Outage Probability in Arbitrarily-Shaped Finite Wireless Networks

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1 Abstract

We propose a general framework for analytically computing the outage probability at any arbitrary location of an arbitrarily-shaped finite wireless network:

- Our **reference link power gain-based (RLPG-based) framework** exploits the distribution of the fading power gain between the reference transmitter and receiver.
- The node locations are modeled by a Binomial point process and fading channels are modeled by independently and identically distributed (i.i.d.) Nakagami-m fading.
- The boundary effects are accurately accounted for using the probability distribution function of the distance of a random node from the reference receiver.

The analysis illustrates the location dependent performance in finite wireless networks and highlights the importance of accurately modeling the boundary effects.

2 System Model

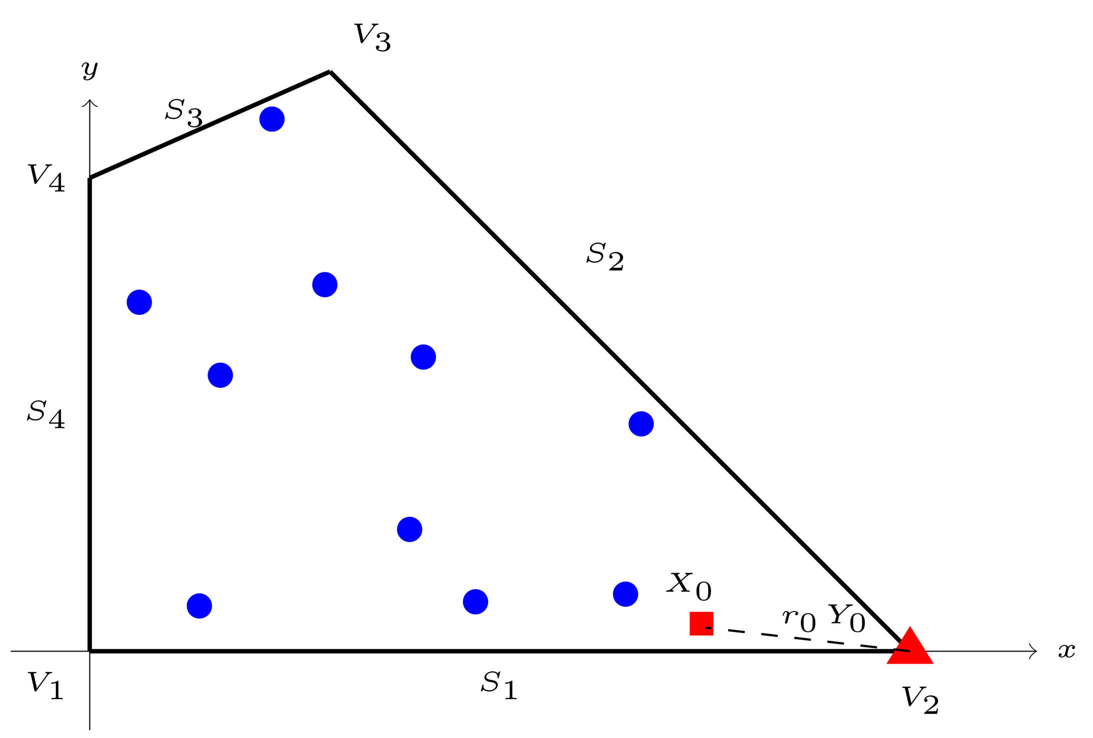

Figure 1: Illustration of an arbitrary location of a reference receiver in an arbitrarily-shaped finite wireless network, with side lengths $S_1 = \sqrt{3}W$, $S_2 = \sqrt{3}W$, $S_3 = \sqrt{7-3\sqrt{3}-\sqrt{6}}W$ and $S_4 = W$ and vertices $V_1 = \pi/2$, $V_2 = \pi/4$, $V_3 = 0.6173\pi$ and $V_4 = 0.6327\pi$. (● = interfering node, ▲ = reference receiver, ■ = reference transmitter).

Consider a wireless network with $M+2$ nodes which are located inside an arbitrarily-shaped finite region $\mathcal{A} \in \mathbb{R}^2$, as shown in Figure 1.

- Assumptions:
  - Reference receiver Y_0, ▲ is located anywhere inside the finite region;
  - Reference transmitter X_0, ● is placed at a given distance r_0 from Y_0;
  - M interfering nodes are randomly independently and uniformly distributed (i.u.d.) inside $\mathcal{A}$. The random distance between Y_0 and X_i is denoted by R_i with distance distribution function $f_{R_i}(r_i)$.
  - P_i denotes the transmit power for X_i.
  - Path-loss model is $l(r) = r^{-\alpha}$.
  - G_0 represents the instantaneous power gain between the reference transmitter and receiver pair X_0 and Y_0, which follows the Nakagami-m distribution with integer fading parameter m_0.
  - G_i represents the instantaneous power gain between the interfering node and receiver pair X_i and Y_0, which follows the Nakagami-m distribution with integer fading parameter m.
- The aggregate interference at the reference receiver is

$$I = \sum_{i=1}^{M} P_i G_i R_i^{-\alpha}. \tag{1}$$

- The **outage probability** is defined as the probability that the signal-to-interference-plus-noise ratio is less than a certain threshold β. Thus, the outage at Y_0 is

$$\epsilon = \Pr\left(\frac{P_0 G_0 r_0^{-\alpha}}{N+I} < \beta\right) = \Pr\left(\frac{G_0}{\frac{1}{\rho_0} + \frac{I}{P_0 r_0^{-\alpha}}} < \beta\right), \tag{2}$$

where $\rho_0 = (P_0 r_0^{-\alpha})/N$ is the average signal-to-noise ratio (SNR) and N is the AWGN power.

3 Proposed RLPG-based Framework

- The basic principle of this framework is to find the cumulative distribution function of the reference link's fading power gain, which can then be used to find the outage probability.
- Due to i.u.d. node distribution and i.i.d. fading distribution, we drop the index i in R_i, G_i, $f_{R_i}(r_i)$ and $f_{G_i}(g_i)$ and let $f_{R_i}(r_i) = f_R(r)$ and $f_{G_i}(g_i) = f_G(g)$. Furthermore, the transmit power is normalized to unity.
- The outage probability at reference receiver Y_0 is mathematically given by [1]

$$\epsilon = 1 - \exp\left(-m_0 \frac{\beta}{\rho_0}\right) \sum_{k=0}^{m_0-1} \frac{m_0^k}{k!} \sum_{j=0}^{k} \binom{k}{j} \left(\frac{\beta}{\rho_0}\right)^{k-j} (\beta r_0^{\alpha})^j \sum_{t_1+t_2...+t_M=j} \binom{j}{t_1, t_2, ..., t_M} \prod_{i=1}^{M} \mathrm{E}_{G,R}\{\Omega_{t_i}\}, \tag{3}$$

where the expectation in (3) can be expressed as

$$\mathrm{E}_{G,R}\{\Omega_{t_i}\} = \int_0^{r_{\max}} \frac{m^m (r^{-\alpha})^{t_i} \Gamma(m+t_i)}{\Gamma(m)\,(m + \beta r_0^{\alpha} m_0 r^{-\alpha})^{m+t_i}} f_R(r) dr, \tag{4}$$

where $r_{\max}$ denotes the maximum range of the random variable R.

4 Example

Consider an arbitrarily-shaped convex polygon region as shown in Figure 1. Suppose that Y_0 is located at the vertex V_2. Then, $f_R(r)$ can be expressed as

$$f_R(r) = \frac{1}{|\mathcal{A}|}\begin{cases} \frac{\pi}{4} r, & 0 \le r \le \sqrt{3}W; \\ 0.3673\pi r - r\arccos\left(\frac{\sqrt{3}W}{r}\right) \\ -r\arccos\left(\frac{1.6159W}{r}\right), & \sqrt{3}W \le r \le 2W; \end{cases} \tag{5}$$

Substituting (5) in (4), the expectation can be expressed as

$$\mathrm{E}_{G,R}\{\Omega_{t_i}\} = \psi(0.3673\pi, 2W, t_i) - \psi\left(0.1173\pi, \sqrt{3}W, t_i\right) - \frac{m^m \Gamma(m+t_i)}{|\mathcal{A}|\Gamma(m)} \int_{\sqrt{3}W}^{2W} \frac{(r^{-\alpha})^{t_i}}{(m+\beta r_0^{\alpha} m_0 r^{-\alpha})^{m+t_i}} r \times \left(\arccos\left(\frac{\sqrt{3}W}{r}\right) + \arccos\left(\frac{1.6159W}{r}\right)\right) dr. \tag{6}$$

where $\psi(\cdot,\cdot,\cdot)$ is defined as

$$\psi(\theta, \upsilon, \tau) = \frac{\theta m^m (\beta r_0^{\alpha} m_0)^{-m-\tau} \upsilon^{2+\alpha m} \Gamma(m+\tau)}{|\mathcal{A}|(2+\alpha m)\Gamma(m)} {}_2F_1\left[\frac{2}{\alpha} + m, m+\tau, 1 + \frac{2}{\alpha} + m, -\frac{m\upsilon^{\alpha}}{\beta r_0^{\alpha} m_0}\right]. \tag{7}$$

Finally, the outage probability for the case of the reference receiver located at vertex V_2 can be evaluated by substituting (6) in (3).

5 Results

- **Boundary Effects:** When the nodes are confined within a finite region, the nodes located close to the physical boundaries of the region experience different network characteristics, such as outage probability, compared to the nodes located near the center of the region.

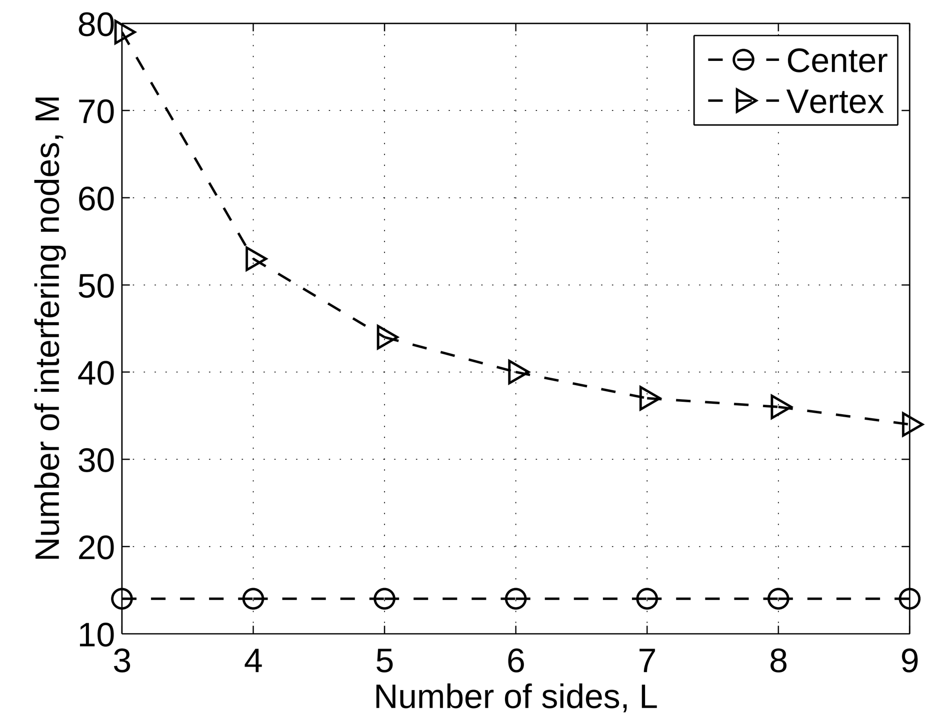

Figure 2: The number of interfering nodes M versus the number of sides L in order to meet a fixed low outage probability constraint of $\epsilon = 0.05$ for the reference node located at the center and the corner, respectively, of a L-sided polygon ($L = 3, 4, 5, 6, 7, 8, 9$).

- **Outage Probability in an Arbitrarily-Shaped Convex Region**

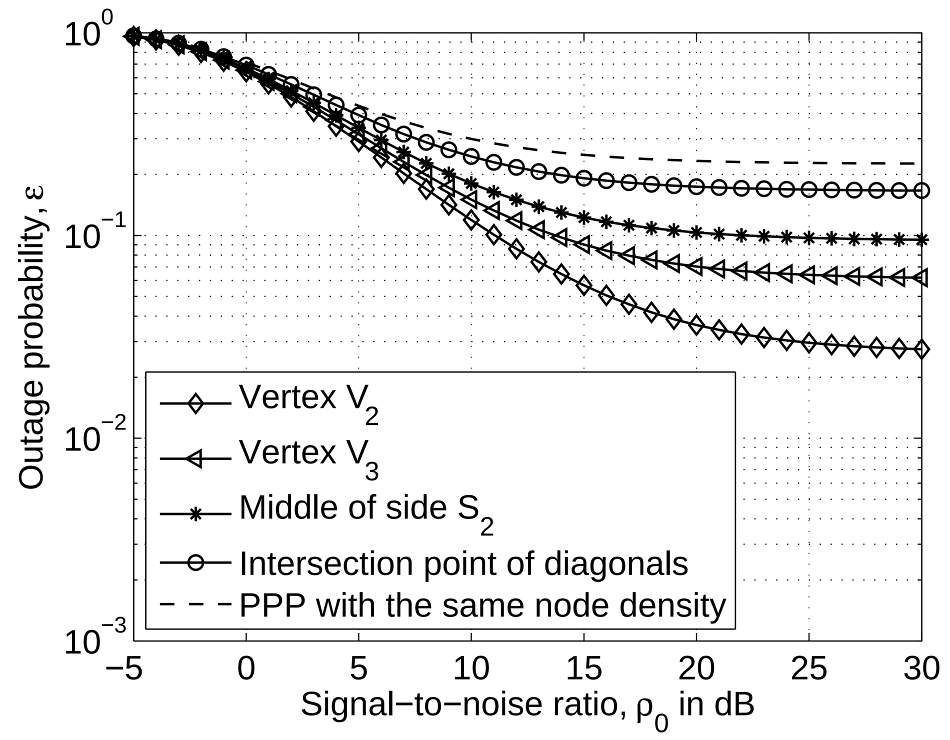

Figure 3: Outage probability, ϵ, versus the signal-to-noise ratio, ρ_0, with arbitrary locations of the reference receiver inside the arbitrarily-shaped finite region defined in Fig. 1 having area $|\mathcal{A}| = 13143$, $M = 10$ interferers, i.i.d. Rayleigh fading channels ($m_0 = m = 1$), path-loss exponent $\alpha = 2.5$ and SINR threshold $\beta = 0$ dB. For the PPP node distribution, the node density is $\lambda = 10/13143 = 7.6086 * 10^{-4}$.

6 References

[1] J. Guo, S. Durrani, and X. Zhou, "Outage Probability in Arbitrarily-Shaped Finite Wireless Networks," IEEE Transactions on Communications, 2014 (to appear).

[2] Mathematica and Matlab code to calculate distance distributions in closed form available at: http://users.cecs.anu.edu.au/~Salman.Durrani/software.html.

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