A Simple Model for Performance Evaluation of a Smart Antenna in a CDMA System

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***Abstract*— Smart antennas are recognized as a key technology to improve the performance of wireless communication systems. In this paper we propose a simple model to analyse the Mean Bit Error Rate (BER) of an IS-95 Code Division Multiple Access (CDMA) system with base station antenna array operating in a Rayleigh fading multipath environment. We present an expression for the Signal to Interference plus Noise (SINR) at the output of the smart antenna receiver, as a function of the system parameters. Good agreement between the results proposed by the model and the simulation results of the actual system is noted.**

I. Introduction

Smart or adaptive arrays are recognized as a key technology to achieve significant improvements in capacity and coverage for wireless communication systems [1]. In this paper we focus on Code Division Multiple Access (CDMA) since the next generation wireless communication systems are based on CDMA. It is well known that the performance of CDMA systems is limited by both multipath fast fading and Multiple Access Interference (MAI) [2]. Hence two dimensional (2D) RAKE receivers, which combine spatial processing via BS antenna array and temporal processing via conventional RAKE receivers, have been proposed to effectively mitigate interference and multipath fading [3].

The exact analytical evaluation of the performance of 2D-RAKE receivers for CDMA systems is still an open subject. However approximations have been proposed, e.g. [3]–[6]. Recently in [7], a new simple method was proposed to analyse the performance of 2D-RAKE receivers. The validity of the new method was illustrated by considering a generic DS-CDMA system, employing BPSK modulation, with BS array antennas.

In this paper, we develop a simple model for performance evaluation of 2D-RAKE receivers for IS-95 CDMA cellular systems. We utilise the BER performance results of a conventional RAKE receiver (without antenna array) in [8] and apply the method proposed in [7] to analyse the Mean BER performance of an IS-95 CDMA system with BS array antenna in a Rayleigh fading multipath environment. We present an expression for the Signal to Interference plus Noise (SINR) ratio as a function of the number of users, number of multipaths, number of antennas and signal to noise levels. We also present simulation results to confirm the accuracy of the analytical results generated by the proposed model.

This paper is organised as follows. The system and channel model is presented in Section II. The performance analysis for a conventional single antenna RAKE receiver is outlined in Section III. The BER approximation procedure is briefly reviewed and applied to a smart antenna for IS-95 CDMA system in a Rayleigh fading multipath channel in Section IV. The analysis and simulation results are compared in Section V. Finally conclusions are presented in Section VI.

II. System and Channel Model

We consider a BS serving a single 120° sector cell. Without loss of generality, we assume that the BS employs a Uniform Linear Array (ULA) of N identical omni-directional antenna elements, with inter-element spacing of $d = \lambda/2$. Let K denote the total number of Mobile Stations (MS) in the system, which are randomly distributed in the azimuthal direction, along the arc boundary of the sector cell in the far field of the array. The $k = 1$ user represents the desired user. The location of each MS is characterized by its Angle of Arrival (AOA) θ_k, which is conventionally measured from the array broadside. We refer to $\theta = 0°$ as the broadside direction.

We assume the MS's follow specifications of the IS-95 CDMA reverse link [2]. We consider the uncoded system for simplicity. The transmitted signal $s_k(t)$ of the kth user can be written as

$$\begin{aligned} s_k(t) &= W_k^{(m)}(t)\, a_k^{(I)}(t)\cos(\omega_c t) + W_k^{(m)}(t-T_o) \\ &\quad a_k^{(Q)}(t-T_o)\sin(\omega_c t) \end{aligned} \quad (1)$$

where $W_k^{(m)}(t)$ is the mth M-ary Walsh Symbol ($m = 1, 2, \ldots, M = 64$) of the kth user, $a_k^{(I)}(t) = a_k(t)\, a^{(I)}(t)$, $a_k^{(Q)}(t) = a_k(t)\, a^{(Q)}(t)$, $a^{(I)}(t)$ is the In-phase (I) channel spreading sequence, $a^{(Q)}(t)$ = Quadrature (Q) channel spreading sequence, $a_k(t)$ is the kth user long code sequence, T_o is the half chip delay for OQPSK signals, $\omega_c = 2\pi f_c$ and f_c is the carrier frequency.

The signals transmitted by the K users pass thorough a Rayleigh fading multipath channel and are received by the BS. The channel impulse response between the lth multipath of the kth user and the nth antenna is given as [9]

$$\begin{aligned} h_{k,l,n}(t) &= \sqrt{\frac{\Omega_{k,l}}{S}} \sum_{s=1}^{S} e^{[j(\phi_{k,l}^{(s)} + 2\pi\cos(\Psi_{k,l}^{(s)}) f_D t)]} \\ &\quad \times\ e^{[-j\mathcal{K}d(n-1)\sin\theta_k]}\, \delta(t-\tau_l) \end{aligned} \quad (2)$$

where S is the number of subpaths for each resolvable path, $\Omega_{k,l}$ is the mean path power of the lth multipath, τ_l is the multipath delay, $\mathcal{K} = 2\pi/\lambda$ is the wave number, f_D is the Doppler frequency, $\phi_{k,l}^{(s)}$ is random phase of each subpath, assumed to be uniformly distributed over $[0, 2\pi]$ and $\Psi_{k,l}^{(s)}$ is the Angle of Departure (AOD) for each sub-path relative to the motion of the mobile, which is also assumed to be uniformly distributed over $[0, 2\pi]$ [10].

Under these assumptions, the total received signal at the nth element of the BS array is given by

$$x_n(t) = \sum_{k=1}^{K}\sum_{l=1}^{L}\Big[\beta_{k,l}\, W_k^{(m)}(t-\tau_{k,l})\, a_k^{(I)}(t-\tau_{k,l}) \cos(\omega_c t + \varphi_{k,l,n}) + \beta_{k,l}\, W_k^{(m)}(t-T_o-\tau_{k,l}) a_k^{(Q)}(t-T_o-\tau_{k,l})\sin(\omega_c t+\varphi_{k,l,n})\Big] + \eta_n(t) \quad (3)$$

where $s_k(t)$ is the signal transmitted by kth user given by Eq. (1), $\beta_{k,l}$ is the modulus of the complex channel amplitude, $\varphi_{k,l,n}$ is the overall phase of the lth path of the kth user at the nth antenna, $\tau_{k,l} = \Gamma_k + \tau_l$, Γ_k is the random delay of the kth user due to the asynchronous nature of the CDMA system and $\eta(t)$ is the noise which is assumed to be AWGN.

The BS implements a smart antenna system that consists of a combined beamformer and RAKE receiver (2D-RAKE receiver). The block diagram of the smart antenna receiver incorporating the array antenna is shown in Fig. 1. The receiver uses non-coherent demodulation to recover the desired signal. The received signal at each antenna element is first down converted. To detect the lth path, the signal is despread using the PN sequence of the respective MS and synchronized to the delay of the lth path. A beamformer is then constructed for each resolvable multipath and the signal after PN despreading is combined by the beamformer. The smart antenna output is given by

$$z_{k,l}(t) = (\mathbf{w}_{k,l})^H\, \mathbf{y}_{k,l} \quad (4)$$

where $(\cdot)^H$ denotes Hermitian transpose operation, $\mathbf{y}_{k,l}$ is the post PN-despread signal vector and $\mathbf{w}_{k,l}$ is the beamforming weight vector. We assume that the weights are determined using the Maximum Signal to Noise Ratio (MSNR) beamforming criteria [5] and that the ideal solution for the weight vectors is known i.e. the weight vectors are set to the channel response vector for the desired user. This provides the best case system performance.

After smart antenna combining, the beamformer outputs are Walsh correlated. The decision variable obtained from the lth multipath of the kth user is given by

$$u_{k,l}(q) = (Z_{k,l}^{(I)})^2 + (Z_{k,l}^{(Q)})^2 \quad (5)$$

where $Z_{k,l}^{(I)}$ and $Z_{k,l}^{(Q)}$ denote the output of the Walsh correlators. The overall decision variable is obtained by equal gain combining of all the decision variables from the L multipaths as $u_k(q) = \sum_{l=1}^{L} u_{k,l}(q)$. Finally the receiver makes a hard decision on the mth symbol by using the following rule: $\hat{m} = \arg\max_{q=1,\ldots,M}\{u_k(q)\}$.

III. Performance of Conventional RAKE Receiver

For the case of a single antenna (conventional) receiver, it can be shown that the output of the qth Walsh correlator $(q = 1, 2, \cdots, M = 64)$ can be written as [8], [11]

$$Z_{k,l}^{(I)}(q) = \begin{cases}\beta_{k,l}\cos\varphi_{k,l} + I_{k,l}^{(I)} + M_{k,l}^{(I)} + N_{k,l}^{(I)} & ;\text{if } q = m\\ I_{k,l}^{(I)} + M_{k,l}^{(I)} + N_{k,l}^{(I)} & ;\text{else}\end{cases} \quad (6)$$

$$Z_{k,l}^{(Q)}(q) = \begin{cases}\beta_{k,l}\sin\varphi_{k,l} + I_{k,l}^{(Q)} + M_{k,l}^{(Q)} + N_{k,l}^{(Q)} & ;\text{if } q = m\\ I_{k,l}^{(Q)} + M_{k,l}^{(Q)} + N_{k,l}^{(Q)} & ;\text{else}\end{cases} \quad (7)$$

where $N_{k,l}^{(I)}$ and $N_{k,l}^{(Q)}$ denote the noise terms, $M_{k,l}^{(I)}$ and $M_{k,l}^{(Q)}$ denote the MAI terms, $I_{k,l}^{(I)}$ and $I_{k,l}^{(Q)}$ denote the self interference terms.

Using the Gaussian approximation [12], it can be shown that the multiple access interferences terms $M_{k,l}^{(I)}$ and $M_{k,l}^{(Q)}$ can be modelled as zero mean Gaussian random processes with variance

$$\sigma_M^2 = \frac{E_s}{3N_c}\sum_{k=2}^{K}\sum_{l=1}^{L} E[(\beta_{k,l})^2] \quad (8)$$

where $E_s = E_b \log_2(M)$ is the symbol energy and N_c is the spreading factor.

Also $I_{k,l}^{(I)}$ and $I_{k,l}^{(Q)}$ are the respective self interferences due to the desired users own multipaths and can be modelled as zero mean Gaussian random processes with variance

$$\sigma_I^2 = \frac{E_s}{3N_c}\sum_{l=2}^{L} E[(\beta_{k,l})^2] \quad (9)$$

The noise terms $N_{k,l}^{(I)}$ and $N_{k,l}^{(Q)}$ are mutually independent zero mean Gaussian random processes with variance $\sigma_N^2 = \frac{N_o}{2}$.

The overall decision variable is a sum of squares of two Gaussian random variables, each with variance

$$\begin{aligned}\sigma_T^2 &= \sigma_N^2 + \sigma_I^2 + \sigma_M^2\\ &= \frac{N_o}{2} + \frac{E_s}{3N_c}\left\{\sum_{l=2}^{L} E[(\beta_{k,l})^2] + \sum_{k=2}^{K}\sum_{l=1}^{L} E[(\beta_{k,l})^2]\right\}\end{aligned} \quad (10)$$

We assume that uniform power delay profile is used to characterize the multipath fading channel and that all users have the same average signal power at the receiver due to perfect power control, i.e. $\Omega_1 = \Omega_2 = \cdots = \Omega_L = \Omega/L$. Therefore, Eq. (10) becomes

$$\begin{aligned}\sigma_T^2 &= \frac{N_o}{2} + \frac{E_s}{3N_c}\,\frac{\Omega}{L}\,(L-1) + \frac{E_s}{3N_c}\,\frac{\Omega}{L}\,(K-1)L\\ &= \frac{N_o}{2} + \frac{E_s}{3N_c}\,\frac{\Omega}{L}\,(KL-1)\end{aligned} \quad (11)$$

The SINR can thus be written as

$$\rho = \frac{\Omega}{L}\,\frac{E_s}{2\,\sigma_T^2} = \frac{\gamma}{1 + \frac{2}{3\,N_c}\,\gamma\,(KL-1)} \quad (12)$$

where $\gamma = \frac{\Omega}{L}\frac{E_s}{N_o} = \frac{\Omega}{L}\frac{E_b}{N_o}\log_2(M)$.

Using the SINR and the statistics of the decision variable, it can be shown that the mean bit error probability for

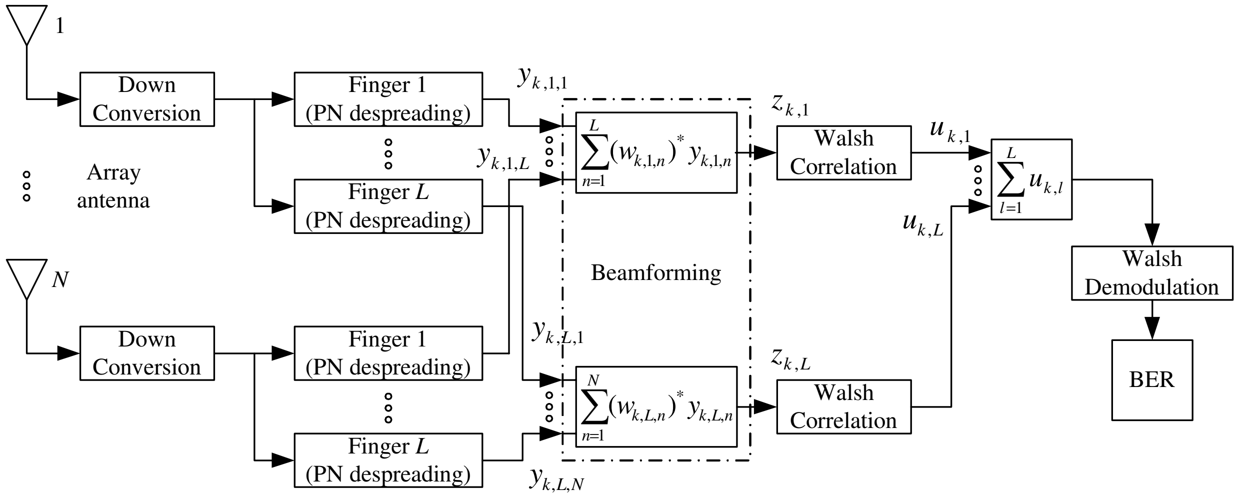

Fig. 1. Block diagram of the smart antenna receiver.

a conventional RAKE receiver (i.e. single antenna without beamforming) in Rayleigh fading over $L-$fold multipath diversity with Equal Gain Combining (EGC) is given by [2]

$$P_b^{(1D)} = \frac{M/2}{M-1}\sum_{m=1}^{M-1}\binom{M-1}{m}\frac{(-1)^{m+1}}{(1+m+m\rho)^L} \times \sum_{l=0}^{m(L-1)} \mathcal{C}_l(m)\binom{L+l-1}{l}\left(\frac{1+\rho}{1+m+m\rho}\right)^l \quad (13)$$

where ρ is given by Eq. (12) and the coefficients $\mathcal{C}_l(m)$ can be computed recursively using [2]

$$\mathcal{C}_l(m) = \begin{cases} \frac{1}{l}\sum\limits_{q=1}^{l}\binom{l}{q}[(m+1)q-l]\,\mathcal{C}_{l-q}(m) & ;\ l \le L-1, \\ \frac{1}{l}\sum\limits_{q=1}^{L-1}\binom{l}{q}[(m+1)q-l]\,\mathcal{C}_{l-q}(m) & ;\ l \ge L-1. \end{cases} \quad (14)$$

with $\mathcal{C}_0(m) = 1$ for all m.

IV. Performance of 2D-RAKE Receiver

We first briefly review the BER approximation procedure here and then apply it to analytically determine the Mean BER performance of the IS-95 CDMA system with an adaptive array antenna at the BS in Rayleigh fading channels [7].

A. BER Approximation

The approximation proposes to adapt the single antenna performance bounds to array antenna systems by manipulating those terms in the error probability formulas for single antenna receivers that account for the noise and MAI.

The procedure partitions the interferers into two categories – in-beam and out-of-beam – based on whether their Angles of Arrivals lie inside or outside the beam formed toward the desired user. The antenna array beampattern is approximated by a piece-wise function representing in-beam and out-of-beam performance. Because of the piece-wise approximation, the energy of each in-beam interferer is a random variable uniformly distributed within $[\frac{1}{2}, 1]$ (half power beamwidth region). Hence a correction factor of $f = 3/4$ (average value) is applied for in-beam interferers. For the purpose of evaluation of the error probability, the in-beam interferers are counted as interference while the out-of-beam users increase the additive noise level [13], [14].

B. BER Analysis

Let the modified variances of the noise, self interference and MAI be denoted as $\bar{\sigma}_N^2$, $\bar{\sigma}_I^2$ and $\bar{\sigma}_M^2$ respectively.

Let κ denote the number of in-beam interferers. Then number of out-of-beam interferers $= K - \kappa - 1$. Hence we have

$$\bar{\sigma}_M^2 = \overbrace{\left\{\alpha_o \frac{E_s}{3N_c}\frac{\Omega}{L}(K-\kappa-1)L\right\}}^{\text{out-of-beam}} + \overbrace{\left\{f\frac{E_s}{3N_c}\frac{\Omega}{L}\kappa L\right\}}^{\text{in-beam}} \quad (15)$$

where α_o is the attenuation factor for out-of-beam interferers and f is a correction factor for in-beam interferers.

Since the spatial channel is based on one AOA only, the self interference due to desired user's multipaths cannot be attenuated by beamforming. Hence we have

$$\bar{\sigma}_I^2 = \frac{E_s}{3N_c}\frac{\Omega}{L}(L-1) \quad (16)$$

We know that in comparison with the power of the desired signal, the noise power at the output of the antenna array is reduced by N times, where N is the number of antenna elements [15]. Hence $\bar{\sigma}_N^2 = \frac{\sigma_N^2}{N}$.

The modified SINR expression is thus given by $\rho = \frac{E_s}{2\bar{\sigma}_T^2}$, where $\bar{\sigma}_T^2 = \bar{\sigma}_N^2 + \bar{\sigma}_I^2 + \bar{\sigma}_M^2$ is the total variance. Substituting the values and simplifying, we get

$$\rho = \frac{\gamma}{\left(\frac{1}{N} + \frac{2\gamma}{3N_c}(L-1) + \alpha_o\frac{2\gamma}{3N_c}(K-\kappa-1)L + f\frac{2\gamma}{3N_c}\kappa L\right)} \quad (17)$$

where $\gamma = \frac{\Omega}{L}\frac{E_s}{N_o} = \log_2(M)\frac{\Omega}{L}\frac{E_b}{N_o}$.

Assuming a uniform distribution of interferers in an angular sector of $120°$, we can obtain the average bit error probability of 2D-RAKE receiver (combined beamformer and RAKE receiver) as [7]

$$P_b^{(2D)} = \sum_{\kappa=0}^{K-1} \chi\, \eta^{\kappa} \binom{K-1}{\kappa} P_b^{(1D)} \tag{18}$$

where $P_b^{(1D)}$ is the probability of bit error for the single antenna (conventional) receiver given by Eq. (13) with ρ is given by Eq. (17), χ and the probability of an in-beam interferer η, are defined as [13]

$$\eta = \frac{(2\,\theta_{BW})}{\Delta\theta} \tag{19}$$

$$\chi = (1-\eta)^{K-\kappa-1} \tag{20}$$

where $\Delta\theta = 120°$ is the total coverage angle of the sector and $2\theta_{BW}$ is the total beamwidth towards the desired user. The values of $2\theta_{BW}$ and α_o used in this work are given in [13].

V. Results

We perform system simulations to confirm the validity of the analytical model presented in the previous section. The mean BER is collected and averaged over $M_c = 100$ drops [9] in the simulations. A 'drop' is defined as a simulation run for a given number of MS's and BS over 125 frames, which corresponds to the time required by the desired user to traverse the entire azimuth range $[-60°, 60°]$. For other users, their AOAs are assumed to be uniformly distributed over $[-60°, 60°]$. The overall channel gain is normalised to one for each user. The main simulation parameters are summarized in Table I.

The analytical results in Figs. 2-5 are generated using (18) together with (13) and (17). Fig. 2 shows the Mean BER vs. Number of users K for $N = 1$ antenna, $E_b/N_o = 10$ dB, assuming $L = 1, 2, 3$ Rayleigh fading multipaths respectively. We see that the simulation results (markers) show good agreement with analytical results (lines).

Fig. 3 shows the Mean BER vs. Number of users K for $N = 4, 6, 8$ antennas, $E_b/N_o = 10$ dB, assuming $L = 1, 2$ Rayleigh fading multipaths respectively. Comparing with the reference curves for single antenna in Fig. 2, we see that beamforming considerably improves the performance of the system. It can also be seen that as the number of users increases, the BER model provides a more accurate match with the simulations.

Finally, Fig. 4 shows the Mean BER vs. E_b/N_o (dB) for $N = 6, 8$ antennas, $K = 15$ users, assuming $L = 1, 2, 3$ Rayleigh fading multipaths respectively while Fig. 5 shows the Mean BER vs. Number of antennas N, for $E_b/N_o = 10$ dB, $K = 15$ users, assuming $L = 1, 2, 3$ Rayleigh fading multipaths. It can be seen again that the simulation results agree well with the analytical model.

VI. Conclusions

In this paper, we have presented an analytical procedure to analyse the performance of an IS-95 CDMA system with base station antenna array in a Rayleigh fading multipath environment. In order to confirm the validity of the analytical model, simulations under different system conditions have been performed. The obtained results have shown that the simple model accurately predicts the performance of smart antennas for IS-95 CDMA system in Rayleigh fading channels.

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TABLE I

MAIN SIMULATION PARAMETERS

Parameter	Description/Value
Carrier frequency	900 MHz
Chip time	$T_c = \frac{1}{1228800}$ s
Bits/frame	576
Frame length	20 ms
Spreading gain	$N_c = 256$
Base Station synchronisation	Asynchronous operation
Channel estimation	perfect
Power control	ideal
Detection	Non-coherent
Number of Multipaths	$L = 1, 2, 3$
Power Delay Profile	Uniform
User mobility	$0.01°$ per snapshot
PDF in AOD	Uniform
Doppler frequency	$f_D = 100$ Hz
Number of subpaths	$S = 15$

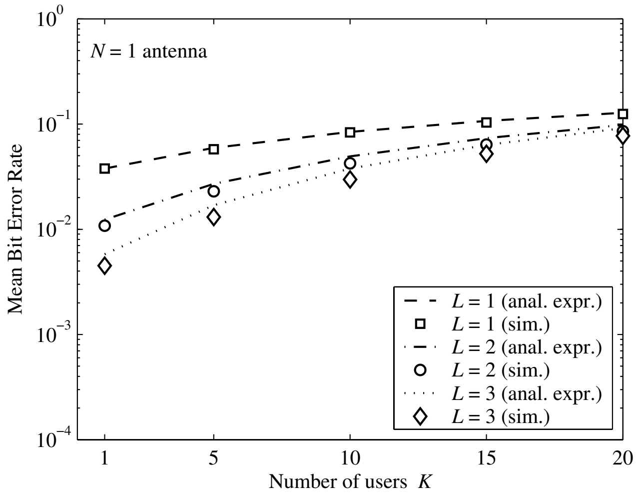

Fig. 2. Mean BER vs. Number of users K for $N = 1$ antenna, $E_b/N_o = 10$ dB, assuming $L = 1, 2, 3$ Rayleigh fading multipaths respectively (anal. expr.: lines, simulations: markers).

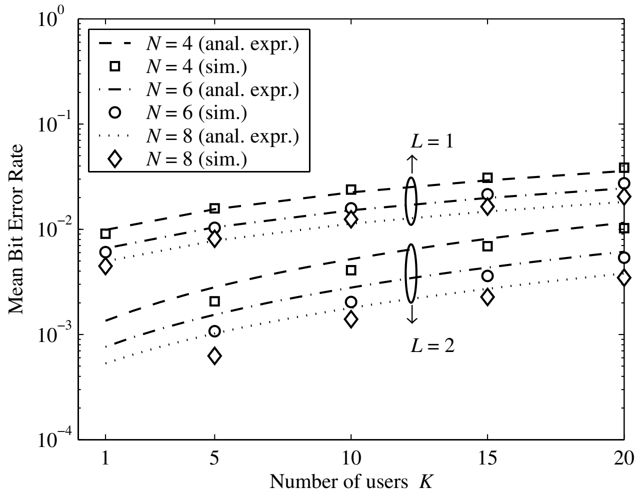

Fig. 3. Mean BER vs. Number of users K for $N = 4, 6, 8$ antennas, $E_b/N_o = 10$ dB, assuming $L = 1, 2$ Rayleigh fading multipaths respectively (anal. expr.: lines, simulations: markers).

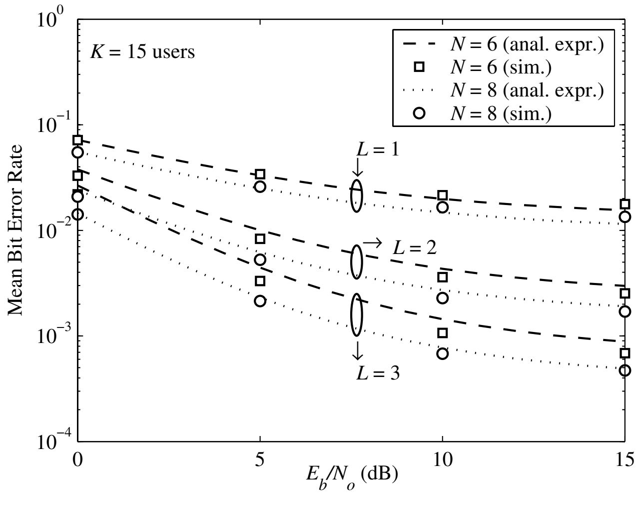

Fig. 4. Mean BER vs. E_b/N_o (dB) for $N = 6, 8$ antennas, $K = 15$ users, assuming $L = 1, 2, 3$ Rayleigh fading multipaths respectively (anal. expr.: lines, simulations: markers).

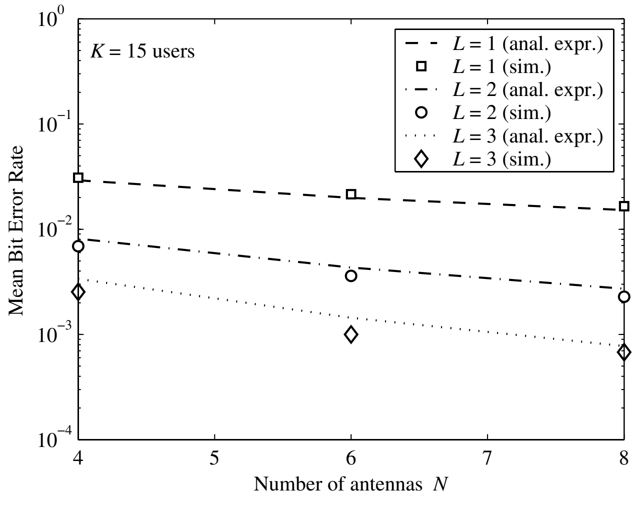

Fig. 5. Mean BER vs. Number of antennas N, for $E_b/N_o = 10$ dB, $K = 15$ users, assuming $L = 1, 2, 3$ Rayleigh fading multipaths respectively (anal. expr.: lines, simulations: markers).