Machine-Type Communication with Random Access and Data Aggregation: A Stochastic Geometry Approach

Jing Guo*, Salman Durrani*, Xiangyun Zhou*, and Halim Yanikomeroglu†
*Research School of Engineering, The Australian National University, Canberra, ACT 2601, Australia.
Emails: {jing.guo, salman.durrani, xiangyun.zhou}@anu.edu.au.
†Department of Systems and Computer Engineering, Carleton University, Ottawa, ON K1S 5B6, Canada.
Email: halim@sce.carleton.ca.

Abstract— **Enabling machine-type communication (MTC) over large scale cellular networks is a promising solution to handling the emerging MTC traffic. To enable a massive number of machines to connect to the base station, random access mechanisms and data aggregation have been largely studied separately in the literature. In this paper, we use stochastic geometry to investigate MTC over cellular with access class barring enhanced random access and data aggregation. We present an approximate yet accurate and tractable analytical framework for characterizing the MTC performance in terms of the machine type device (MTD) success probability, average number of successful MTDs and probability of successful preamble utilization. We validate the proposed model by comparison with simulations. Our results show that while the provision of more resources for the relaying phase benefits MTC, the provision of more preambles in the random access is not always beneficial to MTC. Thus, system parameters need to be chosen carefully to benefit the MTC traffic.**

I. Introduction

Machine-type communication (MTC) communication requires connection of a large number of machines to the access point, with each machine sporadically transmitting a small amount of data [1]. Enabling MTC communication over cellular is a promising approach to handle the emerging traffic because of the wide coverage area and high performance provided by the existing celluar infrastructure [2].

One key challenge in realizing MTC over cellular is enabling massive number of machines (at least 10× greater than the current number of cellular subscribers [2]) to efficiently connect to the base stations (BSs). Two main approaches have emerged in the literature as a solution to this problem: (i) modifying and improving the operation of the current random access channel [3]. For example, access class barring (ACB) is an effective approach to reduce congestion in the random access by limiting the number of contending machines [4]; (ii) data aggregation, where instead of connecting to the BSs directly, machines first connect to the aggregators and the aggregators relay the aggregated data to the BS [5, 6].

Recently, data aggregation has received much attention in the literature [5–12]. In [7–10], the authors have considered only a single BS or a single aggregator, i.e., the performance over large scale networks was not assessed. For large scale networks with multiple aggregators and BSs, MTC can be impacted by the interference due to the concurrent transmission. Some papers have examined MTC with data aggregation over large scale networks. The distribution of the signal-to-interference ratio (SIR) was derived for a single-hop wireless network in [11]. However, collisions on the random access channel were not considered. A slotted ALOHA-based cluster random access method was proposed in [12]. However, the performance was analyzed using simulations only. Using stochastic geometry, an analytical model for data aggregation was proposed in [5], while data aggregation and resource scheduling at aggregators was analyzed in our prior work in [6]. However, both [5] and [6] assumed that all machines have been granted access to the aggregators and did not model the random access procedure.

Contributions: In this work, we focus on characterizing the MTC with random access and data aggregation over cellular networks, i.e., machines access the aggregator via random access and then the aggregators relay the aggregated data to BSs. To reduce the traffic overload at the aggregators, we include the ACB mechanism to enhance the random access operation [4, 13, 14]. The major contributions of this paper are:

- Using stochastic geometry, we provide an approximate yet accurate analytical framework for characterizing MTC with random access and data aggregation.
- We derive analytical expressions for the machine type device (MTD) success probability, average number of successful MTDs and the probability of successful preamble utilization.
- Our numerical results show that the ACB mechanism is more beneficial when the number of preambles is small and the relaying phase's resources (defined as the multiplication of transmission time and bandwidth for the relaying phase) are large. Also, another important finding is that while the provision of more resources for the relaying phase benefits MTC, the provision of more preambles in the random access is not always beneficial

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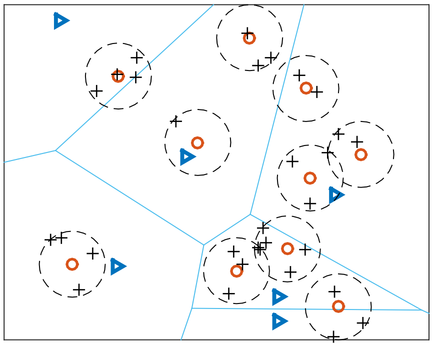


Fig. 1. Illustration of the system model (▲ = BSs, ○ = aggregators, + = MTDs, dashed line is the serving zone).

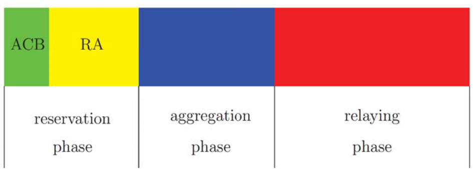


Fig. 2. Illustration of the MTD transmission model.

to MTC. Thus, system parameters need to be chosen carefully to benefit the MTC traffic.

II. System model

Consider a cellular uplink model for MTC where the MTDs first transmit packets to the aggregators and then the aggregators relay the aggregated data to BSs. We model the locations of BSs and aggregators as independent homogeneous poisson point processes (HPPPs) in a two dimensional Euclidean space $\mathbb{R}^2$, denoted as Φ_B with density λ_B and ϕ_a with density λ_a, respectively. As for the locations of MTDs that need to be served, we assume it follows a Matérn cluster point process (MCPP), where their parents are the aggregators. That is to say, for each aggregator, it has a serving zone and MTDs to be served in a serving zone is following a PPP. The considered model is applicable to smart utility metering and industry automation use cases for massive MTC over cellular, where the authorized MTDs are either static or have low mobility and are served by their designated aggregators [6].

The data size for all MTDs to be served are assumed to be same, denoted as D. Let R_s and r_m denote the radius of the serving zone and the distance between a MTD and its serving aggregator, respectively. According to the definition of MCPP, r_m has the distribution of $f(r_m) = \frac{2r_m}{R_s^2}$ and K has the probability mass function (PMF) of $\Pr(K = k) = \frac{1}{k!}\bar{m}^k \exp(-\bar{m})$. The network model is illustrated in Fig. 1.

A. MTD transmission model

The data transmission of a MTD is divided into three phases, namely reservation phase, aggregation phase and relaying phase. The transmission model is illustrated in Fig. 2. Note that we consider the one-shot transmission scheme (i.e., if one MTD's data is failed to transmit in one realization, it will not appear in the following realizations). The phases work as follows:

- At the start of the reservation phase, the ACB scheme takes place to reduce the collisions. Each MTD generates a random number within $[0, 1]$ and compares the random number with the probability factor p. If the random number is lower than p, this MTD is allowed to initiate the RA procedure. Otherwise, this MTD's data is dropped. In this work, we consider the basic ACB scheme, where p is constant and it is known to each MTD.
- Random access (RA) is the next stage in the reservation phase. Each MTD that passes ACB randomly selects a preamble among N available preambles in order to build a connection with its aggregator. We assume that when multiple MTDs within the same serving zone select the same preamble, a destructive collision happens, where their data is dropped and not aggregated at the aggregator. Those MTDs whose preambles are not chosen by others within the serving zone are called non-dropped MTDs.
- In the aggregation phase, the non-dropped MTDs transmit data to their serving aggregators via the orthogonal channel resources. We assume that there are total number of N orthogonal channels, which corresponds to N preambles and are reused across the whole network. Due to the concurrent transmission, some MTDs's signals are too weak and experience channel outage. Consequently, their data cannot be successfully aggregated at the aggregators. Those MTD that are not in channel outage are called served MTDs.
- In the relaying phase, the aggregators transmit the aggregated data to its closest BS. That is to say, aggregators in $\mathbb{R}^2$ are separated by the Voronoi cells formed by BSs. This implies that the distance between an aggregator and its serving BS r_a has the distribution of $f(r_a) = 2\pi\lambda_B r_a \exp(-2\pi\lambda_B r_a^2)$. Note that we assume that only the aggregator that has gathered data will transmit and these aggregators are known as active aggregators. Otherwise the aggregator keeps silent. Furthermore, the active aggregators within the same Voronoi cell are assumed to access the BS in a round-fashion, e.g., they equally share the available bandwidth [15]. We assume there are T transmission time and W available bandwidth which is reused for all BSs for the relaying phase.

B. Channel model

We consider the path-loss plus Rayleigh fading channel model. Then, the instantaneous power received at a receiver is $p_t g r^{-\alpha}$, where p_t is the transmit power, g is the independently and identically distributed (i.i.d.) fading power with exponential distribution, r is the random distance between a transmitter and a receiver and α is the path-loss exponent. We use h and g to denote the fading on the desired link and interfering link, respectively. In addition, we consider an interference-limited scenario and assume that all MTDs and aggregators employ the full inversion power control. Thus, the transmit power for a transmitter can be simplified as r^{α}, where r is the distance from the transmitter to its desired receiver and the receiver sensitivity is ignored as it will not impact the network performance.

C. Assumptions for tractable analysis

In this work, we make the following assumptions for tractable analysis. The accuracy of these assumptions will be verified in the results section.

Assumption 1. In general, the three phases are closely coupled. For example, the total data transmitted by the aggregator depends on the instantaneous number of accessing MTDs, the number of collided preambles, etc. For analytical tractability, we assume that each phase is independent.

Assumption 2. In the aggregation phase, the interference generated on each channel within the same serving zone is partially spatially-correlated, because of the considered MCPP (i.e., the interfering MTDs are clustered). To the best of our knowledge, it is a very challenging problem to analytically model this spatial-correlation. Instead, similar to [6], we assume that interference on each channel is independent.

Assumption 3. The location of active aggregators is correlated, i.e., those aggregators that are close to each other are more likely to be inactive since their channels experience severe interference. For analytical tractability, we approximate the location of active aggregators as a HPPP with density $\lambda_a' = (1 - \Pr(K_1 = 0))\lambda_a$, where $\Pr(K_1 = 0)$ is the probability of being inactive for an aggregator.

Assumption 4. Based on **Assumption 3** and [16], the distribution of the number of active aggregators associated to the same BS, N_a can be approximated as

$$\Pr(N_a = n_a) = \frac{3.5^{3.5}\Gamma[n_a + 3.5](\frac{\lambda_a'}{\lambda_B})^{n_a}}{\Gamma[3.5]\Gamma[n_a + 1](3.5 + \frac{\lambda_a'}{\lambda_B})^{n_a+3.5}} \quad . \tag{1}$$

Assumption 5. In the relaying phase, we consider the orthogonal access for each aggregator within each BS's coverage area. Hence, only one aggregator per cell can use the certain resource block and generate the interference to the typical BS. The location of interfering aggregators can be approximated as a HPPP with density $(1-p_{\text{void}})\lambda_B$ [15], where $p_{\text{void}} = \Pr(N_a = 0)$ is the void probability that accounts for the probability that BS has zero active aggregator.

Based on these assumptions, we formulate the performance metrics and the analytical results in the next two sections.

III. MTC Analytical Framework and Metrics

To investigate the MTC performance, we consider three metrics in this work, namely the MTD success probability, average number of successful MTDs and probability of successful preamble utilization. The definition of first two metric is adapted from [8], while the definition of the last metric is adapted from [6].

Definition 1. MTD success probability is the probability that a MTD's data can be successfully received at the BS. Based on our system set-up, the MTD's data is said to be delivered successfully if and only if

- The typical MTD passes the ACB and there is no collision on its selected preamble in the reservation phase;
- The typical MTD does not experience the channel outage in the aggregation phase;
- The typical MTD's serving aggregator does not experience the channel outage in the relaying phase.

Using **Assumption 1**, the MTD success probability is mathematically approximated by [6]

$$\bar{p}_{\text{suc}} \approx \bar{p}_{\text{nodrop}} \times \bar{p}_{\text{suc1}} \times \bar{p}_{\text{suc2}}, \tag{2}$$

where $\bar{p}_{\text{nodrop}}$ is the average probability that a typical MTD passes the ACB and has no collision on its preamble, $\bar{p}_{\text{suc1}}$ is the average probability that the typical MTD is not in channel outage probability (equivalently, the channel success probability of the aggregation phase) and $\bar{p}_{\text{suc2}}$ is the average probability that an aggregator is not in channel outage probability (equivalently, the channel success probability of the relaying phase).

Definition 2. Average number of successful MTDs is defined as the average number of MTDs whose data are successfully relayed to a BS.

It can be expressed as [6]

$$\bar{K}_{\text{suc}} = \sum_{k_1=1}^{N} k_1 \Pr(K_1 = k_1) p_{\text{suc2}}(k_1), \tag{3}$$

where K_1 denotes the number of served MTDs (i.e., data has been successfully aggregated at the aggregator), $\Pr(K_1 = k_1)$ is its corresponding PMF and $p_{\text{suc2}}(k_1)$ is conditional channel success probability given that the aggregator has k_1 MTDs to be served.

Definition 3. Probability of successful preamble utilization allows us to assess how much benefit having more preambles brings to MTC. It is defined as the probability that a preamble is only selected by one MTD and this MTD's data is finally received at the BS.

Using **Assumption 1**, we can express this metric as [6]

$$\bar{p}_{\text{utility}} \approx \bar{p}_O \times \bar{p}_{\text{suc1}} \times \bar{p}_{\text{suc2}}, \tag{4}$$

where $\bar{p}_O$ is the probability that a typical preamble is only selected by a MTD.

The detailed analysis of the key factors (i.e., $\bar{p}_{\text{nodrop}}$, $\bar{p}_O$, $\bar{p}_{\text{suc1}}$, $\Pr(K_1 = k_1)$ and $\bar{p}_{\text{suc2}}$) that determine the three metrics is presented in the following section.

IV. MTC Analysis

In this section, we provide the analysis of the key factors that determine the MTC performance metrics in Section III.

A. Access class barring and random access analysis

From the system model in Section II, a MTD can transmit its data to its serving aggregator if and only if it passes the access class barring and selects a non-collision preamble. By the probability theory, we can have $\bar{p}_{\text{nodrop}}$ and $\bar{p}_O$ given as follows.

Lemma 1: Based on the system model in Section II, the average non-drop (i.e., MTD passes ACB and RA) probability is

$$\bar{p}_{\text{nodrop}} = p\frac{\exp\left(-\frac{\bar{m}}{N}p\right) - \exp\left(-\bar{m}\right)}{1-\frac{p}{N}}. \tag{5}$$

Lemma 2: Based on the system model in Section II, the average preamble non-collision probability is

$$\bar{p}_O = p\frac{\bar{m}}{N}\exp\left(-p\frac{\bar{m}}{N}\right). \tag{6}$$

Proof: In this work, the number of MTDs to be served in a serving zone is following a Poisson distribution with mean $\bar{m}$, i.e., $\Pr(K=k) = \frac{1}{k!}\bar{m}^k\exp(-\bar{m})$. For a MTD, the probability that it passes ACB is p and the probability that it passes ACB and selects a preamble is $\frac{p}{N}$. Then given that k MTDs in a serving zone, the probability that a typical MTD selects a non-collided MTD is $\left(1-\frac{p}{N}\right)^{k-1}$. Hence, we can write the average non-drop probability as

$$\begin{aligned}\bar{p}_{\text{nodrop}} &= \sum_{k=1}^{\infty} p\left(1-\frac{p}{N}\right)^{k-1}\Pr(K=k)\\ &= p\frac{\exp\left(-\frac{\bar{m}}{N}p\right)}{1-\frac{p}{N}}\sum_{k=1}^{\infty}\left(1-\frac{p}{N}\right)^{k}\bar{m}^k\frac{\exp\left(-\bar{m}+\frac{\bar{m}}{N}p\right)}{k!}\\ &= p\frac{\exp\left(-\frac{\bar{m}}{N}p\right)}{1-\frac{p}{N}}\left(1-\exp\left(-\bar{m}+\frac{\bar{m}}{N}p\right)\right),\end{aligned} \tag{7}$$

where the last comes from the fact that $\sum_{k=0}^{\infty}\left(\bar{m}-\bar{m}\frac{p}{N}\right)^k\frac{\exp\left(-\bar{m}+\frac{\bar{m}}{N}p\right)}{k!} = 1 = \exp\left(-\bar{m}+\frac{\bar{m}}{N}p\right) + \sum_{k=1}^{\infty}\left(\bar{m}-\bar{m}\frac{p}{N}\right)^k\frac{\exp\left(-\bar{m}+\frac{\bar{m}}{N}p\right)}{k!}$.

Similarly, for a typical preamble, given that k MTDs in a serving zone, the probability that it is only selected by a preamble is $\binom{k}{1}\frac{p}{N}\left(1-\frac{p}{N}\right)^{k-1}$. Then averaging this probability over the distribution of the number of MTDs to be served, we have

$$\begin{aligned}\bar{p}_O &= \sum_{k=1}^{\infty}\binom{k}{1}\frac{p}{N}\left(1-\frac{p}{N}\right)^{k-1}\bar{m}^k\frac{\exp\left(-\bar{m}\right)}{k!}\\ &= \frac{p}{N}\bar{m}\exp\left(-p\frac{\bar{m}}{N}\right)\sum_{k=1}^{\infty}\left(\bar{m}-\bar{m}\frac{p}{N}\right)^{k-1}\frac{\exp\left(-\bar{m}+\frac{\bar{m}}{N}p\right)}{(k-1)!}\\ &= \frac{p}{N}\bar{m}\exp\left(-p\frac{\bar{m}}{N}\right)\sum_{t=0}^{\infty}\left(\bar{m}-\bar{m}\frac{p}{N}\right)^{t}\frac{\exp\left(-\bar{m}+\frac{\bar{m}}{N}p\right)}{t!}\\ &= \frac{p}{N}\bar{m}\exp\left(-p\frac{\bar{m}}{N}\right).\end{aligned} \tag{8}$$

■

B. Channel success probability of the aggregation phase

The channel success probability of the aggregation phase for a typical link is mathematically defined as

$$\bar{p}_{\text{suc1}} = \Pr\left(\frac{h}{\sum_{x\subset\Phi_{\text{MTD}}^{\text{served}}} g r_s^{\alpha} x^{-\alpha}} > \gamma_1\right), \tag{9}$$

where γ_1 is the SIR threshold on the aggregation phase, x denotes both the location and the interfering MTD which occupies the typical channel.

In order to characterize the channel success probability for a typical link, we first need to know the point process of interfering MTDs on the typical link, i.e., $\Phi_{\text{MTD}}^{\text{served}}$. From the independent thinning property [17], the location of aggregators whose typical channel is occupied (i.e., its corresponding preamble is selected without collision) is following the HPPP with density $\bar{p}_O\lambda_a$. According to the definition of the MCPP, the location of interfering MTDs is a transformed version of these aggregators. Thus, from the displacement theorem, the location of interfering MTDs on the typical channel is a PPP with density $\bar{p}_O\lambda_a$. Using stochastic geometry, we obtain the channel success probability in the following theorem.

Theorem 1: Based on the system model in Section II, the aggregation phase's average channel success probability experienced at a typical aggregator for a certain channel is

$$\bar{p}_{\text{suc1}} = \exp\left(-\bar{p}_O\lambda_a\pi\frac{R_s^2}{2}\Gamma\left[1+\frac{2}{\alpha}\right]\Gamma\left[1-\frac{2}{\alpha}\right]\gamma_1^{\frac{2}{\alpha}}\right). \tag{10}$$

Proof: The proof is the same as Appendix B in [6]. It is omitted here for brevity. ■

C. Distribution of served MTDs

The PMF of the number of served MTDs K_1 is an essential element since it governs both the average number of successful MTDs and the channel success probability of the relaying phase. The number of served MTDs K_1 is determined by the number of non-dropped MTDs K' and whether each channel experiences outage or not. Using **Assumption 2** and probability theory, we obtain the PMF of K_1 in the following corollary.

Corollary 1: Based on the system model in Section II, the PMF of the number of served MTDs, K_1, for a typical aggregator is approximated by

$$\begin{aligned}\Pr(K_1=k_1) \approx \sum_{k'=k_1}^{N}&\left(\binom{k'}{k_1}\bar{p}_{\text{suc1}}^{k_1}(1-\bar{p}_{\text{suc1}})^{k'-k_1}\right)\\ &\times\binom{N}{k'}\bar{p}_O^{k'}(1-\bar{p}_O)^{N-k'},\end{aligned} \tag{11}$$

where $\bar{p}_{\text{suc1}}$ and $\bar{p}_O$ are given in (10) and (6), respectively.

Proof: See Appendix A.

D. Channel success probability of the relaying phase

For an active aggregator with K_1 served MTDs, its aggregated data can be successfully decoded by its associated BS if and only if its SIR meets the following condition

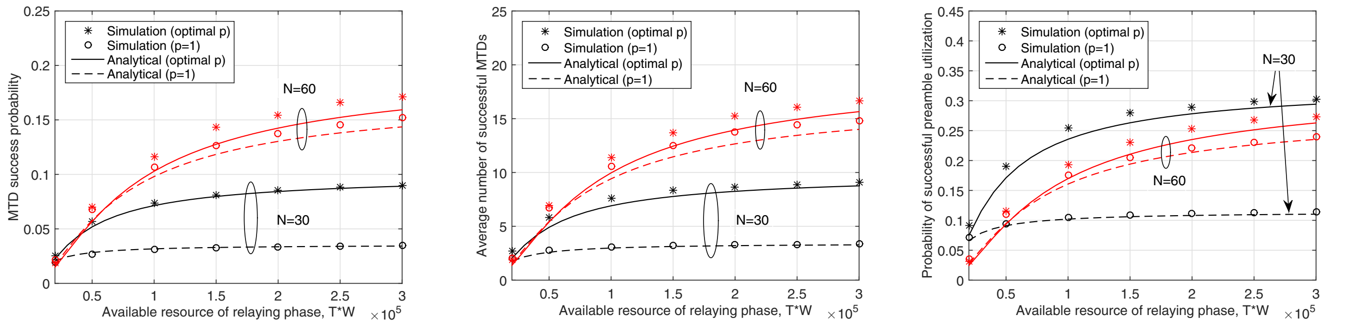
$$DK_1 \leq \frac{TW}{N_a}\log\left(1+\frac{h}{\sum_{y\subset\Phi_a^{\text{intf}}} g r_a^{\alpha} y^{-\alpha}}\right), \tag{12}$$

where y denotes both the location and the interfering aggregators, Φ_a^{intf} is point process of interfering aggregators and N_a is number of active aggregators associated to the same BS.

TABLE I

SUMMARY OF THE MTC ANALYTICAL FRAMEWORK AND METRIC.

metric	general form	$\bar{p}_{\text{nodrop}}$	$\bar{p}_O$	$\bar{p}_{\text{suc1}}$	$\bar{p}_{\text{suc2}}$, $p_{\text{suc2}}(k_1)$	$\Pr(K_1 = k_1)$
MTD success probability	(2)	(5)		(10)	(14)&(15)	(11)
average number of successful MTDs	(3)				(15)	(11)
probability of successful preamble utilization	(4)		(6)	(10)	(14)&(15)	(11)



(a) MTD success probability $\bar{p}_{\text{suc}}$.

(b) Average number of successful MTDs $\bar{K}_{\text{suc}}$.

(c) Probability of successful preamble utilization $\bar{p}_{\text{utility}}$.

Fig. 3. Available resources of the relaying phase, TW, versus the considered metrics with different number of preambles N. Optimal $p = \frac{3}{10}$ for $N = 30$ and $p = \frac{3}{5}$ for $N = 60$.

Then the average channel success probability of the relaying phase at a typical BS is

$$\bar{p}_{\text{suc2}} = \mathbb{E}_{K_1,N_a}\left\{\Pr\left(\frac{h}{\sum_{y\subset\Phi_a^{\text{intf}}} g r_a^{\alpha} y^{-\alpha}} \geq 2^{\frac{DK_1N_a}{TW}} - 1\right)\right\}. \tag{13}$$

Using the stochastic geometry, we can obtain the approximation of $\bar{p}_{\text{suc2}}$ in the following proposition.

Proposition 1: Based on the system model described in Section II, the average channel success probability experienced at a typical BS from one of its associated active MTD is approximated as

$$\bar{p}_{\text{suc2}} = \sum_{k_1=1}^{N} p_{\text{suc2}}(k_1) \frac{\Pr(K_1 = k_1)}{1 - \Pr(K_1 = 0)}, \tag{14}$$

$$p_{\text{suc2}}(k_1) = \sum_{n_a=1}^{\infty} \exp\left(\frac{2\,{}_2F_1\left[1, 1-\frac{2}{\alpha}, 2-\frac{2}{\alpha}, 1-2^{\frac{Dk_1n_a}{W}}\right]}{\left((1-p_{\text{void}})\left(2^{\frac{Dk_1n_a}{W}}-1\right)\right)^{-1}(2-\alpha)}\right) \times \frac{\Pr(N_a = n_a)}{1 - \Pr(N_a = 0)}, \tag{15}$$

where $p_{\text{void}} = \left(1 + \frac{\lambda_a'}{3.5\lambda_B}\right)^{-3.5}$ is the void probability, $p_{\text{suc2}}(k_1)$ is the conditional channel success probability of the relaying phase which is conditioned on the number of active channels for an aggregator, $\Pr(K_1 = k_1)$ is presented in (11) and $\Pr(N_a = n_a)$ is given in (1).

Proof: See Appendix B.

E. Summary

Using the above derived results, we can summarise the analytical framework and metrics as shown in Table I.

V. RESULTS

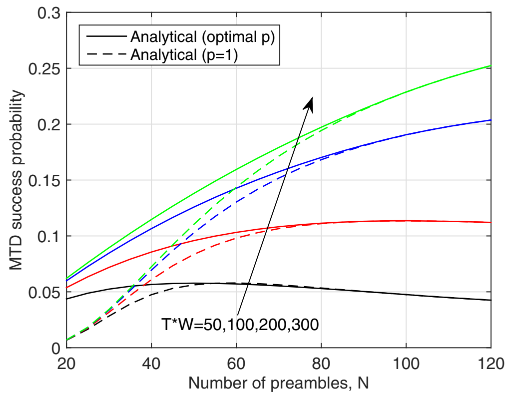
In this section, we present numerical and simulation results to characterize the MTC performance. In order to eliminate the boundary effects, the simulation results are generated by distributing nodes inside a disk region with radius 6 km over 10^4 simulation runs. Unless specified otherwise, the following values of the main system parameters are used: $\lambda_B = \frac{1}{\pi 500^2}$ m^{-2}, $\lambda_a = 10^{-4.5}$ m^{-2}, $R_s = 50$ m, $\alpha = 3$, $\bar{m} = 100$, $\gamma_1 = 0$ dB and $D = 100$ bits. As for the ACB probability factor, we consider the optimal p value, which is $p = \min\{1, \frac{N}{\bar{m}}\}$ [13].

A. Model validation

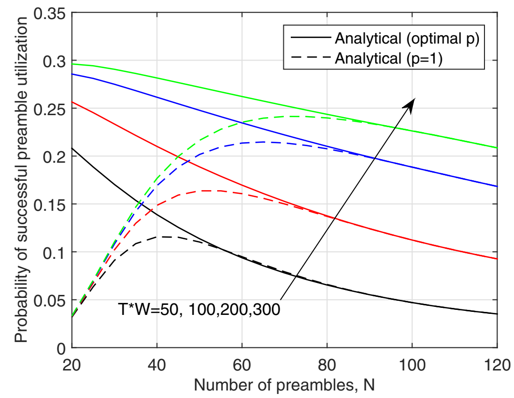
Fig. 3 plots the available resource of the relaying phase versus the performance metrics defined in Section III with different number of preambles. Note that the available resource of the relaying phase is defined as the multiplication of transmission time and bandwidth for the relaying phase. By comparing the analytical results with the simulation results, we can see that our analytical results match the simulation results fairly well, which validates our analysis. The small gap between the analytical and simulation results come from the assumptions we made in Section II-C. However, unlike the simulations which are time-consuming, our derived results allow us to compute the results quickly.

B. Impact of relaying phase

From Fig. 3, we can see that, the provision of more available resources for the relaying phase can benefit the MTC, because all three metrics increase with the increasing of TW. This is expected, since increasing the available resource of the relaying phase improves the channel success probability of the relaying phase. The figure shows that when the number of preambles is relatively small (i.e., the curves for $N = 30$), increasing the resource for the relaying phase beyond a certain



(a) MTD success probability $\bar{p}_{\mathrm{suc}}$.



(b) Probability of successful preamble utilization $\bar{p}_{\mathrm{utility}}$.

Fig. 4. Number of preambles, N, versus (a) MTD success probability and (b) probability of successful preamble utilization.

point brings diminishing returns, since the curves become virtually flat. This is because, when TW is very large, the MTC performance is mainly governed by the reservation and aggregation phases. For example, for the case of $N = 30$, N is too small compared to $\bar{m}$ such that most MTDs are dropped.

Comparing Fig. 3(a) and Fig. 3(b), we can see that the results for $\bar{p}_{\mathrm{suc}}$ and $\bar{K}_{\mathrm{suc}}$ have very similar shape and approximately differ by a scaling factor in the y-axis. We extensively have tested other sets of system parameters and find that $\bar{K}_{\mathrm{suc}} \approx \bar{m} \times \bar{p}_{\mathrm{suc}}$. Hence, in this following, we do not show the curves for $\bar{K}_{\mathrm{suc}}$ for the sake of brevity.

C. Impact of reservation and aggregation phases

Fig. 4(a) plots the number of preambles versus MTD success probability with/without optimal ACB, respectively. Fig. 4(a) shows that, when the number of preambles N is small, which leads to a larger number of MTDs contending for access, ACB leads to the improvement in $\bar{p}_{\mathrm{suc}}$. As N becomes large, the benefit of ACB diminishes and the curves for optimal p and $p = 1$ (i.e., no ACB) merge.

Fig. 4(b) plots the number of preambles versus the probability of successful preamble utilization with/without optimal ACB, respectively. We can see that when ACB is not used, there is an optimal N which maximizes $\bar{p}_{\mathrm{utility}}$. However, when optimal ACB is used, $\bar{p}_{\mathrm{utility}}$ decreases as N increases. This is because $\bar{p}_O$ is constant under the optimal ACB. Hence, $\bar{p}_{\mathrm{utility}}$ is totally governed by $\bar{p}_{\mathrm{suc2}}$. Once there is no ACB (i.e., $p = 1$), $\bar{p}_O$ increases with the increasing of N, since more number of preambles can reduce the collision at the preambles. The interplay of $\bar{p}_O$ and $\bar{p}_{\mathrm{suc2}}$ determines the performance of $\bar{p}_{\mathrm{utility}}$.

From the discussion above, we can conclude that providing more preambles is not always beneficial to MTC. This is because of the inherent coupling between the random access and the data aggregation. In addition, ACB mechanism is more beneficial, when the number of preambles is small and the relaying phase's resources are large. Thus system parameters need to be chosen carefully to benefit MTC traffic.

VI. Conclusions

In this paper, we have studied MTC over cellular networks with random access and data aggregation. To reduce the preamble collision in the random access (i.e., from MTDs to aggregators), we have included the ACB in the random access. We have derived the analytical expressions for the key metrics, namely the MTD success probability, average number of successful MTDs and probability of successful preamble utilization. Our results illustrate that the system parameters need to be chosen carefully to benefit the MTC traffic. Future work can consider random access procedures with backoff and retransmission, as specified in [18, 19].

Appendix

Appendix A: Proof of Corollary 1

Proof: Using **Assumption 2**, given that there are total number of k' non-dropped MTDs within a serving zone, the conditional PMF of K_1 can be approximated as a Binomial distribution [6]. It can be written as

$$\Pr(K_1 = k_1 | K') \approx \begin{cases} 0, & K' < k_1; \\ \binom{K}{k_1} \bar{p}_{\mathrm{suc1}}^{k_1} (1 - \bar{p}_{\mathrm{suc1}})^{K' - k_1}, & K' \geq k_1. \end{cases} \tag{16}$$

We then average this conditional probability over the distribution of the number of non-dropped MTDs and can obtain the PMF of K_1, i.e.,

$$\Pr(K_1 = k_1) = \mathbb{E}_{K'} \left\{ \Pr(K_1 = k_1 | K') \right\}. \tag{17}$$

The exact distribution of K' can be found in [14], which is quite complicated. For simplicity, similar to [8], we approximate it as a Binomial distribution, i.e., ignoring the dependence for the non-collided preambles. Hence, the PMF of K' is given by

$$\Pr(K' = k') \approx \binom{N}{k'} \bar{p}_O^{k'} (1 - \bar{p}_O)^{N - k'}. \tag{18}$$

Substituting (16) and (18) into (17), we arrive at the result in (11). ■

APPENDIX B: PROOF OF PROPOSITION 1

Proof: To compute the average channel success probability for the relaying phase, let us first compute the conditional channel success. We assume that the typical BS is located at the origin. Given K_1 number of served MTDs for an active aggregator and N_a number of active aggregators associated with the typical BS N_a, the conditional channel success probability is given by

$$p_{\text{suc2}}(K_1, N_a) = \mathbb{E}_{I_2}\left[\exp(-\gamma_2 I_2)\right] = \mathcal{M}_{I_2}(s)|_{s=\gamma_2}, \quad (19)$$

where $\gamma_2 \triangleq 2^{\frac{DK_1N_a}{TW}} - 1$, $I_2 = \sum_{y\subset\Phi_a^{\text{intf}}} g r_a^{\alpha} y^{-\alpha}$ is the aggregate interference from active aggregators occupying the same resource [15] and $\mathcal{M}_{I_2}(s)$ is its moment generate function (MGF).

According to **Assumption 4**, Φ_a^{intf} is approximated as a HPPP and the MGF of the interference at the relaying phase is then approximated as $\mathcal{M}_{I_2}(s) = \exp\left(-2(1-p_{\text{void}})s\frac{{}_2F_1\left[1,1-\frac{2}{\alpha},2-\frac{2}{\alpha},-s\right]}{\alpha-2}\right)$. A detail derivation is provided in [6]. Due to the space constraint, we skip it here.

After averaging (19) over N_a and K_1, we can arrive the result presented in Proposition 1. Note that the extra term in denominator $(1-\Pr(N_a=0))$ comes from the fact the derived average channel success probability is calculated for the link from the active aggregator to BS, which requires $K_1 \geq 1$ and $N_a \geq 1$. ■

REFERENCES

[1] C. Bockelmann, N. Pratas, H. Nikopour, K. Au, T. Svensson, C. Stefanovic, P. Popovski, and A. Dekorsy, "Massive machine-type communications in 5G: Physical and MAC-layer solutions," *IEEE Commun. Mag.*, vol. 54, no. 9, pp. 59–65, Sep. 2016.

[2] Z. Dawy, W. Saad, A. Ghosh, J. G. Andrews, and E. Yaacoub, "Towards massive machine type cellular communications," *IEEE Wireless Commun.*, vol. 24, no. 1, pp. 120–128, Feb. 2017.

[3] A. Laya, L. Alonso, and J. Alonso-Zarate, "Is the random access channel of LTE and LTE-A suitable for M2M communications? A survey of alternatives," *IEEE Commun. Surveys Tuts.*, vol. 16, no. 1, pp. 4–16, First quarter 2014.

[4] S. Duan, V. Shah-Mansouri, Z. Wang, and V. W. S. Wong, "D-ACB: Adaptive congestion control algorithm for bursty M2M traffic in LTE networks," *IEEE Trans. Veh. Technol.*, vol. 65, no. 12, pp. 9847–9861, Dec. 2016.

[5] D. Malak, H. S. Dhillon, and J. G. Andrews, "Optimizing data aggregation for uplink machine-to-machine communication networks," *IEEE Trans. Commun.*, vol. 64, no. 3, pp. 1274–1290, Mar. 2016.

[6] J. Guo, S. Durrani, X. Zhou, and H. Yanikomeroglu, "Massive machine type communication with data aggregation and resource scheduling," *IEEE Trans. Commun.*, 2017, to appear. [Online]. Available: http://ieeexplore.ieee.org/document/7937902/

[7] D. Niyato, P. Wang, and D. I. Kim, "Performance modeling and analysis of heterogeneous machine type communications," *IEEE Trans. Wireless Commun.*, vol. 13, no. 5, pp. 2836–2849, May 2014.

[8] G. Rigazzi, N. K. Pratas, P. Popovski, and R. Fantacci, "Aggregation and trunking of M2M traffic via D2D connections," in *Proc. IEEE ICC*, Jun. 2015, pp. 2973–2978.

[9] G. Miao, A. Azari, and T. Hwang, "E^2-MAC: Energy efficient medium access for massive M2M communications," *IEEE Trans. Commun.*, vol. 64, no. 11, pp. 4720–4735, Nov. 2016.

[10] A. Kumar, A. Abdelhadi, and C. Clancy, "A delay efficient MAC and packet scheduler for heterogeneous M2M uplink," in *Proc. IEEEGlobecom Workshops*, Dec. 2016, pp. 1–5.

[11] T. Kwon and J. M. Cioffi, "Random deployment of data collectors for serving randomly-located sensors," *IEEE Trans. Wireless Commun.*, vol. 12, no. 6, pp. 2556–2565, Jun. 2013.

[12] Z. Zhao, S. Szyszkowicz, T. Beitelmal, and H. Yanikomeroglu, "Spatial clustering in slotted ALOHA two-hop random access for machine type communication," in *Proc. IEEE Globecom*, Dec. 2016, pp. 1–6.

[13] M. Koseoglu, "Lower bounds on the LTE-A average random access delay under massive M2M arrivals," *IEEE Trans. Commun.*, vol. 64, no. 5, pp. 2104–2115, May 2016.

[14] J. Lee, J. Guo, and S. Durrani, "Analytical framework for access class barring in machine type communication," in *Proc. IEEE PIMRC*, Oct. 2017, pp. 1–6.

[15] S. Singh, X. Zhang, and J. G. Andrews, "Joint rate and SINR coverage analysis for decoupled uplink-downlink biased cell associations in HetNets," *IEEE Trans. Wireless Commun.*, vol. 14, no. 10, pp. 5360–5373, Oct. 2015.

[16] J. Ferenc and Z. Neda, "On the size distribution of Poisson voronoi cells," *Phys. A*, vol. 385, no. 2, pp. 518–526, 2007.

[17] M. Haenggi, *Stochastic Geometry for Wireless Networks*. Cambridge University Press, 2012.

[18] 3GPP TR 37.868, "Study on RAN improvements for machine-type communications," Sep. 2011.

[19] 3GPP TS 36.321 V13.2.0, "Evolved universal terrestrial radio access (E-UTRA); medium access control (MAC) protocol specification," Aug. 2016.