Two-Way Training: Optimal Power Allocation for Pilot and Data Transmission

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***Abstract*—In this letter, we consider multiple-input single-output (MISO) systems with two-way training based transmission. We focus on the long-term system performance and study the optimal power allocation between reverse training, forward training and data transmission. We derive closed-form solutions for the optimal power allocation using high signal-to-noise ratio (SNR) approximations, and show that they achieve near optimal performance in terms of symbol error rate (SER) for different modulation schemes over a wide range of SNR values.**

***Index Terms*—Two-way training, channel estimation, power allocation, symbol error rate.**

I. Introduction

THE use of multiple antennas significantly increases the data throughput in wireless communication systems, especially when the channel state information (CSI) is known at both the receiver and the transmitter [1]. The transmitter CSI can usually be obtained using various feedback transmission schemes [2]. Recently, a multi-stage training method was proposed to allow the transmitter to estimate its outgoing channel gains using pilot transmissions from both the transmitter and the receiver without using feedback [3]. This method is designed for asymmetric channels where the outgoing and incoming channels have different characteristics. When the channels are symmetric, such as in time-division duplex (TDD) systems, a simpler training method named two-way training was proposed in [4] for block-fading single-input multiple-output (SIMO) systems. In this scheme, the transmitter acquires the outgoing CSI using the pilots sent from the receiver (*i.e.*, reverse training) and performs block-wise power adaptation. After that, the receiver estimates the effective channel gains using the pilots sent from the transmitter (*i.e.*, forward training). The two-way training scheme was also considered in multi-user transmissions in [5].

For multiple-input single-output (MISO) systems, the major benefit of two-way training is the reduction in the overhead of acquiring CSI at both the transmitter and the receiver. To further improve the performance in resource-constrained systems, it is crucial to optimize the power allocation between pilot and data transmission. While solutions to this optimization problem have been found for systems with traditional one-way training [6, 7], no such solution exists for systems with two-way training. This issue is addressed in this letter. We study block-fading MISO TDD communications systems with two-way training. We derive the linear minimum mean square error (LMMSE) estimator for the forward training. We study the optimal power allocation between the reverse training, forward training, as well as the data transmission under various power constraints. The main contributions of this work are:

- We propose a lower bound on an average received signal-to-noise ratio (SNR) as an objective function for power optimization. The proposed objective function is easy to optimize numerically and yields solutions that are independent of the number of transmit antennas.
- Using the average received SNR lower bound, we derive closed-form solutions at high SNR for optimal power allocation with two-way training under three different power constraint scenarios. These solutions are shown to achieve near optimum symbol error rate (SER) over a wide range of SNR values, especially when moderate to high-order modulations are used.
- We also consider systems with reverse training only and obtain an analytical solution for optimal power allocation. We find that two-way training provides no or marginal performance gain over reverse training only, at low SNR or when low-order modulations are used.

Notations: Boldface letters denote vectors. $[\cdot]^*$ and $[\cdot]^\dagger$ denotes the complex conjugate and conjugate transpose operation, respectively. $E\{\cdot\}$ denotes the mathematical expectation.

II. System Model

We consider a flat-fading wireless communication system with N_t transmit antennas and a single receive antenna. For simplicity, we refer to the transmitter as the base station (BS) and the receiver as the user terminal (UT). The received signal at the UT is given by $y = \boldsymbol{h}\boldsymbol{x} + n$, where $\boldsymbol{x}$ is the $N_t \times 1$ transmitted symbol vector from the BS, $\boldsymbol{h}$ is the $1 \times N_t$ channel gain vector and n is the noise at the UT. We assume that both n and the elements of $\boldsymbol{h}$ are independent and identically distributed (i.i.d.) zero-mean circularly symmetric complex Gaussian (ZMCSCG) with unit variance. We also assume that the forward and reverse channels are symmetric, *i.e.*, channel reciprocity holds. The symmetric channel assumption can be justified for TDD channels in which the forward and reverse

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transmissions share a common frequency band, *e.g.*, IEEE 802.11 standards [4].

A. Two-way Training Based Transmission

Similar to [5], we consider a four-stage TDD transmission scheme, in which the total duration for transmission of L symbols is less than the channel coherence time. Hence we assume that the channel gains remain constant over L symbol periods. In *Stage 1 (reverse training)*, the UT sends one pilot with power $\mathcal{P}_r$ and the BS estimates the channel using LMMSE estimator. Using channel reciprocity, the BS obtains the estimates of its outgoing channel gains. We denote the channel estimates and the estimation errors by $\hat{\boldsymbol{h}}$ and $\tilde{\boldsymbol{h}}$, respectively, and $\boldsymbol{h} = \hat{\boldsymbol{h}} + \tilde{\boldsymbol{h}}$. The variance of each element in $\tilde{\boldsymbol{h}}$ is given by $\sigma_e^2 = \frac{1}{\mathcal{P}_r+1}$. In *Stage 2 (precoder design)*, the BS treats $\hat{\boldsymbol{h}}$ as the true channel and designs the beamforming vector $\boldsymbol{w}$ as $\boldsymbol{w} = \frac{\hat{\boldsymbol{h}}^\dagger}{\|\hat{\boldsymbol{h}}\|} = \frac{\hat{\boldsymbol{h}}^\dagger}{\sqrt{\hat{\boldsymbol{h}}\hat{\boldsymbol{h}}^\dagger}}$ to maximize the SNR at the UT. For simplicity, we assume that the duration of the second stage is negligible. In *Stage 3 (forward training)*, the BS sends one pilot denoted by p with power $|p|^2 = pp^* = \mathcal{P}_f$ via the beamforming vector $\boldsymbol{w}$ and the UT estimates the effective channel using LMMSE estimator. This will be discussed in detail in the next section. During *Stage 4 (data transmission)*, the BS transmits QPSK or M-QAM modulated information symbols s_i, $i = 1, 2, ..., L-2$ with power $E\{s_i s_i{}^*\} = \mathcal{P}_d$ via the beamforming vector $\boldsymbol{w}$. Since the noise and channel variances are normalized to unity, we will also refer to $\mathcal{P}_r$, $\mathcal{P}_f$ and $\mathcal{P}_d$ as the reverse training SNR, forward training SNR and data SNR, respectively.

III. Forward Channel Estimation

During the forward training stage, the received signal at the UT is given by

$$y = \boldsymbol{h}\boldsymbol{w}p + n = (\|\hat{\boldsymbol{h}}\| + \tilde{\boldsymbol{h}}\boldsymbol{w})p + n = fp + n, \tag{1}$$

where $f = \|\hat{\boldsymbol{h}}\| + \tilde{\boldsymbol{h}}\boldsymbol{w}$ denotes the effective channel for the forward transmission. Unlike the reverse training where the Gaussian channel makes the LMMSE estimator equivalent to the MMSE estimator, the complicated non-Gaussian distribution of f makes MMSE estimator in the forward training mathematically intractable. Therefore, we consider the widely-used LMMSE channel estimation for the forward training. Denoting the first and second order statistics of f by $\mu_1 = E\{f\}$ and $\mu_2 = E\{|f|^2\}$, the LMMSE channel estimation is given by [8]

$$\hat{f} \;=\; \mu_1 + \frac{\sigma_{fy}^2}{\sigma_{yy}^2}(y - \mu_1 p), \tag{2}$$

where $\sigma_{fy}^2 = E\{fy^*\} - \mu_1^2 p^* = (\mu_2 - \mu_1^2)p^*$ and $\sigma_{yy}^2 = E\{yy^*\} - \mu_1^2\mathcal{P}_f = (\mu_2 - \mu_1^2)\mathcal{P}_f + 1$. To find the values of μ_2 and μ_1, we proceed as follows. Since the elements of $\boldsymbol{h}$ are i.i.d. Gaussian random variables, its LMMSE estimate $\hat{\boldsymbol{h}}$ also has i.i.d. Gaussian elements, and hence $\|\hat{\boldsymbol{h}}\|^2$ has a Gamma distribution with parameters $(N_t, 1-\sigma_e^2)$. Therefore, we have

$$\begin{aligned}\mu_2 &= E\{\|\hat{\boldsymbol{h}}\|^2\} + E\{\tilde{\boldsymbol{h}}\boldsymbol{w}\boldsymbol{w}^\dagger\tilde{\boldsymbol{h}}^\dagger\} \\ &= N_t(1-\sigma_e^2) + \sigma_e^2 = N_t\frac{\mathcal{P}_r}{\mathcal{P}_r+1} + \frac{1}{\mathcal{P}_r+1}.\end{aligned} \tag{3}$$

Furthermore, μ_1 can be calculated using the probability density function of $g = \|\hat{\boldsymbol{h}}\|^2$ as

$$\begin{aligned}\mu_1 = E\{g^{1/2}\} &= \int_0^\infty g^{1/2} g^{N_t-1}\frac{e^{-g/(1-\sigma_e^2)}}{(1-\sigma_e^2)^{N_t}\Gamma(N_t)}\mathrm{d}g \\ &= \frac{1}{\sqrt{(1-\sigma_e^2)}\Gamma(N_t)}\int_0^\infty\Big(\frac{g}{1-\sigma_e^2}\Big)^{N_t-1/2} e^{-g/(1-\sigma_e^2)}\mathrm{d}g,\end{aligned} \tag{4}$$

where $\Gamma(z) = \int_0^\infty t^{-1+z}e^{-t}\mathrm{d}t$ is the Gamma function. Letting $t = g/(1-\sigma_e^2)$, (4) reduces to

$$\begin{aligned}\mu_1 &= \frac{\sqrt{(1-\sigma_e^2)}}{\Gamma(N_t)}\int_0^\infty t^{N_t-1/2}e^{-t}\mathrm{d}t \\ &= \sqrt{\frac{\mathcal{P}_r}{\mathcal{P}_r+1}}\frac{\Gamma(N_t+1/2)}{\Gamma(N_t)}.\end{aligned} \tag{5}$$

Denoting the variance of f as $\sigma_f^2 = \mu_2 - \mu_1^2$, the variance of the channel estimation error $\tilde{f} = f - \hat{f}$ is given by

$$\sigma_{\tilde{f}}^2 = E\{|\tilde{f}|^2\} = \frac{\sigma_f^2}{\sigma_f^2\mathcal{P}_f + 1}, \tag{6}$$

where $\sigma_f^2 = \frac{k\mathcal{P}_r + 1}{\mathcal{P}_r + 1}$ and $k = N_t - \left(\frac{\Gamma(N_t+1/2)}{\Gamma(N_t)}\right)^2$.

We see that k characterizes the effect of N_t on $\sigma_{\tilde{f}}^2$. By evaluating k for different N_t, we see that $0.232 < k < 0.250$ for $1 < N_t < 100$. Therefore, the value of N_t has little impact on $\sigma_{\tilde{f}}^2$, which implies that adding extra antennas at the BS does not change the forward channel estimation error.

IV. Optimal Power Allocation with Two-way Training

In this section, we study the optimal power allocation for two-way training based transmission. The problem of optimizing power allocation can be formulated in different ways according to the given power constraints and the degrees of freedom in the system design. We provide a comprehensive study by solving the power optimization problem in three different scenarios.

The received signal at the UT during data transmission is given as $y = \hat{f}s + \tilde{f}s + n$, and hence the received SNR for a particular channel realization is given by

$$\rho = \frac{\mathcal{P}_d|\hat{f}|^2}{1 + \mathcal{P}_d|\tilde{f}|^2}.$$

Due to the complicated nature of the distribution of ρ, closed-form expressions for long-term system performance measures, such as information capacity and SER, are generally very difficult to obtain, which makes the problem of power optimization mathematically intractable. Instead of directly dealing with ρ, we define a measure of the average received SNR as

$$\rho_{\text{ave}} = \frac{\mathcal{P}_d E\{|\hat{f}|^2\}}{1 + \mathcal{P}_d E\{|\tilde{f}|^2\}}.$$

Using $E\{|\hat{f}|^2\} > E\{\hat{f}\}E\{\hat{f}^*\}$, we obtain a lower bound on ρ_{ave} as

$$\begin{aligned}\rho_{\text{ave}}^{\text{LB}} &= \frac{\mathcal{P}_d\mu_1^2}{1 + \mathcal{P}_d\sigma_{\tilde{f}}^2} \\ &= \nu\frac{\mathcal{P}_d\mathcal{P}_r[(k\mathcal{P}_r+1)\mathcal{P}_f + \mathcal{P}_r + 1]}{(\mathcal{P}_r+1)[(k\mathcal{P}_r+1)(\mathcal{P}_f+\mathcal{P}_d) + \mathcal{P}_r + 1]},\end{aligned} \tag{7}$$

which is obtained using (5) and (6), and $\nu = \left[\frac{\Gamma(N_t+1/2)}{\Gamma(N_t)}\right]^2$. We simply refer to $\rho_{\text{ave}}^{\text{LB}}$ in (7) as the *average SNR lower bound* and propose to use it as the objective function to obtain solutions for power optimization. We are interested in how optimal these solutions are for the SER performance.

A. Optimizing Reverse and Forward Training

In the first scenario, we study the optimal power allocation between reverse and forward training for a given total training power. The data transmission is assumed to have a fixed SNR, *i.e.*, $\mathcal{P}_d$ is fixed. This study allows us to investigate the relative importance of reverse training and forward training. Note that the power optimization does not depend on the block length L. We denote the average training SNR as $\mathcal{P}_{rf}$ and the ratio of power allocated to forward training as α. Then, the power constraint can be written as

$$2\mathcal{P}_{rf} = \mathcal{P}_r + \mathcal{P}_f.$$

Therefore, we have the following relationships.

$$\mathcal{P}_f = \alpha 2\mathcal{P}_{rf}, \qquad \mathcal{P}_r = (1-\alpha)2\mathcal{P}_{rf}. \tag{8}$$

In the high SNR regime for training, we assume that $\mathcal{P}_r \gg 1$ and $\mathcal{P}_f \gg 1$ (which implies $k\mathcal{P}_r\mathcal{P}_f \gg \mathcal{P}_r$). We also assume that $k\mathcal{P}_r\mathcal{P}_f \gg \mathcal{P}_d$ which is valid when either $\mathcal{P}_r$ or $\mathcal{P}_f$ is much higher than $\mathcal{P}_d$. Therefore, the average SNR lower bound $\rho_{\text{ave}}^{\text{LB}}$ in (7) can be approximated as

$$\begin{aligned}\rho_{\text{ave}}^{\text{LB}} &\approx \nu\mathcal{P}_d \frac{\mathcal{P}_r}{\mathcal{P}_r+1}\frac{\mathcal{P}_f}{\mathcal{P}_f+\mathcal{P}_d}\\ &\approx \frac{2\nu\mathcal{P}_d\mathcal{P}_{rf}\alpha(1-\alpha)}{2\alpha(1-\alpha)\mathcal{P}_{rf}+(1-\alpha)\mathcal{P}_d+\alpha}.\end{aligned} \tag{9}$$

Letting the first derivative of $\rho_{\text{ave}}^{\text{LB}}$ in (9) w.r.t. α be zero, one can solve for the optimal α as

$$\alpha = \begin{cases} \frac{1}{2}, & \text{for } \mathcal{P}_d = 1 \\ \frac{\mathcal{P}_d - \sqrt{\mathcal{P}_d}}{\mathcal{P}_d - 1}, & \text{for } \mathcal{P}_d \neq 1 \end{cases} \tag{10}$$

Numerical Results: In the numerical results in this and subsequent subsections, we carry out Monte-Carlo simulation of a two-way training based communication according to Section II-A with a simulation length of 5×10^6 symbols. The receiver performs minimum distance detection on each received symbol. The SER is then computed as the ratio of the number of incorrectly detected symbols and the total number of transmitted symbols. The optimal ratios of power allocation, *e.g.*, α, which minimize the SER are found from a linear search between 0 and 1 with a step size of 0.01. We will mainly use 16-QAM modulation which is a widely-considered constellation.

Fig. 1 shows the optimal power ratio to forward training α versus average training SNR $\mathcal{P}_{rf}$ for different data SNR $\mathcal{P}_d$. We see that the values of α which minimize the SER for 16-QAM modulation follow the same trend as the values of α which maximize $\rho_{\text{ave}}^{\text{LB}}$ in (7), and the mismatch increases as $\mathcal{P}_d$ increases. This mismatch is mainly due to the fact that maximizing $\rho_{\text{ave}}^{\text{LB}}$ does not necessarily result in the optimal distribution of ρ which minimizes the SER, *e.g.*, it does not minimize the probability of ρ taking very small values.

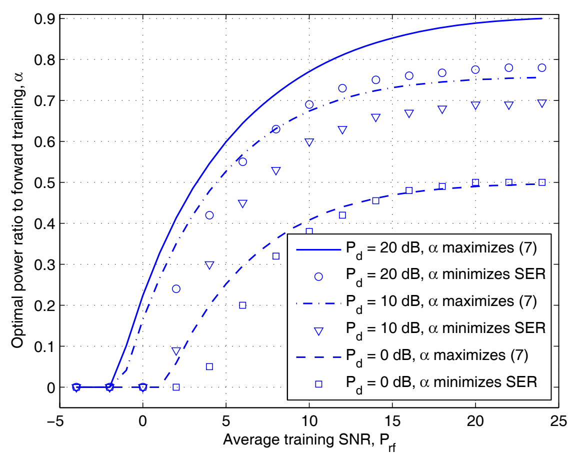

Fig. 1. Optimal power ratio to forward training α vs. average training SNR $\mathcal{P}_{rf}$ for systems with $N_t = 4$ transmit antennas and different values of data SNR $\mathcal{P}_d$. Lines indicate the values of α that maximize the average SNR lower bound $\rho_{\text{ave}}^{\text{LB}}$ in (7) and the markers indicate the values of α that minimize the SER for 16-QAM modulation found via Monte-Carlo simulations. We see that the optimal values of α which maximize $\rho_{\text{ave}}^{\text{LB}}$ are reasonably close to those which minimize the SER, especially at low data SNR.

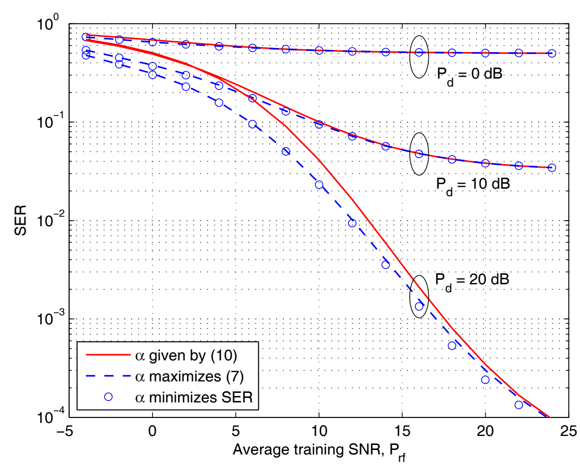

Fig. 2. SER for 16-QAM modulation vs. average training SNR $\mathcal{P}_{rf}$ for systems with $N_t = 4$ transmit antennas and different values of the data SNR $\mathcal{P}_d$. The values of α used are found from Monte-Carlo simulations by minimizing the SER, maximizing the average SNR lower bound $\rho_{\text{ave}}^{\text{LB}}$ in (7), as well as the closed-form solution given in (10). We see that the closed-form solution for α can be used to achieve near optimum SER at moderate to high training SNR.

Furthermore, we see that more power should be allocated to reverse (forward) training when $\mathcal{P}_{rf}$ is low (high). In particular, we see that forward training should not be used when $\mathcal{P}_{rf}$ is sufficiently low, *e.g.*, $\mathcal{P}_{rf} < 0$ dB. As $\mathcal{P}_{rf}$ increases the optimal α is reasonably close to the value given in (10) for low to moderate $\mathcal{P}_d$, *e.g.*, $\alpha = 0.50$ and 0.76 for $\mathcal{P}_d = 0$ dB and 10 dB, respectively.

Fig. 2 shows the SER for 16-QAM modulation versus average training SNR $\mathcal{P}_{rf}$. For comparison, we use the values of α found by minimizing the SER, maximizing $\rho_{\text{ave}}^{\text{LB}}$ given in (7), as well as the closed-form solution given in (10). We see that the values of α that maximize $\rho_{\text{ave}}^{\text{LB}}$ also achieves the near optimum SER performance. Furthermore, the closed-form solution for α derived from the high SNR approximation can be used to achieve near optimum SER at moderate to high training SNR, *e.g.*, when $\mathcal{P}_{rf} > 10$ dB for $\mathcal{P}_d = 10$ dB and

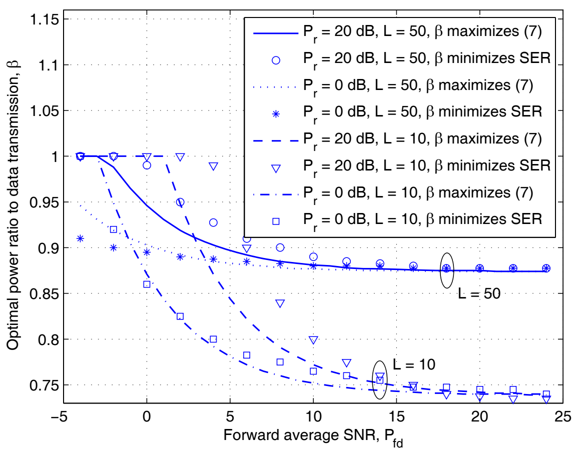

Fig. 3. Optimal power ratio to data transmission β vs. forward average SNR $\mathcal{P}_{fd}$ for systems with $N_t = 4$ transmit antennas and different values of the reverse training SNR $\mathcal{P}_r$ and block length L. Lines indicate the values of β that maximize the average SNR lower bound $\rho_{\text{ave}}^{\text{LB}}$ in (7) and the markers indicate the values of β that minimize the SER for 16-QAM modulation found via Monte-Carlo simulations. We see that the optimal values of β which maximize $\rho_{\text{ave}}^{\text{LB}}$ are reasonably close to those which minimize the SER.

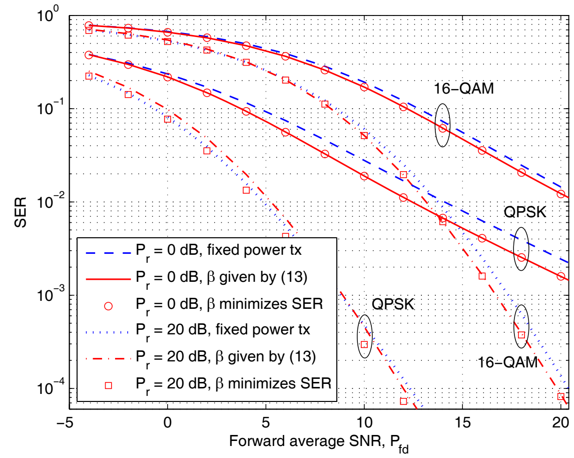

Fig. 4. SER vs. forward average SNR $\mathcal{P}_{fd}$ for systems with $N_t = 4$ transmit antennas, reverse training SNR of $\mathcal{P}_r = 0, 20$ dB, and block length of $L = 50$. The values of β used are found from Monte-Carlo simulations by minimizing the SER, as well as the closed-form solution given in (13). The SER with fixed power transmission, *i.e.*, $\mathcal{P}_f = \mathcal{P}_d = \mathcal{P}_{fd}$, is also included for comparison. We see that the closed-form solution for β can be used to achieve near optimum SER over a wide range of SNR. We have also observed the same results for $L = 10$, hence they are not shown for brevity.

$\mathcal{P}_{rf} > 20$ dB for $\mathcal{P}_d = 20$ dB. We have also investigated the SER performance with other modulations. For example, the SER achieved by using the closed-form solution for α is within 0.5 dB, 1 dB and 1.4 dB from the optimum SER for 32-QAM, 16-QAM and 8-QAM, respectively, when $\mathcal{P}_{rf} > 20$ dB for $\mathcal{P}_d = 20$ dB. This suggests that the closed-form solution is more accurate for higher order modulations.

B. Optimizing Forward Transmission

In the second scenario, the BS tries to optimize the power allocation between forward training and data transmission for a given total forward transmit power budget. The reverse training is assumed to have a fixed SNR, *i.e.*, $\mathcal{P}_r$ is fixed. We denote the average SNR for the forward link as $\mathcal{P}_{fd}$ and the ratio of power allocated to data transmission as β. Then, the power constraint can be written as

$$\mathcal{P}_{fd}(L-1) = \mathcal{P}_f + \mathcal{P}_d(L-2).$$

Therefore, we have the following relationships.

$$\mathcal{P}_d = \beta\mathcal{P}_{fd}(L-1)/(L-2), \quad \mathcal{P}_f = (1-\beta)\mathcal{P}_{fd}(L-1). \quad (11)$$

In the high SNR regime for forward transmission, we assume that $\mathcal{P}_d \gg 1$ and $\mathcal{P}_f \gg 1$. Therefore, the average SNR lower bound $\rho_{\text{ave}}^{\text{LB}}$ in (7) can be approximated as

$$\begin{aligned} \rho_{\text{ave}}^{\text{LB}} &\approx \frac{\nu\mathcal{P}_r}{\mathcal{P}_r+1}\frac{\mathcal{P}_d\mathcal{P}_f}{\mathcal{P}_d+\mathcal{P}_f} \\ &= \frac{\nu\mathcal{P}_r\mathcal{P}_{fd}}{\mathcal{P}_r+1}\frac{(L-1)\beta(1-\beta)}{\beta+(L-2)(1-\beta)}. \end{aligned} \quad (12)$$

Letting the first derivative of $\rho_{\text{ave}}^{\text{LB}}$ in (12) w.r.t. β be zero, one can solve for the optimal β as

$$\beta = \phi - \sqrt{\phi(\phi-1)}, \quad \text{where } \phi = \frac{L-2}{L-3}. \quad (13)$$

It is clear that the optimal forward power allocation at high SNR given in (13) is independent of the reverse training SNR. This result implies that the power optimization in the forward link is independent of the reverse link conditions, which is an important message for system designers.

Numerical Results: Fig. 3 shows the optimal power ratio to data transmission β versus the forward average SNR $\mathcal{P}_{fd}$ for different reverse training SNR $\mathcal{P}_r$ and block lengths L. We see that the values of β which minimize the SER for 16-QAM modulation follow the same trend as the values of β which maximize $\rho_{\text{ave}}^{\text{LB}}$ in (7), and the mismatch occurs when $\mathcal{P}_{fd}$ is low to moderate. This mismatch is mainly due to the fact that maximizing $\rho_{\text{ave}}^{\text{LB}}$ does not necessarily result in the optimal distribution of ρ which minimizes the SER. Similar to the first scenario, we see that forward training should not be used when $\mathcal{P}_{fd}$ is sufficiently low. As $\mathcal{P}_{fd}$ increases the optimal value of β converges to the value given in (13), *e.g.*, $\beta = 0.739$ and 0.874 for $L = 10$ and 50, respectively, and the value is independent of the reverse training SNR $\mathcal{P}_r$.

Fig. 4 shows the SER versus forward average SNR $\mathcal{P}_{fd}$. For comparison, we use the values of β found by minimizing the SER as well as the closed-form solution given in (13). We see that the closed-form solution for β can be used to achieve near optimum SER over a wide range of SNR. We also include the SER using fixed power transmission, *i.e.*, $\mathcal{P}_f = \mathcal{P}_d = \mathcal{P}_{fd}$. We observe that power optimization only provides around 0.7 dB gain over fixed power transmission for 16-QAM modulation. Therefore, the use of fixed power transmission achieves reasonably good SER performance. On the other hand, the SNR gain by using power optimization is around 1.8 dB for QPSK modulation for $\mathcal{P}_r = 0$ dB, while this gain reduces to around 0.8 dB for $\mathcal{P}_r = 20$ dB.

C. Optimizing Overall Transmission

In the third scenario, the system designer has the most degrees of freedom and tries to optimize the power allocation between reverse training, forward training and data transmission under a total transmit power constraint. We denote the average SNR for overall (reverse and forward) transmission

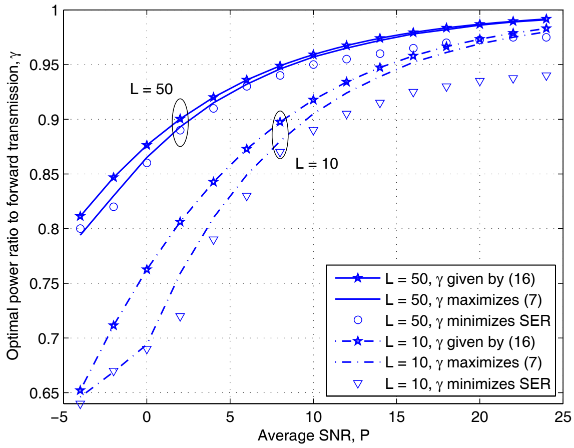

Fig. 5. Optimal power ratio to forward transmission γ vs. average SNR $\mathcal{P}$ for systems with $N_t = 4$ transmit antennas and block length of $L = 10$ and $L = 50$. The values of γ which minimize the SER for 16-QAM modulation found via Monte-Carlo simulations as well as those maximize the average SNR lower bound $\rho_{\text{ave}}^{\text{LB}}$ in (7) are shown. The closed-form solutions for the optimal γ derived from high SNR approximation in (16) are also included. We see that the optimal values of γ given by the closed-form solution in (16) are reasonably close to those which minimize the SER.

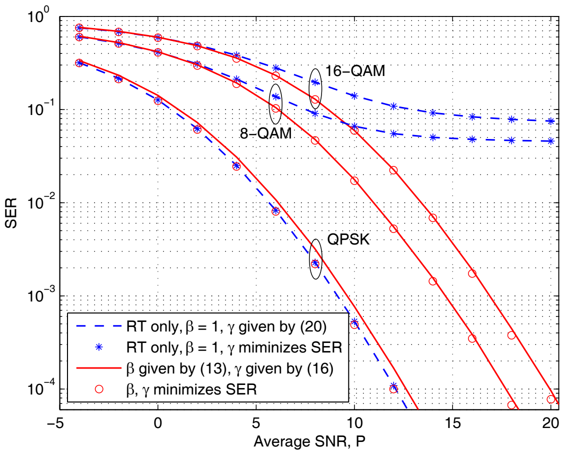

Fig. 6. SER vs. average SNR $\mathcal{P}$ for systems with $N_t = 4$ transmit antennas and block length of $L = 50$. The values of β and γ used are found from Monte-Carlo simulations by minimizing the SER as well as the closed-form solutions given in (13) and (16). The SER with only reverse training (RT) is also included for comparison. We see that the closed-form solutions for β and γ can be used to achieve near optimum SER over a wide range of SNR when moderate to high-order modulation is used, *e.g.*, 8-QAM and 16-QAM. When low-order modulation is used, *e.g.*, QPSK, the systems with RT achieves almost the same SER as those with two-way training. We have also observed the same results for $L = 10$, hence they are not shown for brevity.

as $\mathcal{P}$, and the ratio of power allocated to forward transmission as γ. We also use β as defined in the second scenario. The power constraint can be written as

$$\mathcal{P}L = \mathcal{P}_r + \mathcal{P}_f + \mathcal{P}_d(L-2).$$

Therefore, we have the following relationships.

$$\begin{aligned} \mathcal{P}_r &= (1-\gamma)\mathcal{P}L, \\ \mathcal{P}_d &= \beta\gamma\mathcal{P}L/(L-2), \\ \mathcal{P}_f &= (1-\beta)\gamma\mathcal{P}L. \end{aligned} \quad (14)$$

In the high SNR regime, we apply all the assumptions stated in the previous two scenarios, and hence the optimal power allocation satisfies the relationships given in (10) and (13). Using (8) and (10), we have[1] $\mathcal{P}_r = \frac{\mathcal{P}_f}{\sqrt{\mathcal{P}_d}}$. And using (11), we have $\mathcal{P}_f = \frac{1-\beta}{\beta}\mathcal{P}_d(L-2)$. Therefore, we obtain

$$\begin{aligned} \mathcal{P}L &= \mathcal{P}_r + \mathcal{P}_f + \mathcal{P}_d(L-2) \\ &= \frac{1-\beta}{\beta}\sqrt{\mathcal{P}_d}(L-2) + \frac{1}{\beta}\mathcal{P}_d(L-2), \end{aligned} \quad (15)$$

from which one can easily solve for $\mathcal{P}_d$. Then using the relationship between $\mathcal{P}_d$ and γ in (14), the optimal power allocation strategies at high SNR can be obtained as

$$\gamma = \frac{L-2}{2\mathcal{P}L\beta}\left[(1-\beta)^2 + \frac{2\mathcal{P}L\beta}{L-2} - (1-\beta)\sqrt{(1-\beta)^2 + \frac{4\mathcal{P}L\beta}{L-2}}\right], \quad (16)$$

where β is given in (13). Similar to the previous two scenarios, the optimal power allocation at high SNR given in (16) is independent of the number of transmit antennas.

[1]Here we omit the result for the special case of $\mathcal{P}_d = 1$ for brevity.

Numerical Results: Fig. 5 shows the optimal power ratio to forward transmission γ versus the average SNR $\mathcal{P}$ for different block lengths L.[2] The general trend is that a larger ratio of power should be allocated to the forward transmission as $\mathcal{P}$ increases. Similar to the previous two scenarios, we see that the values of γ which maximize $\rho_{\text{ave}}^{\text{LB}}$ in (7) are close to the values of γ that minimize the SER for 16-QAM modulation. We also see that the closed-form solution of γ given in (16) is reasonably accurate over a wide range of SNR, and the mismatch increases as $\mathcal{P}$ increases. This mismatch is mainly due to the fact that maximizing $\rho_{\text{ave}}^{\text{LB}}$ does not necessarily result in the optimal distribution of ρ which minimizes the SER, *e.g.*, it does not minimize the probability of ρ taking very small values.

Fig. 6 shows the SER versus the average SNR $\mathcal{P}$. The values of β and γ used in this plot are found both from minimizing the SER and from the closed-form solutions in (13) and (16). We see that the closed-form solutions for β and γ can be used to achieve near optimum SER over a wide range of SNR for 8-QAM and 16-QAM modulation. For QPSK modulation, the closed-form solutions result in a slight SER degradation of around 0.6 dB. In fact, we will see in the next section that the system without forward training, *i.e.*, $\mathcal{P}_f = 0$, can achieve near optimal SER performance for QPSK modulation. With $\mathcal{P}_f = 0$, the high SNR assumptions used in deriving the closed-form solutions for β and γ are no longer valid, which results in the observed SER degradation for QPSK modulation.

V. Optimal Power Allocation with Reverse Training Only

The systems with only reverse training can be regarded as a special case of two-way training based systems. In this

[2]For brevity, we omit the results on the optimal values of β as they are very similar to those shown in Fig. 3.

section, we provide an analytical solution for the optimal power allocation between the reverse training and the data transmission under a total transmit power budget. Since forward training is not used, the duration of data transmission becomes $L-1$. Similar to Section IV-C, we denote the average SNR for both links as $\mathcal{P}$ and the ratio of power allocated to the forward (data) transmission as γ. Therefore, we have the follow relationships

$$\mathcal{P}_d = \gamma \mathcal{P} L/(L-1), \qquad \mathcal{P}_r = (1-\gamma)\mathcal{P} L. \tag{17}$$

With reverse training only, the UT does not know the effective channel. However, the UT can accurately obtain the mean value of the effective channel μ_1, since it is a long-term statistic which changes much more slowly than the channel gain [9, 10]. Therefore, the average received SNR at the UT is given by

$$\rho_{\text{ave}} = \frac{\mathcal{P}_d \mu_1^2}{1+\mathcal{P}_d \sigma_f^2} = \frac{\nu \mathcal{P}_d \mathcal{P}_r}{k\mathcal{P}_d\mathcal{P}_r + \mathcal{P}_d + \mathcal{P}_r + 1} \tag{18}$$

$$= \frac{\nu \mathcal{P}^2 L^2 \gamma(1-\gamma)}{k\mathcal{P}^2 L^2 \gamma(1-\gamma) + \mathcal{P} L\gamma + \mathcal{P} L(L-1)(1-\gamma) + L-1}, \tag{19}$$

where (18) is obtained using (5) and (6), and (19) is obtained using (17). Letting the first derivative of ρ_{ave} in (19) w.r.t. γ be zero, one can solve for the optimal γ as

$$\gamma = \theta - \sqrt{\theta(\theta-1)}, \ \text{ where } \ \theta = \frac{(\mathcal{P} L+1)(L-1)}{\mathcal{P} L(L-2)}. \tag{20}$$

It is clear that the optimal power allocation given in (20) is independent of the number of transmit antennas, which is an important message for system designers.

Numerical Results: For comparison with two-way training based transmission, Fig. 6 also includes the SER performance for systems using only reverse training. We see that the closed-form solution for γ given in (20) achieves near optimum SER over a wide range of SNR. When moderate to high-order modulation is used, *e.g.*, 8-QAM and 16-QAM, the use of two-way training achieves a significant SER reduction over reverse training at moderate to high SNR. However, when low-order modulation is used, *e.g.*, QPSK, it is clear that the systems with two-way training achieve no or marginal performance gain over those with only reverse training. This result suggests that reverse training is sufficient at low operating SNR or when low-order modulations are used.

VI. Conclusion

We studied MISO systems with two-way training based transmission. We derived the LMMSE channel estimation for the forward training. We investigated the optimal power allocation which optimizes the SER performance. An average SNR lower bound was used to obtain closed-form solutions to the optimal power allocation at high SNR. These solutions were shown to achieve near optimum SER performance over a wide range of SNR values. In addition, we found the optimal power allocation for systems with reverse training only. We numerically showed that two-way training provides no or marginal performance gain over reverse training only, at low SNR or when low-order modulations are used. Future work can extend this analysis to a multi-user setup. The results in this work directly apply in scenarios where single-user beamforming is used, while similar analysis based on signal to interference plus noise ratio is suggested for other precoding schemes.

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