Statistical Properties of a Parametric Channel Model for Multiple Antenna Systems

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  - MIMO Channel Models
  - Motivation
- **Reference Channel Model**
  - Statistical Properties
- **Parametric Channel Model**
- **Results**
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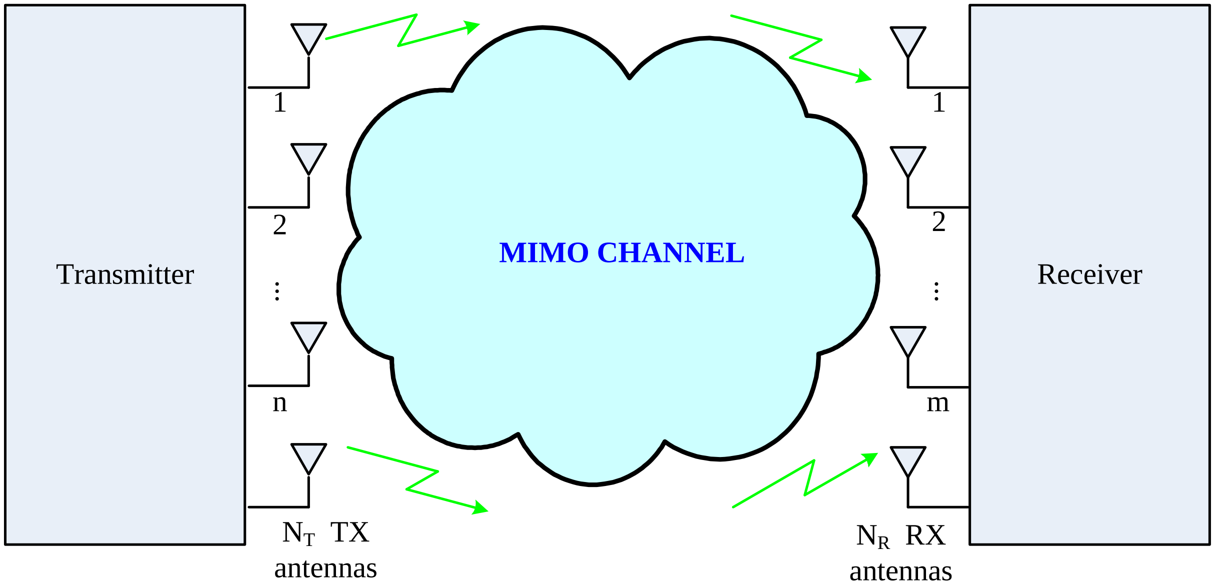
- **Conclusions**

Introduction

▷ **MIMO Channel Models** can be classified as follows:[†]



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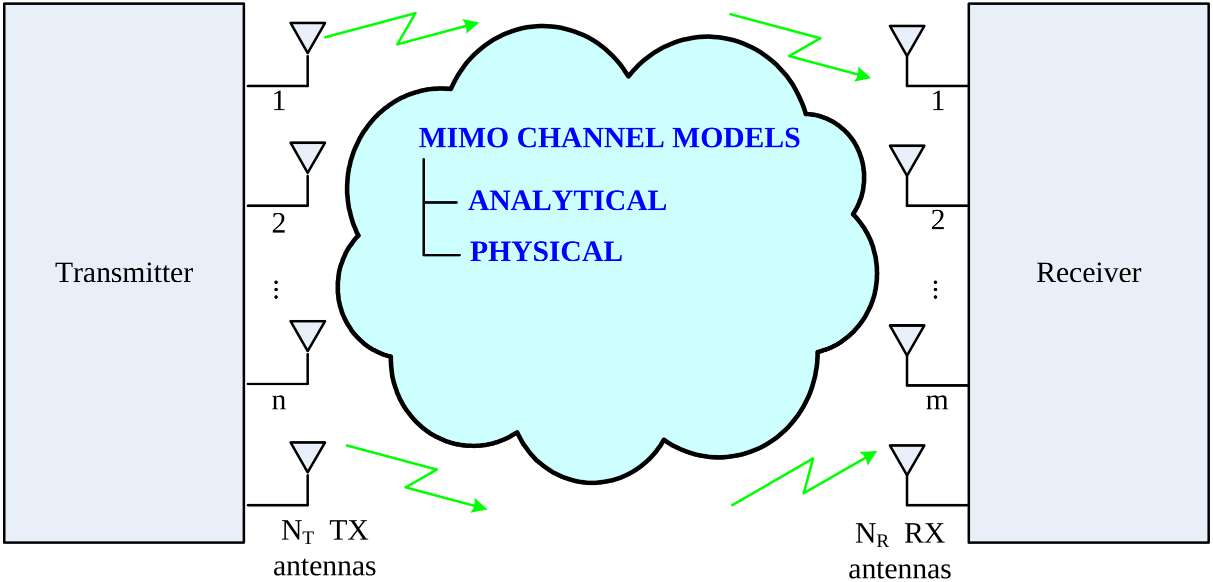

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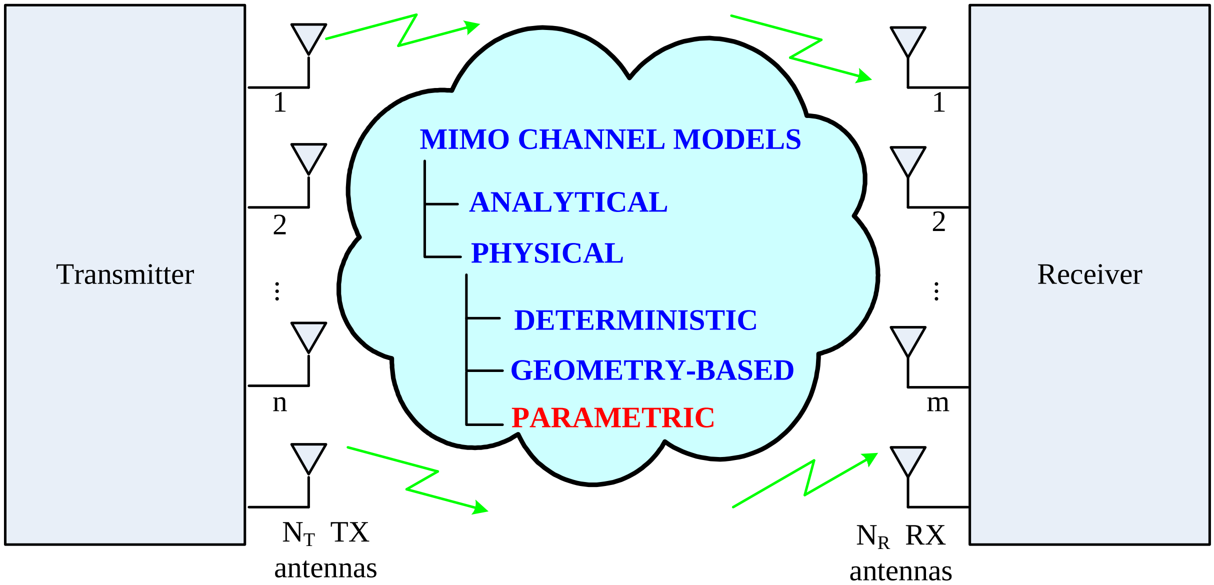

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Parametric Channel Model

- Parametric channel models use important **physical parameters** such as phases, delays, doppler frequency, angle of departure (AOD), angle of arrival (AOA) and angle spread to provide a description of the MIMO channel.

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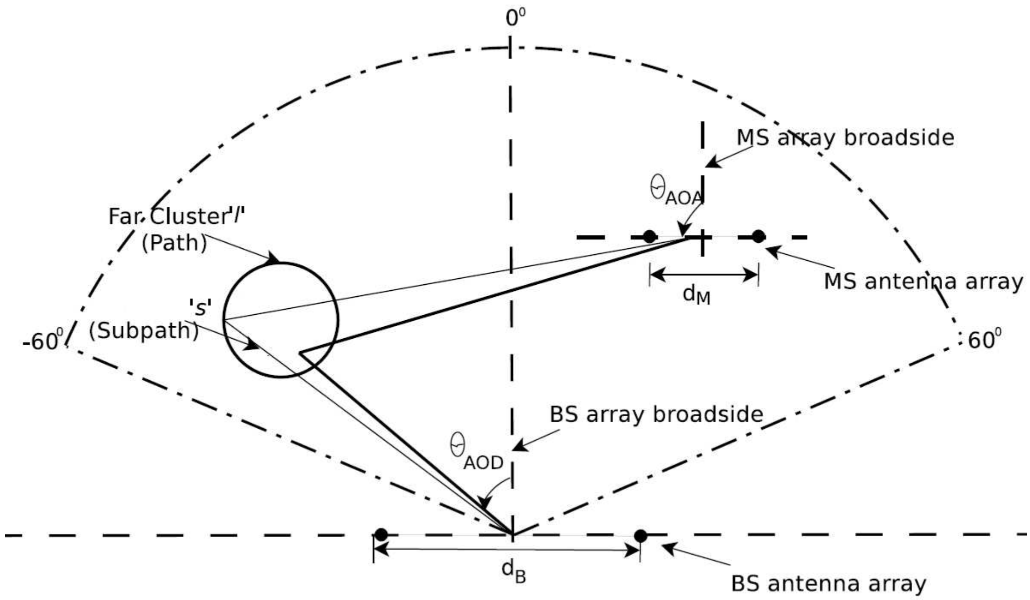
- How many subpaths are sufficient to accurately capture the statistical properties of the MIMO wireless channel?

Wireless Propagation Environment

▷ We consider a MIMO system in an **urban macro-cell environment**.



Reference Channel Model

- The **channel impulse response** between MS antenna m and BS antenna n for user k's path l can be written as

$$h_{k,l}^{m,n}(t) = (h_I)_{k,l}^{m,n}(t) + j(h_Q)_{k,l}^{m,n}(t)$$

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- For isotropic scattering, the **temporal correlation** properties are summarized below:

$$\begin{aligned}
R_{h_I h_I}(\tau) &= E[h_I(t)h_I(t+\tau)] = J_0(2\pi f_D \tau) \\
R_{h_Q h_Q}(\tau) &= E[h_Q(t)h_Q(t+\tau)] = J_0(2\pi f_D \tau) \\
R_{h_I h_Q}(\tau) &= E[h_I(t)h_Q(t+\tau)] = 0 \\
R_{hh}(\tau) &= E[h(t)h^*(t+\tau)] = J_0(2\pi f_D \tau) \\
R_{|h|^2|h|^2}(\tau) &= 4 + 4J_0^2(2\pi f_D \tau)
\end{aligned}$$

Reference Channel Model

- The **Level Crossing rate** is defined as the rate at which the fading envelope crosses a specified threshold in the positive slope

$$L_{|h|} = \sqrt{2\pi} f_D \rho e^{-\rho^2}$$

- The **Average Fade Duration** is the average duration of time that the fading envelope remains below a specified

$$T_{|h|} = \frac{e^{\rho^2} - 1}{\rho \sqrt{2\pi} f_D}$$

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Reference Channel Model

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  - For urban macro-cellular environment, median angular spread: $5° - 20°$.

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Reference Channel Model

- The **spatial envelope correlation coefficient** ρ_s, between the pth and qth antenna elements for a ULA, is given by

$$\rho_s(p,q) = |R_s(p,q)|^2 = |\Re\{R_s(p,q)\} + j\Im\{R_s(p,q)\}|^2$$

- **Spatial Correlation at BS**

$$\Re\{R_s(p,q)\} = J_0(z_{pq}) + 2C_g \sum_{v=1}^{\infty} J_{2v}(z_{pq}) \cos(2v\theta_{AOD}) e^{(-2v^2\sigma_{AOD}^2)} \Re\left\{ \operatorname{erf}\left(\frac{\pi + j2v\sigma_{AOD}^2}{\sqrt{2}\sigma_{AOD}} \right) \right\}$$

$$\Im\{R_s(p,q)\} = 2C_g \sum_{v=0}^{\infty} J_{2v+1}(z_{pq}) \sin[(2v+1)\theta_{AOD}] e^{\left[\frac{-(2v+1)^2\sigma_{AOD}^2}{2}\right]} \Re\left\{ \operatorname{erf}\left(\frac{\pi + j(2v+1)\sigma_{AOD}^2}{\sqrt{2}\sigma_{AOD}} \right) \right\}$$

- **Spatial Correlation at MS**

$$\Re\{R_s(p,q)\} = J_0(z_{pq}) + 2\sum_{v=1}^{\infty} J_{2v}(z_{pq}) \cos(2v\theta_{AOA}) \operatorname{sinc}(2v\Delta)$$

$$\Im\{R_s(p,q)\} = 2\sum_{v=0}^{\infty} J_{2v+1}(z_{pq}) \sin[(2v+1)\theta_{AOA}] \operatorname{sinc}[(2v+1)\Delta]$$

Parametric Channel Model

- The **channel impulse response** can be written as

$$h_{k,l}^{(m,n)}(t) = \sqrt{\frac{\Omega_{k,l}}{S}} \left\{ \sum_{s=1}^{S} \exp[j(\phi_{k,l}^{(s)} + 2\pi f_D t \cos\theta_{k,l,AOA}^{(s)})] \times \exp[-j\kappa d_{\mathrm{M}}(m-1)\sin\theta_{k,l,AOA}^{(s)}] \times \exp[-j\kappa d_{\mathrm{B}}(n-1)\sin\theta_{k,l,AOD}^{(s)}] \right\} \delta(t - \tau_{k,l})$$

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Temporal Parameters

K = users;

L = multipaths;

S = sub-paths/path;

$\Omega_{k,l}$ = mean path power;

$\tau_{k,l}$ = propagation delay;

$\phi_{k,l}^{(s)}$ = random phase;

f_D = Doppler frequency;

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Spatial Parameters

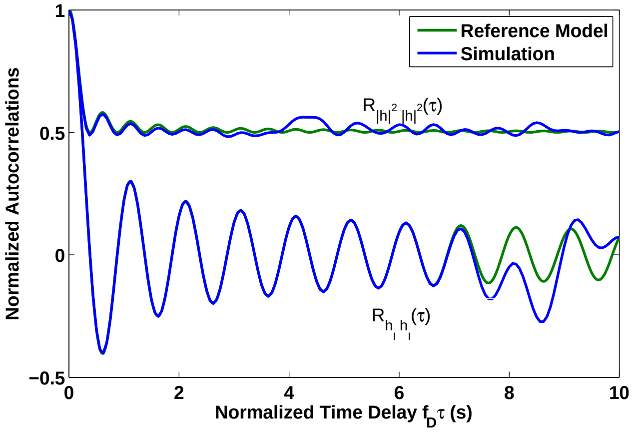
N = No. of antennas;
d = inter-element distance;
$\kappa = 2\pi/\lambda$;
$\theta_{k,l,AOD}^{(s)} = \theta_{k,AOD} + \vartheta_{k,l,AOD}^{(s)}$
$\theta_{k,l,AOA}^{(s)} = \theta_{k,AOA} + \vartheta_{k,l,AOA}^{(s)}$
$\theta_{k,AOA}$ = Mean Angle of Arrival;
$\theta_{k,AOD}$ = Mean Angle of Departure;

Parametric Channel Model

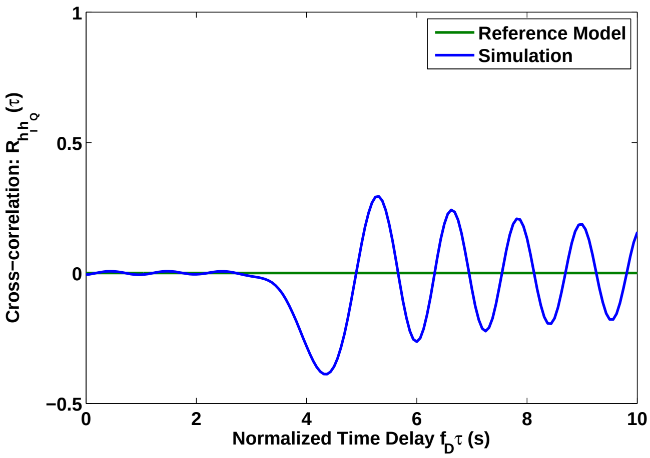
Aspect	Parameter	Value or Description
General	Carrier frequency	$f_c = 2$ GHz
	Number of channel samples	$T = 20000$
	Samples/wavelength	8
Temporal	MS velocity	$v = 60$ km/hr
	Number of paths	$L = 1$
	Number of subpaths	$S = 25$
Antennas	Antenna geometry	ULA
	Number of antennas	$N_{\mathrm{B}} = 2,\ N_{\mathrm{M}} = 2$
	BS Inter-element distance	$d_{\mathrm{B}} = 5\lambda$
	MS Inter-element distance	$d_{\mathrm{M}} = 0.5\lambda$
Spatial	Mean Angle of Arrival	$\theta_{AOA} = 60°$
	pdf in Angle of Arrival	Uniform $[-\pi, \pi]$
	Mean Angle of Departure	$\theta_{AOD} = 0°$
	pdf in Angle of Departure	Gaussian
	BS Angle spread	$\sigma_{AOD} = 5°, 10°, 20°$

Results – Temporal Correlations

Autocorrelations



Cross-correlation



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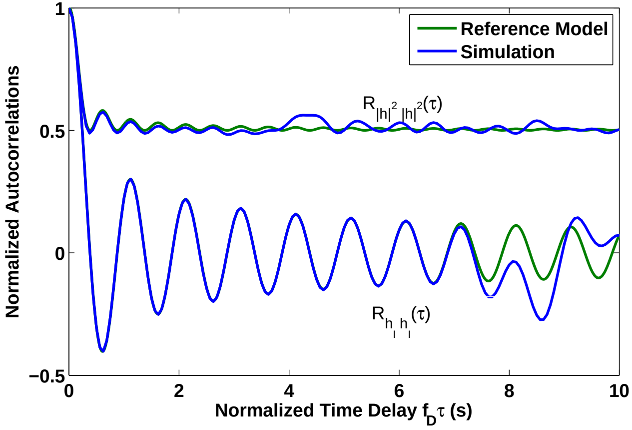

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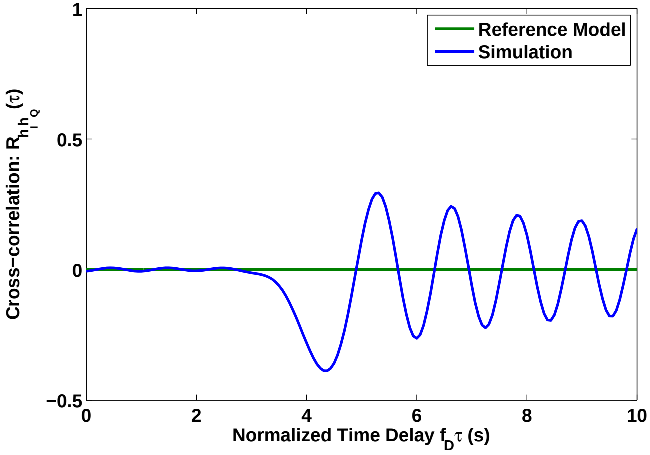
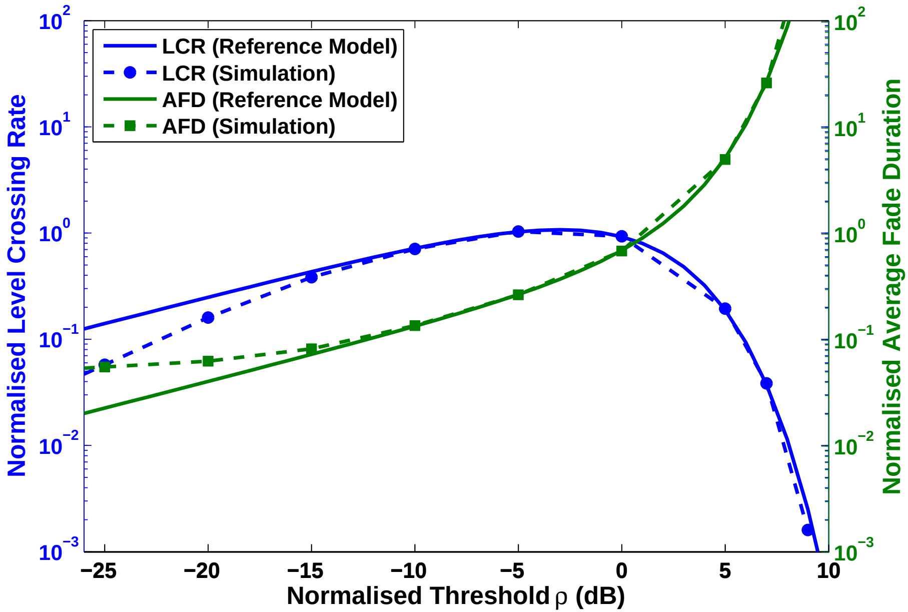


▷ For $S = 25$, simulation results agree with reference results for $0 \leq f_D\tau \leq 3$.

Results – LCR & AFD

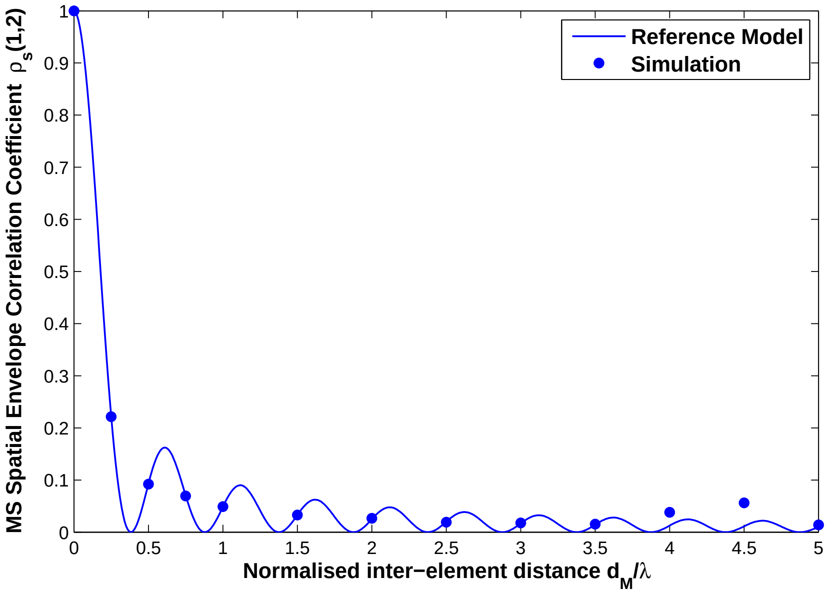
MS velocity $v = 60$ km/hr ($f_D = 111.11$ Hz)



▷ For $S = 25$, simulation results deviate from reference results only for very low threshold values.

Results – MS Spatial Correlation

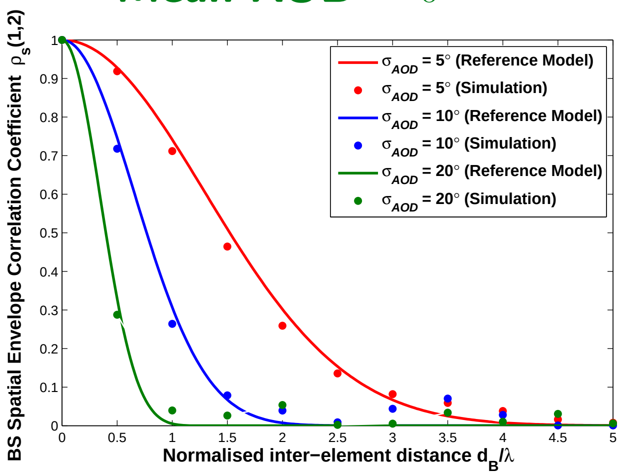
Spatial Correlation Coefficient vs. distance d_M/λ



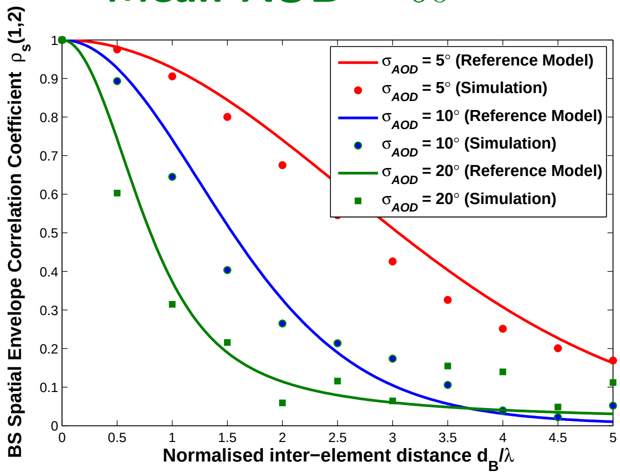
Results – BS Spatial Correlation

Spatial Correlation Coefficient vs. distance d_B/λ

Mean AOD $= 0°$



Mean AOD $= 60°$



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- The proposed channel model can accurately represent the **temporal correlations** for time delays $0 \leq f_D\tau \leq 3$ and **spatial correlations** at MS and BS for inter-element spacings $0 \leq d/\lambda \leq 3$.

- The obtained results have shown that $S = 25$ **subpaths** is sufficient to capture the important statistical properties of MIMO wireless channel.

Thank you for your attention