Performance Analysis of Beamforming in Ricean Fading Channels for CDMA Systems

Salman Durrani and Marek E. Bialkowski

***Abstract*— Adaptive arrays have been proposed as an effective means of mitigating Multiple Access Interference (MAI) and improving the performance of existing and future wireless communication systems. In this paper, we apply the analytical method proposed in [1] to analyse the theoretical Mean Bit Error Rate (BER) of an uncoded IS-95 based Code Division Multiple Access (CDMA) system with an array antenna at the Base Station (BS) in a Ricean fading environment. We present a modified expression for the Signal to Interference plus Noise (SINR) ratio as a function of the number of users, number of antennas and noise levels. We also verify the analytical results by means of Monte Carlo simulations by considering different user and channel scenarios. The simple adapted model is shown to provide good agreement with the simulation results and can be used to rapidly calculate the system performance under a variety of conditions.**

***Index Terms*— Adaptive arrays, Code division multiple access, Bit error rate analysis, Ricean channels.**

I. Introduction

The use of smart or adaptive array antennas for CDMA systems has been in the forefront of wireless research in recent years [2]. In CDMA systems, all users communicate simultaneously in the same frequency band and hence MAI is a major cause of performance degradation. Additionally, the ever present multipath fading significantly degrades the uplink or reverse link performance. A Rake receiver is one way to combat multipath fading and it performs this task in the time domain. An adaptive beamforming antenna system utilizes the space domain and can suppress interfering signals by acting as a spatial filter. Thus by combining an antenna array with Rake reception (2D-RAKE receiver), considerable performance gain can be achieved [3].

The performance of beamforming for CDMA systems in Rayleigh fading channels has been presented by a number of authors, e.g. [4]–[7]. Most performance studies of beamforming for CDMA systems are restricted to computer simulations. The exact analytical evaluation of the error probability in a CDMA system with multiple antennas and beamforming is still an open subject. However approximations have been proposed, e.g. in [3] and [6]. Recently in [1], a new simple method was proposed to arrive at closed form expressions for the mean bit error probability of 2D-RAKE receivers. The validity of the new method was illustrated by considering a simplified DS-CDMA system (employing BPSK modulation), with multiple antennas at the BS in a Rayleigh fading environment.

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In this work, we apply the method proposed in [1] to analyse the Mean BER performance of an IS-95 based CDMA system with an adaptive array antenna at the Base Station (BS) in a Ricean fading environment. This is important since many cellular operators have reported Ricean fading to be the dominant fading distribution seen in cellular systems [8]. We present a modified expression for the Signal to Interference plus Noise (SINR) ratio as a function of the number of users, number of antennas and noise levels. We study the effect of the power of the additional Line of Sight (LOS) component, characterized by the Rice factor, on the system performance. We also present simulation results to confirm the validity and accuracy of the analytical results.

This paper is organised as follows. The signal and channel model is presented in Section I. The Receiver model is outlined in Section II. The BER approximation procedure for 2D-RAKE receivers is briefly reviewed and applied to the IS-95 based CDMA system in Section III. The analytical results are compared with Monte Carlo simulations in Section IV. Finally conclusions are presented in Section V.

II. System Model

In this section, we develop the baseband model for beamforming. We consider the situation that the BS is equipped with a Uniform Linear Array (ULA) comprising N antenna elements. Such an array can form one side of a triangular panel array at the BS, serving one angular sector. The individual antenna elements are assumed to be omni-directional in azimuth, with inter-element spacing of $d = \lambda/2$. The array geometry is illustrated in Fig. 1.

Let K denote the total number of Mobile Stations (MS) in the system, which are randomly distributed in the azimuthal direction, along the arc boundary of a $120°$ sector cell, in the far field of the array. We consider the $120°$ sector for compatibility with the tri-sectored approach used by most of the current systems. The $k = 1$ th user is assumed to be the desired user. The location of each MS is characterized by its Angle of Arrival (AOA) θ^k, which is conventionally measured from the array broadside. We refer to $\theta = 0°$ as the broadside direction.

A. Signal Model

We focus on the reverse link (from MS to BS) of the CDMA system. The MS transmitter follows specifications of IS-95 CDMA reverse link [9]. For simplicity, we ignore the convolutional encoder and interleaver. The transmitted signal $s^k(t)$ of the kth user can be written as

$$s^k(t) = W^k_m(t)\, a^k_I(t) \cos(\omega_c t) + W^k_m(t)\, a^k_Q(t) \sin(\omega_c t) \quad (1)$$

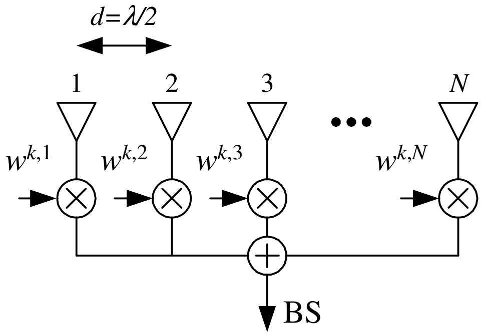

Fig. 1. Block diagram of N element Uniform Linear Array antenna.

where $W_m^k(t)$ is the mth M-ary Walsh Symbol $(m = 1, 2, \ldots, M = 64)$ of the kth user, $a_I^k(t) = a^k(t)a_I(t)$ and $a_Q^k(t) = a^k(t)a_Q(t)$ are the products of the user PN sequences and the I and Q channel PN codes, $a_I(t)$ is the In-phase (I) channel spreading sequence, $a_Q(t) =$ Quadrature (Q) channel spreading sequence, $a^k(t)$ is the kth user long code sequence, $\omega_c = 2\pi f_c$ and f_c is the carrier frequency. The transmitted power of each user is assumed unity.

B. Ricean Channel Model

We consider a parameterized vector model to characterize the wireless channel between a single antenna at the MS and a ULA at the BS [10]. We assume Line Of Sight (LOS) propagation between the MS and BS. In this paper, we do not consider temporal diversity, i.e. we assume that the number of multipaths $L = 1$.

The channel impulse response for kth user at the nth antenna, is given as

$$h^{k,n}(t) = \sqrt{\frac{K_R}{1+K_R}}\, h_{LOS}^{k,n}(t) + \sqrt{\frac{1}{1+K_R}}\, h_{NLOS}^{k,n}(t) \quad (2)$$

where K_R is the Ricean factor which is defined as the ratio of the specular power to the diffused or scattered power, $h_{LOS}^{k,n}(t)$ and $h_{NLOS}^{k,n}(t)$ are the specular and scattered components given by

$$h_{LOS}^{k,n}(t) = e^{[j(\phi_0^k + 2\pi f_D t \cos\theta^k)]}\, e^{[-j\mathcal{K}d(n-1)\sin\theta^k]} \quad (3)$$

$$h_{NLOS}^{k,n}(t) = \frac{1}{\sqrt{S}} \sum_{s=1}^{S} e^{[j(\phi_s^k + 2\pi f_D t \cos\Psi_s^k]}\, e^{[-j\mathcal{K}d(n-1)\sin\theta^k]} \quad (4)$$

where $k = 1, 2, \ldots, K$ is the user index, $n = 1, 2, \ldots, N$ is the antenna index, S is the number of sub-paths for each resolvable path, $\mathcal{K} = 2\pi/\lambda$ is the wave number, d is the inter-element distance, f_D is the maximum Doppler frequency which is the ratio of the mobile velocity v and the wavelength, ϕ_0^k and ϕ_s^k are the random phase, assumed to be uniformly distributed over $[0, 2\pi]$, θ^k is the mean AOA and Ψ_s^k is the Angle of Departure (AOD) for each sub-path relative to the motion of the mobile, modelled by a uniform probability density function [11].

Rewriting Eq. (2), we have

$$h^{k,n}(t) = \beta^{k,n}(t)\, e^{-j\varphi^{k,n}(t)} \quad (5)$$

where $\beta^{k,n}(t) = |h^{k,n}(t)|$ is the modulus of the complex Ricean channel amplitude and $\varphi^{k,n} = \arg\{h^{k,n}(t)\}$ is the phase of the carrier of the kth user at the nth antenna and includes the effects of fast fading, the difference in propagation delays between antennas and the phase difference between the transmitter and the receiver carriers.

In vector notation, the spatial signature or channel response vector for the kth user can be expressed as $N \times 1$ vector as

$$\mathbf{h}^k(t) = [\, h^{k,1} \;\; h^{k,2} \;\; \ldots \;\; h^{k,N} \,]^T \quad (6)$$

where $(\cdot)^T$ denotes transpose operation.

III. Receiver Model

A simplified block diagram of the receiver incorporating the ULA is shown in Fig. 2. The total received signal at the nth antenna is given by

$$x^n(t) = \sum_{k=1}^{K} \left[\beta^{k,n}\, W_m^k(t-\tau^k)\, a_I^k(t-\tau^k)\cos(\omega_c t + \varphi^{k,n}) + \beta^{k,n}\, W_m^k(t-\tau^k)\, a_Q^k(t-\tau^k)\sin(\omega_c t + \varphi^{k,n})\right] + \eta^n(t) \quad (7)$$

where $s^k(t)$ is the signal transmitted by the kth user given by Eq. (1), τ^k is the path delay and $\eta(t)$ is the noise which is assumed to be Additive White Gaussian Noise (AWGN).

The receiver uses non-coherent demodulation to recover the signal. The received signal at each antenna element is first down converted. The signal is then despread using the sequences of the respective MS. The post PN-despread signals can be written in vector notation as

$$\mathbf{y}^k = [\, y^{k,1} \;\; y^{k,2} \;\; \ldots \;\; y^{k,N} \,]^T \quad (8)$$

Next a beamformer is constructed and the beamforming output is given by

$$z^k = (\mathbf{w}^k)^H\, \mathbf{y}^k \quad (9)$$

where $\mathbf{w}^k$ is the beamforming weight vector and $(\cdot)^H$ denotes Hermitian transpose operation. We assume the sub-optimal but computationally simpler Maximum Signal to Noise Ratio (MSNR) Beamforming is performed [6]. Thus the weights are set as $\mathbf{w}^k = \mathbf{h}^k$. We assume that these vector channel coefficients are perfectly known. This provides an upper bound on the system performance.

Finally the beamformer output is Walsh correlated. The output of the qth Walsh correlator $(q = 1, 2, \cdots, M = 64)$ is given by (for details see [12])

$$Z_I^k(q) = \begin{cases} \beta^k \cos\varphi^k + M_I^k + N_I^k & ;\text{if } q = m \\ M_I^k + N_I^k & ;\text{else} \end{cases} \quad (10)$$

$$Z_Q^k(q) = \begin{cases} \beta^k \sin\varphi^k + M_Q^k + N_Q^k & ;\text{if } q = m \\ M_Q^k + N_Q^k & ;\text{else} \end{cases} \quad (11)$$

where N_I^k and N_Q^k denote the noise terms and M_I^k and M_Q^k denote the MAI terms respectively.

The overall decision variable obtained for the kth user is given by

$$u^k(q) = \left(Z_I^k\right)^2 + \left(Z_Q^k\right)^2 \quad (12)$$

The final decision on the mth symbol of the kth user is then obtained by the Maximum Likelihood Criteria as

$$\hat{m} = \max_{q=1,\ldots,M} \{u^k(q)\} \quad (13)$$

IV. Bit Error Rate Analysis

In order to analytically determine the performance of the system, we follow the approximation procedure proposed in [1] and [13]. The approximation proposes to adapt the single antenna performance bounds to array antenna systems by manipulating those terms in the error probability formulas for single antenna receivers that account for the noise and MAI. The procedure divides the number of interferers into two categories – in-beam and out-of-beam – based on whether their direction of arrivals lie inside or outside the beam formed toward the desired user. The attenuation provided by the array antenna to each of the out-of-beam interferes is assumed to be constant. The in-beam interferers are counted as interference while the out-of-beam users increase the additive noise level for the evaluation of the error probability.

A. Variances for the Case of a Single Antenna

Following [14], it can be shown that for a single antenna in Ricean fading environment, the noise terms N_I^k and N_Q^k can be modelled as mutually independent zero mean Gaussian random processes with variance

$$\sigma_N^2 = \frac{N_o}{2} \tag{14}$$

Similarly, the MAI terms M_I^k and M_Q^k can be modelled as a zero mean Gaussian random processes with variance

$$\sigma_M^2 = \frac{E_s}{3N_c}\sum_{k=2}^{K} E[(\beta^k)^2] = \frac{E_s}{3N_c}(K-1) \tag{15}$$

where $E_s = E_b \log_2(M)$ is the symbol energy, E_b is the bit energy, $N_c = 256$ is the spreading factor and the total path power for each user is normalized to unity.

The overall decision variable is a sum of squares of two gaussian random variables, each with variance

$$\sigma_T^2 = \sigma_N^2 + \sigma_M^2 = \frac{N_o}{2} + \frac{E_s}{3N_c}(K-1) \tag{16}$$

The SINR can thus be written as

$$\rho = \frac{E_s}{2\,\sigma_T^2} = \frac{\gamma}{1 + \frac{2}{3N_c}\,\gamma\,(K-1)} \tag{17}$$

where $\gamma = \frac{E_s}{N_o} = \log_2(M)\,\frac{E_b}{N_o}$.

It can be shown that the mean bit error probability for a conventional 1D-RAKE receiver (i.e. single antenna without beamforming) in Ricean fading is given by [15]

$$P_b^{1D}(e) = \frac{M/2}{M-1}\sum_{q=1}^{M-1}\binom{M-1}{q}\frac{(-1)^{q+1}}{(1+q+q\,\delta_1)} \times \exp\left(\frac{-\,\delta_1}{1+q+q\,\delta_2}\right) \tag{18}$$

where the variables δ_1 and δ_2 are given by

$$\delta_1 = \frac{\rho}{1+\frac{1}{K_R}} \tag{19}$$

$$\delta_2 = \frac{\rho}{1+K_R} \tag{20}$$

where ρ is given by Eq. (17) and K_R is Ricean factor.

TABLE I

Equivalent beamforming parameters

Number of antenna elements N	4	6	8
α_o (dB)	-12	-14	-16
$2\,\theta_{BW}$ (degs.)	30°	20°	15°

B. Modified Variances for the Case of Multiple Antennas

Let the modified variances of the noise and MAI be denoted as $\bar{\sigma}_N^2$ and $\bar{\sigma}_M^2$ respectively.

We know that the noise at the output of the antenna array is reduced by N times, where N is the number of antenna elements. Hence,

$$\bar{\sigma}_N^2 = \frac{\frac{N_o}{2}}{N} \tag{21}$$

Let κ denote the number of in-beam interferers. The number of out-of-beam interferers $= K - \kappa - 1$. Hence we have

$$\bar{\sigma}_M^2 = \overbrace{\left\{\alpha_o \frac{E_s}{3N_c}\,(K-\kappa-1)\right\}}^{\text{out-of-beam}} + \overbrace{\left\{f\,\frac{E_s}{3N_c}\,\kappa\right\}}^{\text{in-beam}} \tag{22}$$

where α_o is the attenuation factor for out-of-beam interferers and $f = 3/4$ is a correction factor for in-beam interferers [13].

The modified SINR expression is thus given by

$$\rho = \frac{E_s}{2\,\bar{\sigma}_T^2}$$

where $\bar{\sigma}_T^2 = \bar{\sigma}_N^2 + \bar{\sigma}_M^2$ is the total variance.

Substituting the values and simplifying, we get

$$\rho = \frac{\gamma}{\left(\frac{1}{N} + f\,\frac{2}{3N_c}\,\gamma\,\kappa + \alpha_o\,\frac{2}{3N_c}\,\gamma\,(K-\kappa-1)\right)} \tag{23}$$

where $\gamma = \frac{E_s}{N_o} = \log_2(M)\frac{E_b}{N_o}$.

C. BER Approximation

Using Eqs. (23) and (18) and assuming uniform distribution of interferers in the sector, we can obtain the average bit error probability of 2D-RAKE receiver (combined beamformer and Rake receiver) as

$$P_b^{2D}(e) = \sum_{\kappa=0}^{K-1}\chi\,\eta^{\kappa}\binom{K-1}{\kappa}P_b^{1D} \tag{24}$$

where P_b^{1D} is given by Eq. (18) with ρ is given by Eq. (23), χ and the probability of an in-beam interferer η, are defined as [1], [13]

$$\eta = \frac{(2\,\theta_{BW})}{\Delta\theta} \tag{25}$$

$$\chi = (1-\eta)^{(K-\kappa-1)} \tag{26}$$

where $\Delta\theta = 120°$ is the total coverage angle of the sector and $2\theta_{BW}$ is the total beamwidth towards the desired user. The values of equivalent beamforming parameters $2\theta_{BW}$ and α_o used in this work are given in Table I [1].

TABLE II
MAIN SIMULATION PARAMETERS

Chip time	$T_c = \frac{1}{1228800}$ s
Carrier frequency	900 MHz
Bits/frame	576
Frame length	20 ms
Oversampling factor	$Q = 4$
Pulse shaping	No
Base Station synchronisation	Asynchronous operation
Channel estimation	perfect
Power control	ideal
Detection	Non-coherent
Inter-element distance	$d = \lambda/2$
Angle of Arrival	$-60^\circ \leq \theta \leq 60^\circ$
User mobility	0.01° per snapshot
Number of sub-paths	$S = 15$
PDF in AOD	Uniform
Doppler frequency	$f_D = 100$ Hz

V. RESULTS

Monte Carlo simulations have been carried out to check the accuracy of the BER model. In simulations, the mean BER is collected and averaged over $M_c = 100$ drops [10]. A 'drop' is defined as a simulation run for a given number of MS's and BS over 125 frames, which corresponds to the time required by the desired user to traverse the entire azimuth range $[-60^\circ, 60^\circ]$. For other users, their AOAs are assumed to be uniformly distributed over $[-60^\circ, 60^\circ]$. The main parameters of the simulation model are summarized in Table II.

A. Single User Performance

First we take a look at results for the case of a single user. Fig. 3 shows the Mean BER vs. E_b/N_o (dB) for $N = 6$ antennas, $K = 1$ user with single path assuming Ricean fading with Ricean factor $K_R = 1, 5, 7, 10$ dB respectively. The performance in Rayleigh fading (corresponding to $K_R = -\infty$ dB) and the performance of conventional receiver (i.e. $N = 1$ with no beamforming) are also shown as reference. We can see that beamforming improves the performance of the system. For low values of Ricean factor, the performance is very close to the performance in Rayleigh fading. However for larger Ricean factors, there is a tremendous improvement in the Mean BER. The simulation results (markers) show excellent agreement with theory (lines).

B. Effect of Different Number of Users

Next we examine the case of varying the number of users. Fig. 4 shows the Mean BER vs. Number of users K for $N = 6$ antennas, $E_b/N_o = 10$ dB, assuming Ricean fading with different Ricean factors. The reference curves for single antenna and Rayleigh fading are also shown in the figure. It can be seen from the figure that the BER model provides a good match with simulation results for multi-user scenarios as well.

C. Effect of varying the Noise Level and Number of antennas

Fig. 5 shows the Mean BER vs. E_b/N_o for $N = 6$ antennas, $K = 15$ user with single path assuming Ricean fading with different Ricean factors while Fig. 6 shows the Mean BER vs. Number of antennas N, for $E_b/N_o = 10$ dB, $K = 15$ users assuming Ricean fading with different Ricean factors. It can be seen again that the simulation results agree well with the theory approximation. Thus the simple approximation can be used to predict the performance of the complex system in Ricean fading under different circumstances with reasonable degree of accuracy.

VI. CONCLUSIONS

In this paper, we have applied the analytical procedure proposed in [1] to analyse the theoretical Mean BER of an IS-95 based CDMA system with beamforming array antenna at the BS in Rayleigh and Ricean fading environments. Simulation results have been presented, under a variety of user and channel scenarios, which confirm the validity and accuracy of the analytical results. It has also been shown that the common assumption of Rayleigh (Non Line of Sight) fading gives the worst case performance of the system. Under the Ricean (Line of Sight) fading assumption, which can occur frequently in urban micro-cellular systems, the system performance is improved depending on the power of the LOS component. The simple model provides excellent agreement with the simulation results and can be used to rapidly calculate the system performance under a variety of conditions.

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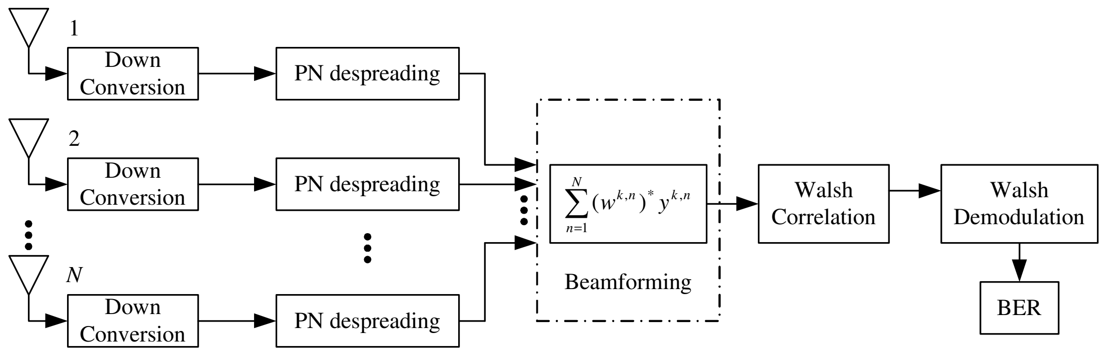

Fig. 2. Receiver Block Diagram.

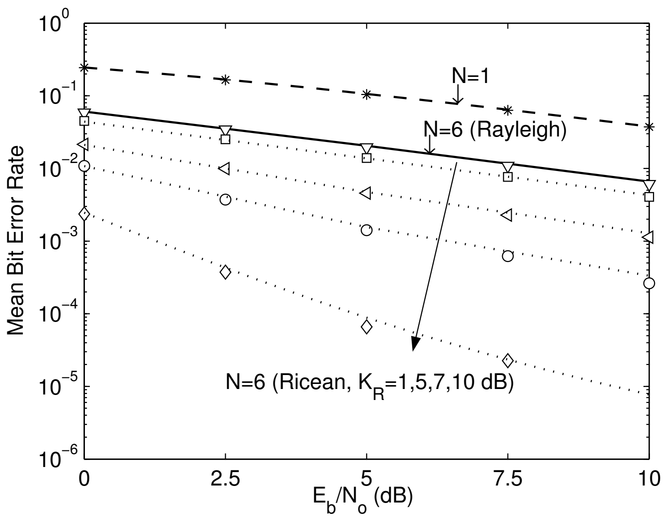

Fig. 3. Mean BER vs. E_b/N_o (dB) for $N = 6$ antennas, $K = 1$ user with single path assuming Rayleigh and Ricean fading channels respectively (theory: lines, simulations: markers).

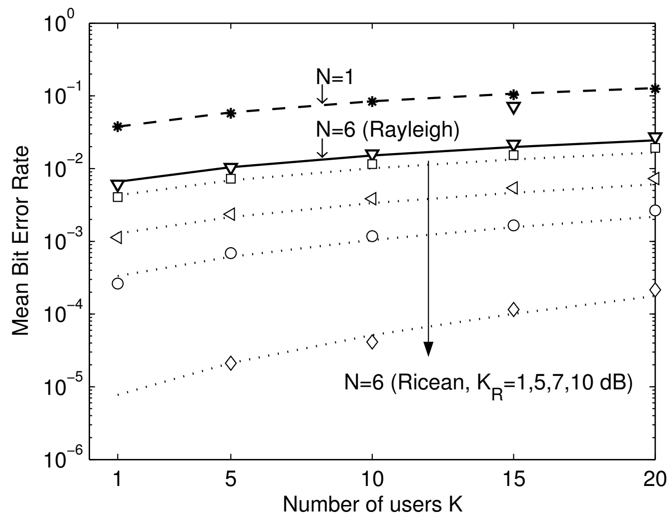

Fig. 4. Mean BER vs. Number of users K for $E_b/N_o = 10$ dB, $N = 6$ antennas, assuming Rayleigh and Ricean fading channels respectively (theory: lines, simulations: markers).

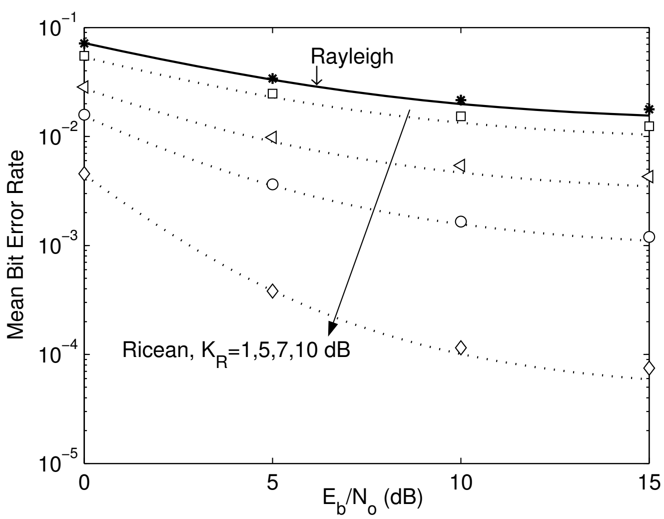

Fig. 5. Mean BER vs. E_b/N_o (dB) for $N = 6$ antennas, $K = 15$ users, assuming Rayleigh and Ricean fading channels respectively (theory: lines, simulations: markers).

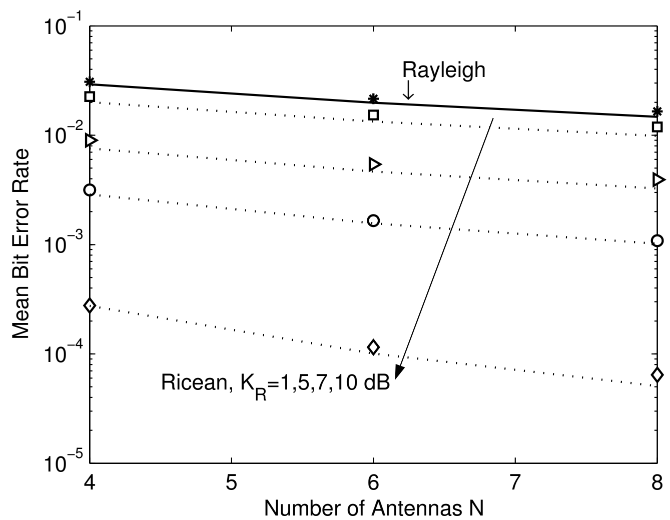

Fig. 6. Mean BER vs. Number of antennas N, for $E_b/N_o = 10$ dB, $K = 15$ users, assuming Rayleigh and Ricean fading channels respectively (theory: lines, simulations: markers).