Connectivity of Ad Hoc Networks: Is Fading Good or Bad?

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Abstract— **In this paper, we investigate the effect of Rayleigh fading on the connectivity of wireless ad hoc networks. We consider static nodes that are equipped with omnidirectional antennas and are randomly distributed in the network according to a uniform distribution. We derive an analytical model for evaluating the impact of Rayleigh fading on the network local connectivity and present an accurate upper bound for 1-connectivity. Furthermore, our numerical results show that the presence of fading can result in an improvement in the overall connectivity, although it reduces the local connectivity of the network.**

I. Introduction

Wireless ad hoc networks are a promising technology for the next generation of wireless communication networks [1]. Due to the lack of infrastructure in these networks, a connection between two nodes is usually established via multiple direct links between intermediate nodes. Due to their multi-hop nature, connectivity is a crucial problem in ad hoc networks. The connectivity of wireless ad hoc networks has been studied assuming both omnidirectional antennas [2]–[10] and beamforming antennas [11]–[13] at each node.

Most studies on the connectivity of wireless ad hoc networks assume a simplistic path loss channel model, which is also referred to as the geometric disk model [2]–[4]. It is assumed that the nodes can communicate with each other if and only if the inter-node distance is smaller than a threshold value. The problems of finding a necessary and sufficient transmission range and the corresponding critical node density for an almost surely connected network were investigated in [2] and [3], respectively. The impact of interference on the connectivity of large-scale ad hoc networks was studied in [4].

In the last few years, the effect of channel randomness has been taken into account in connectivity analysis [5]–[9]. It was shown in [5] that the geometric disk is the hardest shape for high connectivity when compared with irregular shapes. Considering a log-normal shadowing channel model, it was found in [6] that the network local and overall connectivity are higher than those in a path loss channel. Similar conclusions were also reached in [8] and [9].

In the presence of scattering, the amplitude of a radio signal experiences rapid fluctuation over a short period of time or propagation distance. This effect is called small scale or Rayleigh fading. In static networks, such as wireless sensor networks, fading can be understood as channel randomness in the spatial domain which results in randomness in the received signal power for nodes at different locations [7]. The study in [10] performed symbol error rate analysis taking fading and inter-node interference into account. The analytical study on the probability of node isolation in [7] concluded that the presence of Rayleigh fading reduces the connectivity of the network.

In this paper, we investigate the impact of Rayleigh fading on the local and overall connectivity of wireless ad hoc networks with omnidirectional antennas. That is we examine the effect on (i) connectivity from the viewpoint of a single node (probability of node isolation) and (ii) connectivity from the viewpoint of the entire network (path probability and 1-connectivity). Our approach is based on the notion of the effective communication range of a node, which is also adopted in [7], [8]. However, as acknowledged by the authors themselves in [7], the derivation of the node isolation probability in [8] is cumbersome.

The main contributions of this work are:

- In Section III, we present an analytical model for evaluating the impact of Rayleigh fading on the connectivity of static wireless ad hoc networks. In contrast to [7], [8], we present a simpler, intuitive approach to calculating the second order moment of the effective communication range. Also, we provide additional insights into the the effect of fading on both the local and overall connectivity as well as its dependence on the path loss exponent.
- In Section IV, we show that in static ad hoc networks the presence of Rayleigh fading can result in an improvement in the overall connectivity, although it reduces the local connectivity of the network. This important result on overall network connectivity is not revealed from the analysis in [7].

The rest of this paper is organized as follows. The system model and network connectivity metrics used in this work are described in Section II. The analytical model for evaluating the impact of Rayleigh fading on the network connectivity is presented in Section III. The numerical results are shown in Section IV. Finally, conclusions are drawn in Section V.

II. System Model

A. Node Distribution Model

We consider that the nodes in the network are randomly distributed in a two dimensional space of area B according to

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a uniform distribution. The nodes are assumed to be static. The node density is denoted as ρ, so the total number of nodes is $N_0 = \rho B$. Also, the probability of a node located in a smaller area A is given by $p = \frac{A}{B}$. Therefore, the number of nodes X located in A has a binomial distribution with probability mass function given by [14]

$$P(X = x) = \frac{N_0!}{x!(N_0 - x)!} p^x (1-p)^{N_0 - x}. \quad (1)$$

Letting $B \to \infty$ and keeping other parameter unchanged, we can see that $N_0 \to \infty$ and $p \to 0$ in such a way that $\mu = N_0 p$ is constant. Therefore the binomial distribution is approximately equal to a Poisson distribution with parameter μ given by [14]

$$P(X = x) = \frac{\mu^x}{x!} e^{-\mu}. \quad (2)$$

In other words, the node deployment process can be seen as a homogeneous Poisson process, which provides an accurate model for a uniform distribution as the network area approaches infinity.

B. Channel Model

We model the wireless propagation channel using large scale path loss and small scale fading effects. Assuming an isotropic scattering environment, the fading process has a Rayleigh distribution given by [15]

$$f_R(r) = \frac{2r}{\Omega} \exp\left(-\frac{r^2}{\Omega}\right), \quad (3)$$

where $\Omega = E(r^2)$ is a constant. Note that r^2 has a Chi-square distribution. Assuming each node is equipped with an omnidirectional antenna, the received signal power at each node can be modelled as [16]

$$P_R = r^2 \frac{1}{d^\alpha} C P_T, \quad (4)$$

where d is the distance between the transmitting node and the receiving node, α is the path loss exponent, $C = \left(\frac{\lambda}{4\pi}\right)^2$, λ is the wavelength of the propagating signal, and P_T is the transmit power. Note that in typical wireless communication scenarios the path loss exponent α is usually in the range $2-4$, while for some specific scenarios it can be as large as 6 [15].

C. Connectivity Metrics

The following connectivity metrics are used in the discussion of the analytical and numerical results [3], [11]:-

- *Probability of Node Isolation* ($P(\text{iso})$) is defined as the probability that a randomly selected node in an ad hoc network has no connected neighbours. It is a measure of the network local connectivity.
- *1-connectivity* ($P(\text{1-con})$) is defined as the probability that every node pair in the network has at least one path connecting them. It is a measure of the overall network connectivity.
- *Path Probability* ($P(\text{path})$) is defined as the probability that two randomly chosen nodes in an ad hoc network are connected via a direct link or a multi-hop path. It is a measure of the overall network connectivity.

Note that 1-connectivity is a stronger measure of the overall network connectivity than path probability.

III. Theoretical Analysis

In this section, we study the impact of Rayleigh fading on the connectivity of wireless ad hoc networks. In contrast to [7], [8], we present a simpler, intuitive approach to calculating the second order moment of the effective communication range and study the impact of Rayleigh fading on both the local and overall network connectivity.

Without loss of generality, we can normalise (4) with respect to constant C, so that the power attenuation is expressed as

$$\beta(d) = \frac{P_T}{P_R} = \frac{d^\alpha}{r^2}. \quad (5)$$

Assuming a low traffic network with an efficient medium access control (MAC) layer protocol, the inter-node interference is negligible. We define a threshold power attenuation, β_{th}, above which there is no direct connection between the transmitting node and the receiving node. Therefore, the probability of having no direct connection with node separation d is given by

$$\begin{aligned} P(\beta(d) \geq \beta_{\text{th}}) &= P\left(\frac{d^\alpha}{r^2} \geq \beta_{\text{th}}\right) \\ &= P((\beta_{\text{th}} r^2)^{\frac{1}{\alpha}} \leq d). \end{aligned} \quad (6)$$

We define a random variable $\hat{R}$ as

$$\hat{R} = (\beta_{\text{th}} r^2)^{\frac{1}{\alpha}}. \quad (7)$$

Substituting (7) in (6), we have

$$P(\beta(d) \geq \beta_{\text{th}}) = P(\hat{R} \leq d). \quad (8)$$

Hence the random variable $\hat{R}$ can be referred to as the *effective communication range*. That is a node is able to communicate with all other nodes located within a distance of $\hat{R}$. The effective coverage area of a node can thus be considered as a disk with radius $\hat{R}$, centered at the node. Therefore, the effective coverage area is given by $\pi \hat{R}^2$. Assuming the path loss exponent is a constant, we have

$$\begin{aligned} E[\hat{R}^2] &= E\left[(\beta_{\text{th}} r^2)^{\frac{2}{\alpha}}\right] \\ &= (\beta_{\text{th}})^{\frac{2}{\alpha}} E[r^{\frac{4}{\alpha}}], \end{aligned} \quad (9)$$

where $E[\cdot]$ denotes statistical expectation. We can see that the effect of fading in (9) is characterized by $E[r^{\frac{4}{\alpha}}]$.

To evaluate this expectation, we define a random variable $Y = R^{\frac{4}{\alpha}}$. Since $y = r^{\frac{4}{\alpha}}$ is an increasing function of r, the probability density function (PDF) $f_Y(y)$ of the random variable Y can be expressed in terms of $f_R(r)$ as [14]

$$\begin{aligned} f_Y(y) &= f_R(y^{\frac{\alpha}{4}})\frac{d}{dy}y^{\frac{\alpha}{4}}, \\ &= \frac{2y^{\frac{\alpha}{4}}}{\Omega}\exp\left(-\frac{(y^{\frac{\alpha}{4}})^2}{\Omega}\right)\frac{\alpha}{4}y^{\frac{\alpha}{4}-1}, \\ &= \frac{\alpha}{2\Omega}y^{\frac{\alpha}{2}-1}\exp\left(-\frac{y^{\frac{\alpha}{2}}}{\Omega}\right). \end{aligned} \tag{10}$$

Therefore, the expected value of Y can be calculated as

$$\begin{aligned} E[Y] &= \int_0^\infty y f_Y(y) dy \\ &= \int_0^\infty \frac{\alpha}{2\Omega}y^{\frac{\alpha}{2}}\exp\left(-\frac{y^{\frac{\alpha}{2}}}{\Omega}\right)dy \\ &= \int_0^\infty \frac{\alpha}{2}\frac{y^{\frac{\alpha}{2}}}{\Omega}\exp\left(-\frac{y^{\frac{\alpha}{2}}}{\Omega}\right)dy \end{aligned} \tag{11}$$

Let $t = \frac{y^{\frac{\alpha}{2}}}{\Omega}$, hence $y = (\Omega t)^{\frac{2}{\alpha}}$ and $\frac{dy}{dt} = \frac{2}{\alpha}(\Omega)^{(\frac{2}{\alpha})}t^{(\frac{2}{\alpha}-1)}$. Substituting t for y into (11), we get

$$\begin{aligned} E[Y] &= \int_0^\infty \frac{\alpha}{2}t\exp(-t)\frac{2}{\alpha}(\Omega)^{\frac{2}{\alpha}}t^{(\frac{2}{\alpha}-1)}dt \\ &= (\Omega)^{\frac{2}{\alpha}}\int_0^\infty t^{(\frac{2}{\alpha}+1-1)}\exp(-t)\,dt \\ &= (\Omega)^{\frac{2}{\alpha}}\Gamma\left(\frac{2}{\alpha}+1\right) \end{aligned} \tag{12}$$

where $\Gamma(z) = \int_0^\infty t^{z-1}\exp(-t)dt$ is the Gamma function.

Since fading creates fluctuation in the signal but does not change the average power of the received signal determined by path loss, we set $\Omega = E[r^2] = 1$. Therefore the effect of fading on the effective coverage area in (12) reduces to

$$E[r^{\frac{4}{\alpha}}] = \Gamma\left(\frac{2}{\alpha}+1\right) \tag{13}$$

Since $\alpha > 2$ for any practical system, the argument in $\Gamma(\cdot)$ always ranges between 1 and 2. Hence, $E[r^{\frac{4}{\alpha}}] < 1, \forall\,\alpha$. Fig. 1 shows $E[r^{\frac{4}{\alpha}}]$ for a practical range of α from $\alpha = 2$ to $\alpha = 6$ [15]. We see that $E[r^{\frac{4}{\alpha}}]$ decreases from 1 at $\alpha = 2$ until it reaches a minimum value of 0.886 at $\alpha = 4.3$, then it starts to increase slowly towards unity as α increases further. This result suggests that fading reduces the effective coverage area and the maximum reduction happens at $\alpha = 4.3$.

A. Local Connectivity

For a node deployment following a homogeneous Poisson point process with density ρ, the node degree D which is defined as the number of direct connections one node establishes has a Poisson distribution with parameter $\rho\pi E[\hat{R}^2]$ [7]. Therefore, the average node degree $E[D]$ is given by

$$E[D] = \rho\pi E[\hat{R}^2]. \tag{14}$$

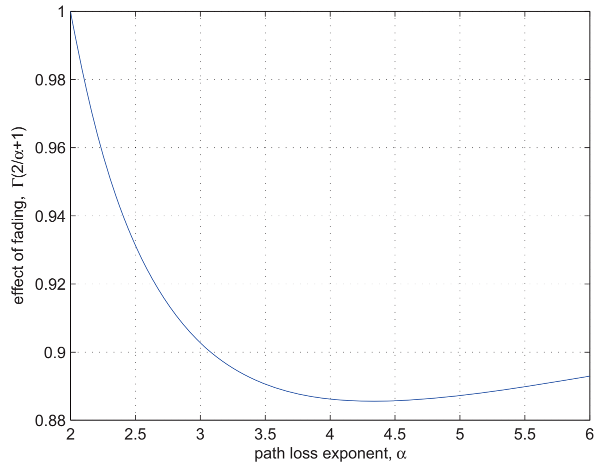

Fig. 1. The effect of fading in (13) for a practical range of path loss exponent.

Using (9), (13) and (14), the probability of node isolation is given by [6]

$$\begin{aligned} P(\text{iso}) &= \exp\{-E[D]\}, \\ &= \exp\{-\rho\pi(\beta_{\text{th}})^{\frac{2}{\alpha}}E[r^{\frac{4}{\alpha}}]\}, \\ &= \exp\left\{-\rho\pi(\beta_{\text{th}})^{\frac{2}{\alpha}}\Gamma\left(\frac{2}{\alpha}+1\right)\right\}. \end{aligned} \tag{15}$$

An equivalent expression for the probability of node isolation is also obtained in [7]. Since $E[r^{\frac{4}{\alpha}}] = \Gamma\left(\frac{2}{\alpha}+1\right)$ is strictly less than 1 for $\alpha > 2$, we can see from (15) that fading always increases the probability of node isolation and hence reduces the local network connectivity.

It must be noted that the reduction in the local connectivity is not always detrimental. For example, the reduction in the node degree may result in a reduction in the interference level, if the inter-node interference needs to be considered, which may improve the connectivity properties of the network.

B. Overall Connectivity

An upper bound for the network 1-connectivity is given by [3]

$$P(\text{1-con}) < \exp\{-\rho A P(\text{iso})\}, \tag{16}$$

where A is the area of the network and $P(\text{iso})$ is given in (15). Before we compare this analytical result with simulation results, we make the following observations regarding the effect of fading on the overall network connectivity:-

1) It is known from the theorems of geometric random graphs [17] that a uniformly distributed network with a path loss channel becomes fully connected at the same moment as when there are no isolated nodes in the network. As a result, the probability of no isolated nodes in (16) provides an upper bound for 1-connectivity and the bound is tight as the probability approaches unity [3]. When shadowing is present, the same result has been found in [6]. Moreover, it has been shown in [6] that the upper bound for a shadowing channel is tighter than

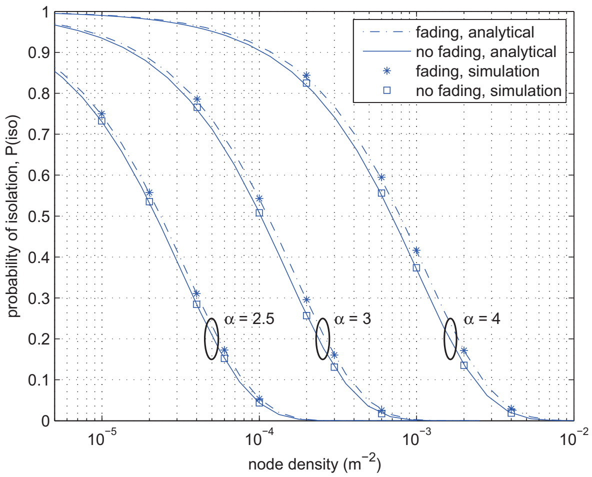

Fig. 2. Probability of node isolation vs. node density with the following system parameters: $\beta_{\text{th}} = 50$ dB, $\alpha = 2.5, 3, 4$ (lines = analytical results from Eq. (15), markers = simulation results).

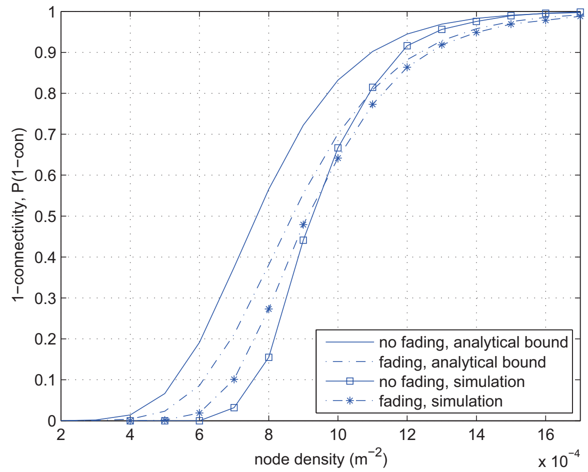

Fig. 3. 1-connectivity vs. node density with the following system parameters: $\beta_{\text{th}} = 50$ dB, $\alpha = 3$, $A = 400^2$ m^2. The analytical upper bounds are calculated from Eq. (16).

that for a path loss channel. Since both shadowing and fading introduce randomness into the geometric graph, we expect that (16) is also a relatively tight upper bound for 1-connectivity in a fading channel.

2) In static networks, the fading process can be considered as channel randomness in the spatial domain, which results in randomness in the received signal power for nodes at different locations. Another mechanism of creating randomness in the signal strength is antenna beamforming. It has been shown in [11], [12] that a reduction in the local network connectivity may result in an improvement in the path probability by the use of beamforming. A beamforming node loses links to closely located neighbours in some directions, while it creates links to nodes that are further away in other directions. It is the long links that improve the overall network connectivity. As the effect of beamforming is very similar to that of fading, we can intuitively expect that the presence of fading may improve the overall network connectivity in terms of path probability.

IV. Numerical Results

In this section, we validate our analytical results by comparison with simulation results and investigate the effect of fading on both the network local connectivity and overall connectivity. The simulations are carried out using Matlab. In the simulations, nodes are randomly distributed according to a uniform distribution on a square of area B m^2. To eliminate border effects, we use the sub-area simulation method [3], i.e., we only compute the connectivity measures for nodes located on an inner square of area A m^2, where A is sufficiently smaller than B. The results are then calculated by averaging over 5000 Monte Carlo simulation trials.

A. Probability of Isolation

Fig. 2 shows the probability of node isolation versus node density for $\beta_{\text{th}} = 50$ dB, and $\alpha = 2.5, 3, 4$. The analytical results (lines) are calculated from (15). We can see that the simulation results (markers) are in excellent agreement with the analytical results in all scenarios. Comparing the probability of isolation between fading and non-fading channels, one can see that fading results in an increase in node isolation. This effect has also been observed in [7]. Furthermore, we see that the increase in the probability of isolation becomes more noticeable as α increases from 2.5 to 4. This agrees with our earlier observation in the discussion of Fig. 1.

B. 1-Connectivity

Fig. 3 shows the 1-connectivity versus node density for $\beta_{\text{th}} = 50$ dB, $\alpha = 3$ and $A = 400^2$ m^2. The analytical upper bounds are calculated from (16). We see that the bound for fading channel is much tighter than the bound for non-fading channel, which agrees with our earlier observation in Section III-B. We can also see that fading increase 1-connectivity when the connectivity is below 0.6, while fading is detrimental for network which is required to be almost surely connected in terms of 1-connectivity (e.g. achieve 1-connectivity = 0.99 which is a very strong condition).

C. Path Probability

Another measure of the overall network connectivity is path probability, which is a moderate metric of overall network connectivity when compared with 1-connectivity. An analytical expression for the path probability is still an open research problem [11]. Hence we use simulations to investigate the effect of Rayleigh fading on path probability.

Fig. 4 shows the path probability versus node density for $\beta_{\text{th}} = 50$ dB, $\alpha = 3$ and $A = 400^2$ m^2. Although we have seen that fading reduces the network local connectivity, it significantly improves the path probability at the same time.

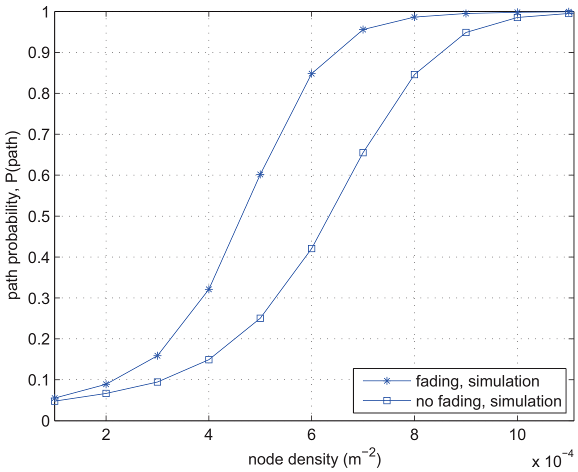

Fig. 4. Path probability vs. node density with the following system parameters: $\beta_{\text{th}} = 50$ dB, $\alpha = 3$, $A = 400^2$ m^2.

For example, the path probability for a fading channel is 0.85 at a node density of 6×10^{-4} m^{-2}, whereas the path probability for a non-fading channel is only 0.42 at the same node density. That is in the particular case considered above, the presence of fading results in a 100% improvement in the overall network connectivity at that node density. This agrees with our expectation in Section III-B.

One may also look at the high connectivity regime in Fig. 4, where the network is required to be almost surely connected in terms of path probability. For example, a path probability of 0.99 can be achieved by having a node density of approximately 8.5×10^{-4} m^{-2} for a fading channel and 10.5×10^{-4} for a non-fading channel. Therefore, in this case, the presence of fading saves about 19% of the number of nodes to establish an almost surely connected network. These important results on path probability cannot be revealed from the local connectivity analysis in [7].

V. Conclusion

In this paper, we have investigated the impact of Rayleigh fading on the local and overall connectivity of static wireless ad hoc networks. In particular, we have analytically characterized the effect of fading on the probability of node isolation and its dependence on the path loss exponent. The analytical results have been verified by comparison with simulation results. Our results have shown that the presence of fading may improve the overall network connectivity although it reduces the local connectivity. In particular, the presence of fading can significantly improve the network connectivity in terms of the path probability. Our future research will focus on the modelling and connectivity analysis for the joint effects of fading and beamforming in wireless ad hoc networks.

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