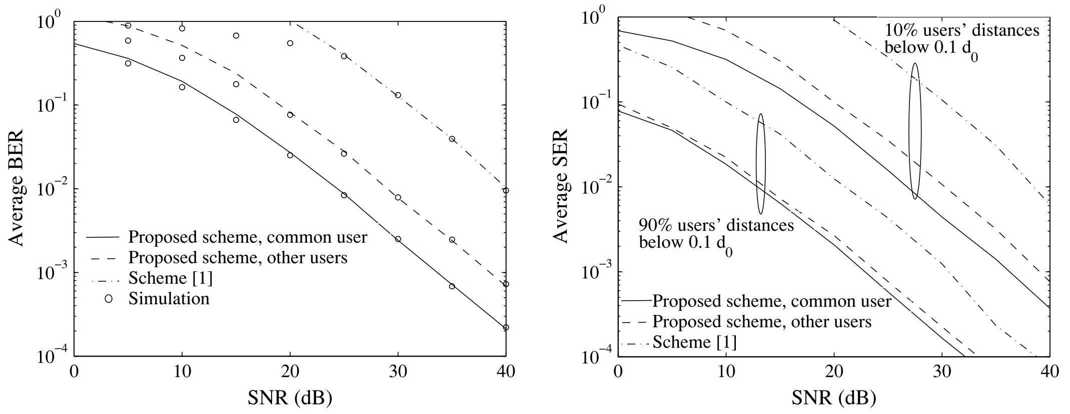
In the proposed pairing scheme, the ith and the ℓth user transmit simultaneously in the $(\ell-1)$th time slot in the multiple access phase. Also, in the broadcast phase, in the $(\ell-1)$th time slot, the relay broadcasts the decoded network coded message to all the users. For the *multiple access phase*, the optimum values of α and β_j in (12) and (13), respectively, are obtained by



(a) Equal channel gain scenario.

(b) Unequal channel gain scenario.

Fig. 5. Average SER for equal and unequal average channel gains in an $L = 10$ user FDF MWRN with different pairing schemes.



(a) Variable channel gain scenario.

(b) Variable channel gain scenario where 10% and 90% users, respectively, have distances below $0.1d_0$.

Fig. 6. Average SER for variable average channel gains in an $L = 10$ user FDF MWRN with different pairing schemes.

setting $\frac{dn}{d\alpha} = 0$ and $\frac{dn'}{d\beta_j} = 0$, where n and n' are given in (7) and (9), respectively. From this, we obtain $\alpha = \frac{P|h_{i,r}|^2+P|h_{\ell,r}|^2}{P|h_{i,r}|^2+P|h_{\ell,r}|^2+N_0}$ and $\beta_j = \frac{P_r|h_{j,r}|^2}{P_r|h_{j,r}|^2+N_0}$. Substituting these values in (13) and (12), (18) and (19) can be derived following the steps in [35] and [9], which are summarized as follows. First, we assume that there exists a rate $\bar{R} < R_{M,\ell-1}$ for which $\Pr(n \notin \mathcal{V})$ (see (7)) is upper bounded by $e^{-N(E_p(\mu))}$, where E_p is the Poltyrev exponent, $\mu = 2^{2(R_{M,\ell-1}-\bar{R})} - O_N(1)$ [35] is the volume to noise ratio of the lattice Λ with respect to the noise n, $O_N(1)$ indicates that the difference between μ and $2^{2(R_{M,\ell-1}-\bar{R})}$ is a first degree function of N and Λ is Poltyrev-good [35]. Then calculating μ and comparing with $2^{2(R_{M,\ell-1}-\bar{R})} - O_N(1)$ gives (18) in Theorem 1. For the *broadcast phase*, (19) in Theorem 1 can be obtained from the point to point channel of the users. The details are omitted here for the sake of brevity. This completes the proof.

Appendix B. Proof of Propositions 1–3

Proof of Proposition 1. For the equal average channel gain scenario, $\sigma^2_{h_{i,r}} = \sigma^2_{h_{\ell,r}} = \sigma^2_{h_{\ell-1,r}} = \sigma^2_{h_{L-\ell+2,r}}$. Thus, the average common rate expressed by (21), (22) and (20d) becomes the same for all the three pairing schemes. This proves Proposition 1.

Proof of Proposition 2. For unequal average channel gain scenario, as explained in Section 3, the transmit power of the ith user needs to be scaled by $(L-1)$ to ensure transmission fairness. As a result, $|h_{i,r}|^2$ can be replaced by $\frac{|h_{i,r}|^2}{L-1}$ in (13). In addition, for a fair comparison with the existing pairing schemes, the transmit power P in the proposed scheme, needs to be multiplied by a factor $(2L-2)$. This is because in the proposed pairing scheme, the common user transmits $(L-1)$ times with power $\frac{P}{L-1}$ and the other $(L-1)$ users transmit once with power P. Hence, the average power per user becomes P. However, for the existing pairing schemes, the average power per user is $\frac{2L-2}{L}P$. Overall, (20d) can be modified by scaling

$\sigma^2_{h_{i,r}}$ with $L-1$ and replacing P with $(2L-2)P$. Thus, the average common rate in (20d) is

$$E_H[R_c] \le \frac{1}{2(L-1)} \log\left(\min\left(\frac{1}{1+\frac{(L-1)\sigma^2_{h_{\ell,r}}}{\sigma^2_{h_{i,r}}}} + \frac{(2L-2)P\sigma^2_{h_{i,r}}}{(L-1)N_0}, \frac{1}{1+\frac{\sigma^2_{h_{i,r}}}{(L-1)\sigma^2_{h_{\ell,r}}}} + \frac{(2L-2)P\sigma^2_{h_{\ell,r}}}{N_0}\right)\right). \tag{33}$$

We consider two cases:

- *case* 1: $\sigma^2_{h_{i,r}} > (L-1)\sigma^2_{h_{\ell,r}}$. In this case, the second quantity on the right hand side of (33) will be the minimum. Then, comparing (33) and (21) shows that $\frac{(2L-2)P\sigma^2_{h_{\ell,r}}}{N_0} > \frac{P\sigma^2_{h_{\ell,r}}}{N_0}$. Thus, the average common rate for scheme [1] will be smaller than that for the proposed pairing scheme, when $\sigma^2_{h_{i,r}} < (L-1)\sigma^2_{h_{\ell-1,r}}$. Similarly, it can be shown that for the pairing scheme in [9], the average common rate is smaller than that for the proposed scheme for $\sigma^2_{h_{i,r}} < (L-1)\sigma^2_{h_{L-\ell+2,r}}$.
- *case* 2: $\sigma^2_{h_{i,r}} < (L-1)\sigma^2_{h_{\ell,r}}$. In this case, the first quantity on the right hand side of (33) will be the minimum. Then comparing (33) and (21) shows that the common rate of scheme [1] will be smaller than that of the proposed pairing scheme, when $\sigma^2_{h_{i,r}} > (L-1)\sigma^2_{h_{\ell-1,r}}$. Similarly, it can be shown that for the pairing scheme in [9], the average common rate is smaller than that of the proposed scheme for $\sigma^2_{h_{i,r}} > (L-1)\sigma^2_{h_{L-\ell+2,r}}$.

Combining the result from the two cases, the proposed pairing scheme will have larger average common rate compared to the two other pairing schemes, which proves Proposition 2.

Proof of Proposition 3. For the variable channel gain scenario, $\sigma^2_{h_{i,r}}$ in (20d) is the largest average channel gain in the system. Thus, from (20d), it can be shown that $\frac{\sigma^2_{h_{i,r}}}{\sigma^2_{h_{\ell,r}}} > \frac{\sigma^2_{h_{\ell,r}}}{\sigma^2_{h_{i,r}}}$ and the second quantity on the right hand side of the inequality in (20d) is the minimum. Then comparing (20d) and (21) would show that $\frac{\sigma^2_{h_{\ell-1,r}}}{\sigma^2_{h_{\ell,r}}} \le \frac{\sigma^2_{h_{i,r}}}{\sigma^2_{h_{\ell,r}}}$. Similarly, from (20d) and (22), it can be shown that $\frac{\sigma^2_{h_{L-\ell+2,r}}}{\sigma^2_{h_{\ell-1,r}}} \le \frac{\sigma^2_{h_{i,r}}}{\sigma^2_{h_{\ell,r}}}$ and $\frac{\sigma^2_{h_{L-\ell+2,r}}}{\sigma^2_{h_{\ell,r}}} \le \frac{\sigma^2_{h_{i,r}}}{\sigma^2_{h_{\ell,r}}}$. However, the impact of either of these ratios on the overall average common rate is small compared to that of the term $\frac{P\sigma^2_{h_{\ell,r}}}{N_0}$ in (20d), (21) and (22). Thus, the common rate for the proposed scheme will be practically the same as that of the existing pairing schemes in [1] and [9], which proves Proposition 3.

Appendix C. Proof of Theorem 2

The achievable rate at the $(\ell-1)$th time slot can be obtained from (17). Since, $\frac{|h_{i,r}|^2}{|h_{i,r}|^2+|h_{\ell,r}|^2} < 1$, the achievable rate at the $(\ell-1)$th time slot will be determined by the achievable rate at the corresponding time slot in the multiple access phase. Then, obtaining the achievable rate in all the time slots and adding them results into (23). The detailed steps are omitted here for the sake of brevity.

Appendix D. Proof of Propositions 4–6

Proof of Proposition 4. For the equal average channel gain scenario, $\sigma^2_{h_{i,r}} = \sigma^2_{h_{\ell,r}} = \sigma^2_{h_{\ell-1,r}} = \sigma^2_{h_{L-\ell+2,r}}$. Thus, the sum rates expressed by (24)–(26) become the same for all the three pairing schemes, which proves Proposition 4.

Proof of Proposition 5. For the unequal average channel gain scenario, if the common user is made to transmit at all the time slots with scaled power, the sum rate can be obtained from (24) with $\sigma^2_{h_{i,r}}$ scaled by $L-1$ and P replaced with $(2L-2)P$. In this case, the average sum rate in (24) becomes

$$E_H[R_s] = \frac{1}{2(L-1)} \sum_{\ell=1,\ell\ne i}^{L} \left(\log\left(\frac{1}{1+\frac{(L-1)\sigma^2_{h_{\ell,r}}}{\sigma^2_{h_{i,r}}}} + \frac{(2L-2)P\sigma^2_{h_{i,r}}}{(L-1)N_0}\right) + \log\left(\frac{1}{1+\frac{\sigma^2_{h_{i,r}}}{(L-1)\sigma^2_{h_{\ell,r}}}} + \frac{(2L-2)P\sigma^2_{h_{\ell,r}}}{N_0}\right)\right). \tag{34}$$

Comparing (34) and (25) shows that $2\sigma^2_{h_{i,r}} > \sigma^2_{h_{\ell-1,r}}$ and $(2L-2)\sigma^2_{h_{\ell,r}} > \sigma^2_{h_{\ell,r}}$. In a similar manner, it can be shown that the average sum rate of the proposed scheme is larger than that of the scheme in [9]. This completes the proof for Proposition 5.

Proof of Proposition 6. For the variable average channel gain scenario, we have $\sigma^2_{h_{i,r}} \geq \sigma^2_{h_{\ell-1,r}}$. Hence, it is clear that $\sum_{\ell=1,\ell\neq i}^{L} \sigma^2_{h_{i,r}} > \sum_{\ell=2}^{L} \sigma^2_{h_{\ell-1,r}}$. Similarly, it can be shown that $\sum_{\ell=1,\ell\neq i}^{L} \sigma^2_{h_{i,r}} > \sum_{\ell=2}^{L} \sigma^2_{h_{L-\ell+2,r}}$. Thus the proposed pairing scheme will have a larger average sum rate (given by (24)), compared to that of the pairing schemes in [1] and [9] (given by (25) and (26), respectively). This proves Proposition 6.

Appendix E. Derivation of $P_{\sqrt{M}-PAM,NC}(i, m)$ in (29)

In this appendix, we derive the probability of incorrectly decoding a PAM network coded signal by building on the symbol mapping idea in [39]. We detail the necessary steps to obtain an exact expression for use in the analysis.

We assume $\sqrt{M}$-PAM signals at the ith and the mth users, such that the users' signals can take values from the set $\mathcal{S} = \{\pm 1, \pm 3, \ldots, \pm(\sqrt{M} - 1)\}$ and we denote each element of the set $\mathcal{S}$ as s. The true network coded signal resulting from the sum of the $\sqrt{M}$-PAM signals have a constellation with $(2\sqrt{M} - 1)$ points, which takes values from the set $\mathcal{S}_{NC} = \{0, \pm 2, \ldots, \pm(2\sqrt{M} - 2)\}$.

In a noiseless environment, the relay maps the network coded signal to a $\sqrt{M}$-PAM signal s in such a way that the same network coded signal is not mapped to different elements of $\mathcal{S}$ (i.e., there is no ambiguity). This can be ensured by mapping the network coded signal into modulo-$\sqrt{M}$ sum of the actual symbols at the ith and the mth user. In a noisy environment, the relay maps the network coded signal into $\hat{s}$ and broadcasts to the users, who decode the signal as $\hat{\hat{s}}$. The end-to-end probability of incorrectly detecting a network-coded signal resulting from $\sqrt{M}$-PAM signals, can be obtained from the sum of the off-diagonal elements of the product of two $\sqrt{M} \times \sqrt{M}$ matrices C and D, with elements $c_{p,q} = P(\hat{s} = q|s = p)$ and $d_{p',q'} = P(\hat{\hat{s}} = q'|\hat{s} = p')$, respectively, where $p, q, p', q' \in [0, \sqrt{M} - 1]$, multiplied by the factor $\sqrt{M}$. That is,

$$P_{\sqrt{M}-PAM,NC}(i, m) = \frac{1}{\sqrt{M}} \left(\sum_{p,q=0}^{\sqrt{M}-1} c_{p,q} \sum_{p',q'=0,p'\neq p,q'\neq q}^{\sqrt{M}-1} d_{p',q'} \right). \tag{35}$$

The coefficients $c_{p,q}$ can be obtained by calculating the probability that the signal received at the relay whose mean (which takes value from the set $\mathcal{S}_{NC}$) should be mapped to $s = p$, falls in the decision region for the signal whose mean is mapped to $s = q$. Thus, $c_{p,q}$ can be expressed as the sum of Q-functions, as follows:

$$c_{p,q} = \begin{cases} \displaystyle\sum_{u=1,u=\text{odd}}^{2(2\sqrt{M}-2)-1} a_{p,q,u} Q(u\sqrt{\gamma_r(i, m)}) & p \neq q \\ 1 + \displaystyle\sum_{u=1,u=\text{odd}}^{2(2\sqrt{M}-2)-1} a_{p,q,u} Q(u\sqrt{\gamma_r(i, m)}) & p = q \end{cases} \tag{36}$$

where, $\gamma_r(i, m)$ represents the SNR of the ith and the mth users' signal at the relay for M-QAM modulation and can be obtained as

$$\gamma_r(i, m) = \frac{P \min(|h_{i,r}|^2, |h_{m,r}|^2)}{E_{av} N_0} \tag{37}$$

where E_{av} is the average energy of symbols for $\sqrt{M}$-PAM modulation (e.g., $E_{av} = 5$ for $M = 16$).

Similarly, the coefficients $d_{p',q'}$ can be obtained by calculating the probability that the signal received at the ith user with mean $s = p'$, falls in the decision region for the signal with mean $s = q'$. Thus,

$$d_{p',q'} = \begin{cases} \displaystyle\sum_{v=1,v=\text{odd}}^{2(\sqrt{M}-1)-1} b_{p',q',v} Q(v\sqrt{\gamma_i}) & p' \neq q' \\ 1 + \displaystyle\sum_{v=1,v=\text{odd}}^{2(\sqrt{M}-1)-1} b_{p',q',v} Q(v\sqrt{\gamma_i}) & p' = q' \end{cases} \tag{38}$$

where $\gamma_i = \frac{P_r |h_{r,i}|^2}{E_{av} N_0}$ represents the SNR at the ith user. The coefficients $a_{p,q,u}$ and $b_{p',q',v}$ for $M = 16$ (or $\sqrt{M} = 4$), have been tabulated in Table 1.

Appendix F. Proof of Theorem 3

The proof follows the steps outlined in [27], which are applicable to any user pairing scheme. However, for the proposed pairing scheme, we need to modify these steps to take into account different error probabilities at the common user and

Table 1

Illustration of the coefficients $a_{p,q,u}$ and $b_{p',q',v}$ for $M = 16$ corresponding to the probability $P(\hat{V}_{i,m} \neq V_{i,m})$ and $P(\hat{\hat{V}}_{i,m} \neq \hat{V}_{i,m})$, respectively.

p, p'	$u\ q$	$a_{p,q,u}$				$v\ q'$	$b_{p',q',v}$			
		$q = 0$	$q = 1$	$q = 2$	$q = 3$		$q' = 0$	$q' = 1$	$q' = 2$	$q' = 3$
$p = 0$	$u = 1$	−7/4	1	0	3/4	$v = 1$	1/4	1/4	0	0
	$u = 3$	0	−1	7/4	−3/4					
	$u = 5$	0	3/4	−1	1/4	$v = 3$	0	−1/4	1/4	0
	$u = 7$	1	−3/4	0	−1/4					
	$u = 9$	−1/4	1/4	0	0	$v = 5$	0	0	−1/4	1/4
	$u = 11$	0	−1/4	1/4	0					
$p = 1$	$u = 1$	1	1	0	0	$v = 1$	1/4	−1/4	1/4	0
	$u = 3$	-1/2	0	−1/2	1					
	$u = 5$	1/2	0	1/2	−1	$v = 3$	−1/4	1/4	−1/4	1/4
	$u = 7$	−1/2	1	−1/2	0					
	$u = 9$	1/2	−1	1/2	0	$v = 5$	0	1/4	0	−1/4
	$u = 11$	0	0	0	0					
$p = 2$	$u = 1$	1	1	−7/4	3/4	$v = 1$	0	1/4	−1/4	1/4
	$u = 3$	7/4	−1	0	−3/4					
	$u = 5$	−1	3/4	0	1/4	$v = 3$	1/4	−1/4	1/4	1/4
	$u = 7$	0	−3/4	1	−1/4					
	$u = 9$	0	1/4	−1/4	0	$v = 5$	−1/4	0	1/4	0
	$u = 11$	1/4	−1/4	0	0					
$p = 3$	$u = 1$	1	0	1	−2	$v = 1$	0	0	1/4	−1/4
	$u = 3$	-1	2	−1	0					
	$u = 5$	1	−2	1	0	$v = 3$	1/4	1/4	−1/4	0
	$u = 7$	0	0	0	0					
	$u = 9$	0	0	0	0	$v = 5$	0	−1/4	0	0
	$u = 11$	0	0	0	0					

the other users. The modified steps have been already presented in Section 6 and the details of these steps are illustrated below:

Step-1: The probabilities of incorrectly decoding a network coded message at the ith and the ℓth user are obtained in (29) and (30), respectively.

Step-2: In the proposed pairing scheme, k error events can occur in two cases:

- A_k: If the decoding user incorrectly extracts exactly k users' messages except the ith user's message. That is, the decoding user (jth user, where $j \in [1, L]$) incorrectly decodes k network coded messages $V_{i,m_1}, V_{i,m_2}, \ldots, V_{i,m_k}$ and correctly decodes the remaining $L - 1 - k$ network coded messages, where $m_1, m_2, \ldots, m_k \in [1, L], m_1 \neq m_2 \neq \cdots \neq m_k \neq j$.
- B_k: If the decoding user incorrectly decodes exactly k users' messages including the ith user's message. This happens when the decoding user (ℓth user, where $\ell \in [1, L], \ell \neq i$) incorrectly decodes $V_{i,\ell}$ and correctly decodes $k - 1$ other network coded messages, $V_{i,m_1}, V_{i,m_2}, \ldots, V_{i,m_{k-1}}$ and incorrectly decodes the remaining $L - 1 - k$ messages, where $m_1, m_2, \ldots, m_{k-1} \in [1, L], m_1 \neq m_2 \neq \cdots \neq m_{k-1} \neq i, \ell$.

Note that, the error case A_k is applicable both for the common user and the other users. However, case B_k is applicable only for users except the common user.

Step-3: The probabilities of the aforementioned error cases for the ith and the ℓth users are

$$P_{i,A_k} = \sum_{m_a=1, m_a \neq i}^{L} \prod_{a=1}^{k} P_{FDF}(i, m_a) \prod_{m_b=1, m_b \neq m_a, i}^{L} \{1 - P_{FDF}(i, m_b)\}. \tag{39}$$

$$P_{\ell,A_k} = \sum_{m_a=1, m_a \neq i, \ell}^{L} \prod_{a=1}^{k} P_{FDF}(\ell, m_a) \prod_{m_b=1, m_b \neq \ell, m_a}^{L} \{1 - P_{FDF}(\ell, m_b)\}. \tag{40}$$

$$P_{\ell,B_k} = \begin{cases} P_{FDF}(\ell, i) \displaystyle\sum_{m_a=1, m_a\neq i,\ell}^{L} \prod_{a=1}^{k-1}\{1 - P_{FDF}(\ell, m_a)\} \prod_{m_b=1, m_b\neq i,\ell,m_a}^{L} P_{FDF}(\ell, m_b) & 1 < k < L-1 \\ P_{FDF}(\ell, i) \displaystyle\prod_{m_b=1, m_b\neq i,\ell}^{L} \{1 - P_{FDF}(\ell, m_b)\} & k = 1 \\ P_{FDF}(\ell, i) \displaystyle\sum_{m_a=1, m_a\neq i,\ell}^{L} \prod_{a=1}^{L-1}\{1 - P_{FDF}(\ell, m_a)\} & k = L-1. \end{cases} \tag{41}$$

Step-4: The probability of k error events for the ith and the ℓth user can be expressed as

$$P(i, k) = P_{i,A_k}, \qquad P(\ell, k) = P_{\ell,A_k} + P_{\ell,B_k}. \tag{42}$$

Step-5: Since, each user decodes $L-1$ other users' messages in an L-user MWRN, there are $L-1$ possible error events. Thus, averaging over all the possible error events, the average SER at the ith and the ℓth user can be obtained as:

$$P_{i,avg} = \frac{1}{L-1}\sum_{k=1}^{L-1} kP_{i,A_k}, \qquad P_{j,avg} = \frac{1}{L-1}\sum_{k=1}^{L-1} k(P_{\ell,A_k} + P_{\ell,B_k}). \tag{43}$$

Step-6: At high SNR, the higher order error terms in (42) can be neglected. Thus, $P_{i,A_k} \approx 0$ and $P_{\ell,A_k} \approx 0$ for $k > 1$ (see (39) and (40)). Similarly, $P_{\ell,B_k} \approx 0$ for $k < L-1$ (see (41)). Thus, at high SNR, (43) can be approximated as

$$P_{i,avg} = \frac{1}{L-1}P_{i,A_1}, \qquad P_{\ell,avg} = \frac{1}{L-1}\left(P_{\ell,A_1} + (L-1)P_{\ell,B_{L-1}}\right). \tag{44}$$

In addition, at high SNR, we can approximate the terms $\{1 - P_{FDF}(i, m_b)\}$, $\{1 - P_{FDF}(\ell, m_b)\}$ and $\{1 - P_{FDF}(\ell, m_a)\}$ in (39)–(41) to be 1. Thus, substituting (39)–(41) in (44), the average SER at the ith and the ℓth user at high SNR can be expressed as

$$P_{i,avg} = \frac{1}{L-1}\sum_{m_1=1, m_1\neq i}^{L} P_{FDF}(i, m_1), \qquad P_{\ell,avg} = \frac{1}{L-1}\left(\sum_{m_1=1, m_1\neq i,\ell}^{L} P_{FDF}(\ell, m_1) + (L-1)P_{FDF}(\ell, i)\right).$$

Finally, replacing m_1 with m in the above equation completes the proof.

Appendix G. Proof of Propositions 7–9

Proof of Proposition 7. For the equal average channel gain scenario, the error probabilities $P_{FDF}(j, 1) = P_{FDF}(j, 2) = \cdots = P_{FDF}(j, L-1) = P_{FDF}$ for all $j \in [1, L]$. Thus, the average SER expressions in (31) and (32) for the proposed pairing scheme can be simplified as:

$$P_{i,avg} = P_{FDF}, \qquad P_{\ell,avg} = \left(\frac{2L-3}{L-1}\right)P_{FDF}. \tag{45}$$

The average SER for the scheme in [1] can be given by [27]:

$$P_{avg} = \frac{L}{2}P_{FDF}. \tag{46}$$

Comparing (45) and (46), we arrive at Proposition 7.

Proof of Proposition 8. For the unequal average channel gain scenario, the average SER expressions for the proposed pairing scheme is given by (31) and (32), with $\gamma_r(i, m) = \frac{(2L-2)P\min\left(\frac{|h_{i,r}|^2}{L-1}, |h_{m,r}|^2\right)}{5N_0}$ and $\gamma_i = \frac{(2L-2)P_r|h_{i,r}|^2}{5N_0}$. For the scheme in [1], the average SER at the jth ($j \in [1, L]$) user can be written as

$$P_{j,avg} = \frac{1}{L-1}\sum_{m=1}^{L-1} mP_{FDF}(j, m), \tag{47}$$

where

$$P_{FDF}(j, m) = 1 - (1 - P_{\sqrt{M}-PAM,NC}(j, m))^2, \tag{48}$$

with $\gamma_r(m) = \frac{P\min(|h_{m,r}|^2, |h_{m+1,r}|^2)}{5N_0}$ and $\gamma_j = \frac{P_r|h_{j,r}|^2}{5N_0}$ in (36) and (38), respectively. Now we consider two cases:

- *case* 1: $E_H[\frac{|h_{i,r}|^2}{L-1}] > E_H[|h_{m,r}|^2]$. In this case,

$$
\begin{aligned}
&E_H\left[\min\left(\frac{(2L-2)P|h_{i,r}|^2}{5(L-1)N_0}, \frac{(2L-2)P|h_{m,r}|^2}{5N_0}\right)\right]\\
&\quad\le \min\left(E_H\left[\frac{(2L-2)P|h_{i,r}|^2}{5(L-1)N_0}\right], E_H\left[\frac{(2L-2)P|h_{m,r}|^2}{5N_0}\right]\right) = E_H\left[\frac{(2L-2)P|h_{m,r}|^2}{5N_0}\right]\\
&\quad\ge \min\left(E_H\left[\frac{P|h_{m,r}|^2}{5N_0}\right], E_H\left[\frac{P|h_{m+1,r}|^2}{5N_0}\right]\right) \ge E_H\left[\min\left(\frac{P|h_{m,r}|^2}{5N_0}, \frac{P|h_{m+1,r}|^2}{5N_0}\right)\right].
\end{aligned}
\tag{49}
$$

  Thus, $E_H[\gamma_r(i, m)] \ge E_H[\gamma_r(m)]$.
- *case* 2: $E_H[\frac{|h_{i,r}|^2}{L-1}] < E_H[|h_{m,r}|^2]$. In this case, $E_H\left[\min\left(\frac{(2L-2)P|h_{i,r}|^2}{5(L-1)N_0}, \frac{(2L-2)P|h_{m,r}|^2}{5N_0}\right)\right] \le E_H[\frac{(2L-2)P|h_{i,r}|^2}{5(L-1)N_0}]$ and since, $|h_{i,r}|^2 > |h_{m,r}|^2, |h_{m+1,r}|^2$, $E_H\left[\min\left(\frac{|h_{m,r}|^2}{5N_0}, \frac{|h_{m+1,r}|^2}{5N_0}\right)\right] \le E_H\left[\frac{(2L-2)P|h_{i,r}|^2}{5(L-1)N_0}\right]$. Thus, $E_H[\gamma_r(i, m)] \ge E_H[\gamma_r(m)]$.

From the above cases, the probability $P_{FDF}(i, m)$ and $P_{FDF}(\ell, m)$ for the proposed scheme would be larger than $P_{FDF}(j, m)$ for scheme [1]. Thus, comparing (31), (32) and (47) shows that the average SER for the proposed scheme would be smaller than that for scheme [1]. This proves Proposition 8.

Proof of Proposition 9. For the variable average channel gain scenario, the average SER expression for the proposed pairing scheme is given by (31) and (32). The average SER for the pairing scheme in [1] is the same as in (47). Now, comparing $P_{FDF}(i, m)$ (from (29)), $P_{FDF}(\ell, m)$ (from (30)) and $P_{FDF}(j, m)$ (from (48)) shows that the only terms which are different in all these probabilities are $\gamma_r(i, m)$ and $\gamma_r(m)$. Note that, if $E[|h_{i,r}|^2] > E[|h_{m+1,r}|^2]$, then $E[\min(|h_{i,r}|^2, |h_{m,r}|^2)] \ge E[\min(|h_{m+1,r}|^2, |h_{m,r}|^2)]$. Thus, $E[\gamma_r(i, m)] \ge E[\gamma_r(m)]$ and in effect, from (29), (30) and (48), the error probability for the new pairing scheme would be less than that for scheme [1]. As a result, the average SER for the proposed scheme is less than that of scheme [1] (in (47)) for both $j = i$ and $j = \ell$, which proves Proposition 9.

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