Analytical Framework for Access Class Barring in Machine Type Communication

Jason Lee, Jing Guo, and Salman Durrani
Research School of Engineering, The Australian National University, Canberra, ACT 2601, Australia.
Corresponding author email: jing.guo@anu.edu.au.

***Abstract*— Access class barring (ACB) is regarded as an efficient and practically implementable method to reduce the traffic overload in cellular networks. In this paper, we present a unified analytical framework to analyze the performance of the fixed ACB scheme for a simple random access procedure (i.e., one-shot transmission model) in machine type communication (MTC) over cellular networks. We derive the exact expressions for the probability of a machine's packet being served by the base station (BS), the average number of machine type devices (MTDs) successfully served by the BS per second and the non-collision slot access probability. We verify the accuracy of the derived expressions by comparison with simulations. Based on the analytical expressions, we then maximize the probability of a MTD's packet being served and obtain the sub-optimal probability factor value for the fixed ACB in closed-form. Our results confirm that, the use of ACB scheme is important for scenarios with high MTD packet arrival rate, which is relevant for massive MTC. The proposed framework allows fine tuning and accurate prediction of the MTC performance with ACB.**

I. Introduction

By allowing ubiquitous connection among massive number of machines, without the human intervention, machine type communication (MTC) is regarded as a key enabler for 5G communication systems [1]. Besides the short-range wireless communication techniques to support the MTC, the cellular network is a major and promising candidate because of its existing infrastructure and high-performance capabilities [2]. In terms of its characteristics, MTC differs greatly from the human-to-human communication, i.e., the number of devices, transmission periodicity, etc. [1–3]. Consequently, the current cellular network system has to be revisited and redesigned to improve its compatibility for managing MTC. Particularly, the enhancement of random access channel becomes a critical issue among the possible challenges, since MTC will lead to the huge data congestion and collision on the random access channel due to the massive number of machines trying to access the base station (BS) [4–6].

In 3GPP, access class barring (ACB) is considered as an effective solution to improve the operation of random access scheme [7]. In ACB, each machine that is trying to access the cellular network first draws a random number and compares it with the probability factor p. Only if the generated number is lower than p, the machine is allowed to attempt the access procedure. Based on the ACB principle, some works have developed modified ACB schemes. For example, the extended ACB was proposed by 3GPP [8], where different probability factors are distributed to machines depending on their service requirements. In [9], the authors combined the virtual resource allocation and dynamic ACB technique and proposed a prioritized random access with dynamic access barring framework. A cooperative ACB scheme was proposed in [10], where the probability factor for each BS was jointly determined instead of individually. Other works concentrated on the optimization of ACB performance via control or estimation techniques. A heuristic algorithm to adaptively change the probability factor was proposed in [7]. In [11], the authors developed a stabilized multi-channel slotted ALOHA algorithm that maintains a finite number of unserved users in the system. The authors in [12] estimated the number of incoming machines using Kalman filtering approach, thereby adjusting the probability factor value.

This work was supported by the Australian Research Council's Discovery Project Funding Scheme (Project number DP170100939).

In this work, we propose an analytical framework to investigate the performance of a simple ACB scheme, known as the fixed ACB (i.e., p is fixed), in MTC over a cellular network under a simplified random access procedure (i.e., one-shot transmission model). The major contributions of this paper are:

- We derive the exact expressions for the probability that a machine's packet is successfully delivered to the BS, the average number of machines successfully served by the BS and the non-collision probability of a slot. These results generalize the analytical results in [13] to include ACB schemes.
- We obtain the closed-form sub-optimal p for the fixed ACB scheme, which maximizes the packet successful delivery probability. It show that the results from the sub-optimal value are very similar to the results using optimal value under the fixed ACB scheme.
- Our derived analytical results confirm that when the MTD packet arrival rate is high, the performance of MTC can be improved using ACB scheme. The proposed framework allows fine tuning and accurate prediction of the MTC performance with ACB.

II. System model

Consider a single cell scenario, with a base station located at the center. The coverage area of the BS is a disk of radius R. There are multiple machine type devices (MTDs) residing in the coverage area. As is done in one of the simulation cases for 3GPP model, we do not consider the inter-cell interference in this work [14]. The MTDs transmit data to the BS using the cellular uplink. A time division multiple access (TDMA) system is considered in this work, where time is divided into frames of duration $L * T$. Each frame has fixed L slots and each slot has the fixed duration T.

We model the traffic arrival at the BS as a Poisson distribution with rate λ packets per second. This is a widely adopted model in the literature [13, 14]. We further assume that, for each frame, at most one packet can be generated by one MTD. Hence, the number of MTDs with packets to deliver per frame, denoted as M, is a Poisson random variable with mean $\lambda_a = \lambda LT$. For example, its probability mass function (PMF) is given by

$$\Pr(M = m) = \frac{\lambda_a^m \exp(-\lambda_a)}{m!}. \quad (1)$$

As for the location of MTDs, for analytical tractability, we assume that each MTD is uniformly distributed in the BS's coverage area. Let r denote the distance between a MTD and BS and its probability density function (PDF) is $f(r) = \frac{2r}{R^2}$.

Based on the TDMA system, each frame for the communication between MTDs and BS can be divided into three phases: access barring, reservation and connection. Let L_B, L_R and L_C denote the number of slots for the accessing barring, reservation and connection phases, respectively. Then, we have

$$L = L_B + L_R + L_C. \quad (2)$$

The detailed description for each phase is presented in the following subsections. Note that we employ the one-shot transmission scheme in this work, i.e., if one packet fails in one frame, it does not appear in the following frames [13].

A. Access Barring Phase

The first part of the frame is used to implement the access class barring scheme. According to the conventional ACB mechanism [7], for the MTD having a packet to transmit, it will generate a random number within 0 and 1. If this number is less than the probability factor p, this MTD is allowed to attempt an access (i.e., contend for an access reservation in the following reservation phase) and these MTDs are called *accessing MTDs*. Otherwise, the access of this MTD is barred in this frame (i.e., the packet is dropped).

The time consumed on the random number generation and comparison is assumed to be negligible. We only account for the time for broadcasting the probability factor p, which varies according to the ACB scheme. In this work, we consider the fixed ACB scheme, where the probability factor p is constant and this information is previously known by each MTD. Hence, the number of slots taken up by broadcasting, L_B, equals to zero[2].

[2]Note that for dynamic ACB (where p is varing), the overhead associated with the slot used to broadcast probability to MTDs has to be taken into account. Hence, L_B is no longer zero. Our derived framework in this work can be extended to include dynamic ACB scheme. This is outside the scope of this work.

B. Reservation Phase

The reservation phase is used to implement the random access for those MTDs that pass the ACB scheme (i.e., indicate their request for packet transmission to the BS). We assume that the reservation phase in each frame has L_C mini-slots and each mini-slot takes up t_{mini}. The accessing MTDs contend for these reservation mini-slots. In order to have the mini-slot as being successfully reserved by an accessing MTD, two requirements must be met, i.e., no collision on the mini-slot and no channel outage between the accessing MTD and BS. Those accessing MTDs satisfying these two requirements are called the *reserved MTDs* and they will be served by the BS later on.

We employ frame slotted ALOHA in this work [13, 15]. Each accessing MTD randomly, independently and uniformly picks a mini-slot to send its connection request. Once more than one accessing MTDs choose the same mini-slot, we assume that none of the requests can be detected on the mini-slots and, thus, the packets of these collided accessing MTDs are dropped.

Even though there is no collision for a mini-slot selected by a MTD, the request of this MTD may still fail to be decoded by the BS if the link experiences channel outage. We model the communication channel as a path-loss plus block Rayleigh fading channel, where fading within one frame keeps the same. Note that non-line-of-sight propagation model is the worst case scenario for machine type communication. The instantaneous signal-to-noise ratio (SNR) on a link is, thus, given by $\frac{p_t g r^{-\alpha}}{\mathcal{N}}$, where p_t is the transmit power of an accessing MTD which is assumed to be the same for all MTDs, g is the fading power gain which follows independent and identical exponential distribution, α is the path-loss exponent and $\mathcal{N}$ is the noise power. The channel outage occurs when the SIR falls below a threshold γ.

At the end of this phase, there is one extra slot remaining for the transmission of feedback information from the BS to MTDs. Again, this channel is assumed to have zero error because of the robust coding [13]. Based on the description, the number of slots for the reservation phase is $L_R = \text{ceil}\left(\frac{L_C \times t_{\text{mini}}}{T}\right) + 1$, where $\text{ceil}(\cdot)$ is the ceiling operator.

C. Connection Phase

This phase is for the packet transmission between reserved MTDs and the BS. Given that there are L_C mini-slots in the reservation phase, there will be L_C slots reserved for the purpose of connection. Since not all the L_C mini-slots are successfully reserved in the reservation phase, only certain number of slots (equal to the number of reserved MTDs) are used for packet transmission. The remaining slots are unused. Note that all these reserved MTDs' packets can be successfully received at the BS in this phase, because the fading is constant within each frame and their SIR has already met the channel requirement from the reservation phase. In other words, these reserved MTDs are no longer in channel outage within the frame.

III. Performance Analysis

To evaluate the MTC performance, in this work, we consider three performance metrics, namely, packet successful delivery probability, average number of served MTDs and slot non-collision probability. Their definitions, along with the detailed analysis, are presented in the following subsections.

A. Packet Successful Delivery Probability

The packet successful delivery probability examines the performance of a MTD and it is the probability that the packet of a MTD can be finally received at the BS.

Proposition 1: Based on the system model in Section II, the packet successful delivery probability can be expressed as

$$P_{\text{delivery}} = p_{\text{suc}} \sum_{m=1}^{\infty} p \left(1 - \frac{p}{L_C}\right)^{m-1} \frac{\lambda_a^m \exp(-\lambda_a)}{m!}, \quad (3)$$

where p_{suc} is shown in (13).

Proof: See Appendix A.

B. Average Number of Served MTDs

Average number of served MTDs is defined as the average number of MTDs that can be successful served by the BS per second.

Proposition 2: Based on the system model in Section II, the average number of served MTDs is

$$N_{\text{suc}} = \frac{\mathbb{E}_N\{N\}}{LT} = \frac{\sum_{n=0}^{L_C} n \times \Pr(N=n)}{LT}, \quad (4)$$

where N denotes the number of successfully served MTDs and is equivalent to the reserved MTDs. These two terms are used interchangeably in this work. $\Pr(N=n)$ is its corresponding PMF, which is given by

$$\Pr(N=n) = \sum_{m_2=0}^{L_C} \left(\Pr(N=n|m_2) \left(\sum_{m=0}^{\infty} \frac{\lambda_a^m \exp(-\lambda_a)}{m!} \times \left(\sum_{m_1=0}^{m} \Pr(M_2=m_2|m_1) \Pr(M_1=m_1|m) \right) \right) \right), \quad (5)$$

where M_1 denote the number of accessing MTDs, M_2 denote the number of accessing MTDs without collision which is called the non-collision MTDs, $\Pr(M_1 = m_1|m)$, $\Pr(M_2 = m_2|m_1)$ and $\Pr(N = n|m_2)$ are presented in (14), (15) and (18), respectively.

Proof: See Appendix B.

C. Slot Non-collision Probability

The slot non-collision probability is defined as the probability that no collision occurs at a mini-slot, which shows the packet congestion at the mini-slot level.

Proposition 3: Based on the system model in Section II, the slot non-collision probability is given by

$$P_{\text{non-collision}} = \sum_{m=1}^{\infty} \frac{L_C - p + mp}{L_C} \left(1 - \frac{p}{L_C}\right)^{m-1} \frac{\lambda_a^m \exp(-\lambda_a)}{m!}. \quad (6)$$

Proof: See Appendix C.

Remark 1: In this paper, we extend the system model in [13] to include fixed ACB scheme, which is not considered in [13]. The first two metrics, P_{delivery} and N_{suc}, are also considered in [13]. The results in this work include the results in [13] as special cases. For example, when we set $p = 1$, (3) reduces to the result in [13, eq.(23)]. When we ignore the term $\Pr(M_1 = m_1|m)$ in (5) and treat the term $\Pr(M_2 = m_2|m)$ as a binomial distribution, (5) reduces to the result in [13, eq.(5)]. In addition, in this paper, we introduce and analyze a new metric $P_{\text{non-collision}}$ to exam how the collision at each slot is minimized by the incorporation of the ACB scheme. This metric has not been studied in [13].

IV. Performance Optimization

According to (3), (5) and (6), the network performance is governed by the selection of probability factor p. In general, a large value of p results in a lot of collided MTDs. By contrast, a small value of p implies the less congestion at the reservation phase, but it also cuts the number of MTDs for contention which leads to the under-utilization of resources. In this section, we aim to find the optimum p^* value which can optimize the network performance.

$P_{\textit{delivery}}$ ***Optimization:*** The following optimization problem maximizes the packet successful delivery probability:

$$\begin{aligned} &\underset{p}{\text{maximize}} && P_{\text{delivery}} \\ &\text{subject to} && 0 < p \leq 1, \end{aligned} \quad (7)$$

where P_{delivery} is presented in (3).

(7) is a relatively simple optimization problem. In order to solve it, we take the first order derivative of P_{delivery} in (3) with respect to p and set this expression to zero. Then, we find the p value satisfying the equation and this p is the optimum value.

The first order derivative of P_{delivery} with respect to p is

$$\begin{aligned} \frac{\mathrm{d}P_{\text{delivery}}}{\mathrm{d}p} &= \frac{\mathrm{d}}{\mathrm{d}p} \left(p_{\text{suc}} \sum_{m=1}^{\infty} p \left(1 - \frac{p}{L_C}\right)^{m-1} \frac{\lambda_a^m \exp(-\lambda_a)}{m!} \right) \\ &= p_{\text{suc}} \left(\sum_{m=1}^{\infty} \frac{\lambda_a^m \exp(-\lambda_a)}{m!} \left(1 - \frac{p}{L_C}\right)^{m-2} \frac{L_C - pm}{L_C} \right). \\ &= p_{\text{suc}} \frac{\exp\left(-\frac{\lambda_a p}{L_C}\right)}{\left(1 - \frac{p}{L_C}\right)^2} \left(\sum_{m=1}^{\infty} \frac{\left(\lambda_a \left(1 - \frac{p}{L_C}\right)\right)^m \exp\left(-\lambda_a \left(1 - \frac{p}{L_C}\right)\right)}{m!} \right. \\ &\quad \left. - \left(1 - \frac{p}{L_C}\right) \frac{p}{L_C} \lambda_a \sum_{m=1}^{\infty} \frac{\left(\lambda_a \left(1 - \frac{p}{L_C}\right)\right)^{m-1} \exp\left(-\lambda_a \left(1 - \frac{p}{L_C}\right)\right)}{(m-1)!} \right) \\ &= p_{\text{suc}} \frac{\exp\left(-\frac{\lambda_a p}{L_C}\right)}{\left(1 - \frac{p}{L_C}\right)^2} \left(1 - \exp\left(-\lambda_a \left(1 - \frac{p}{L_C}\right)\right) \right. \\ &\quad \left. - \left(\frac{p}{L_C} - \frac{p^2}{L_C^2}\right) \lambda_a \right), \end{aligned} \quad (8)$$

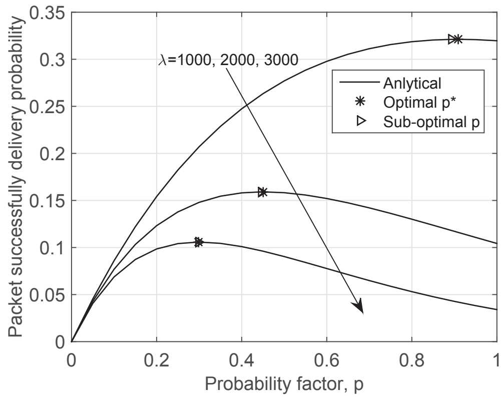


Fig. 1. Packet successful delivery probability P_{delivery} versus the probability factor p with different arrival rate λ.

where the last step comes from the fact that

$$\sum_{m=0}^{\infty}\frac{\left(\lambda_a\left(1-\frac{p}{L_C}\right)\right)^m\exp\left(-\lambda_a\left(1-\frac{p}{L_C}\right)\right)}{m!}=1$$
$$=\sum_{m=1}^{\infty}\frac{\left(\lambda_a\left(1-\frac{p}{L_C}\right)\right)^{m-1}\exp\left(-\lambda_a\left(1-\frac{p}{L_C}\right)\right)}{(m-1)!}. \quad (9)$$

Setting (8) equal to zero, we need to solve p which satisfies

$$1-\exp\left(-\lambda_a\left(1-\frac{p}{L_C}\right)\right)=\left(\frac{p}{L_C}-\frac{p^2}{L_C^2}\right)\lambda_a. \quad (10)$$

Unfortunately, there is no closed-form expression for p which satisfies the above expression. Hence, in general, we have to find p^* using numerical iteration. However, we can work out a sub-optimal p^* in closed-form.

Using Jensen inequality, the exact packet successful delivery probability in (3) can be approximated by

$$P_{\text{delivery}}\approx p\times p_{\text{suc}}\left(1-\frac{p}{L_C}\right)^{\lambda_a-1}. \quad (11)$$

Taking the first order derivative of (11) with respect to p and then finding the p which leads to the expression being to zero, we have the sub-optimal p for the fixed ACB scheme expressed as

$$p_{\text{fixed}}^{sub}=\min\left\{\frac{L_C}{\lambda_a},1\right\}. \quad (12)$$

Fig. 1 plots the packet successful delivery probability versus the probability factor p with different density λ. The adopted system parameters follow Table. I. Note that the results from both the optimal and sub-optimal values are marked. From the figure, we can see that, the results under the sub-optimal value are very close to the results under the optimal value.

N_{suc} Optimization: With regards to the optimal performance for average number of served MTDs, N_{suc}, it is not easy to work out the optimal p^* in closed-form, because of the complicated form of (5). We can obtain the sub-optimal p^* via approximation (i.e., assuming that the number of accessing MTDs per mini-slot follows a Poisson distribution with density $\frac{p\lambda_a}{L_C}$), which is shown to be equivalent to (12). More importantly, packet successful delivery probability can be interpreted as the average probability that a MTD can be successfully served by the BS and it is closely related to the average number of served MTDs. Thus, we believe that our results in (12) can provide the sub-optimal performance for N_{suc}.

TABLE I
MAIN SYSTEM PARAMETER VALUES.

Parameter	Value	Parameter	Value
cell radius R	200 m	path-loss exponent α	3.5
transmit power p_t	-10 dBm	noise power $\mathcal{N}$	-100 dBm
slot duration T	1 ms	mini-slot duration t_{mini}	0.1 ms
SNR threshold γ	1	no. of mini-slots L_C	50

V. RESULTS

In this section, we present numerical results to investigate the performance of the ACB scheme. To validate the numerical results, simulation results are also presented, which are generated using MATLAB and are averaged over 1 million simulation runs. Unless specified otherwise, the values of the main system parameters shown in Table I are used.

A. Model Validation

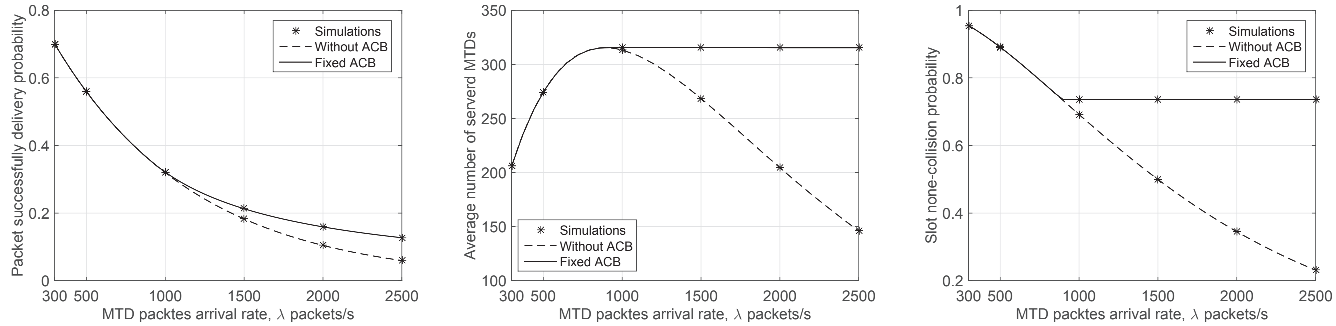
Figs. 2(a), 2(b) and 2(c) plot the MTD packet arrival rate λ versus the packet successful delivery probability, average number of served MTDs and slot non-collision probability for the scenario without ACB and the fixed ACB with sub-optimal p, respectively. As shown in these figures, our analytical results match exactly with the simulation results, which confirms the correctness of our derived expressions.

B. Packet Successful Delivery Probability

From Fig. 2(a), we find that when the arrival rate λ is relatively small, the curve for without ACB overlaps with the curve for fixed ACB. This is because, under such a scenario the number of mini-slots is greater than the average arrival rate per frame. Hence, the fixed ACB does not work according to (12), i.e., $p=1$. When λ becomes large, the ACB scheme begins to kick in and reduces the collision in the reservation phase. Thus, P_{delivery} with fixed ACB is higher than P_{delivery} without ACB for the larger value of λ.

C. Average Number of Served MTDs

Fig. 2(b) shows that the fixed ACB scheme can improve the performance of N_{suc} when the packet arrival rate is high. From this figure, we can also see that, under the fixed ACB scheme, N_{suc} increases at first and then almost becomes flat. This can be explained as follows. When the packet arrival rate is small, there is little traffic congestion and hence, more arrival rate leads to the higher N_{suc}. When λ is large, the ACB begins to play a role and results in almost constant non-collision probability, which maintains the number of accessing MTDs to the same level regardless of the packet arrival rate.



(a) packet successful delivery probability. (b) Average number of served MTDs. (c) Slot non-collision probability.

Fig. 2. MTD packet arrival rate λ versus (a) packet successful delivery probability, (b) average number of served MTDs and (c) slot non-collision probability for the scenario without ACB, and the fixed ACB with sub-optimal p.

D. Slot Non-collision Probability

As shown in Fig. 2(c), without the ACB scheme, the non-collision probability is always decreasing with increasing λ. After incorporating the ACB scheme, $P_{\text{non-collision}}$ remains almost as a constant regardless of how large λ is. It can be easily explained using similar arguments as before.

VI. Conclusions

In this paper, we presented an analytical framework to investigate the performance of ACB for MTC over a cellular network, where ACB is incorporated to improve system performance. The analytical expressions for the packet successful delivery probability, average number of served MTDs and slot non-collision probability were obtained. We also computed the optimal probability factor p for this system. Our numerical results showed that, the use of access barring technique is necessary for the MTC scenarios characterized by high data arrival rate. Future work can consider the scenarios with time-varying traffic demands, more complicated random access procedures with backoff and retransmissions, and joint optimization of random access and data transmission.

Appendix

A. Derivation of Proposition 1

Proof: In order to make the packet of a MTD successfully served, three requirements have to be met, i.e., the MTD passes the ACB scheme, no collision of mini-slot selection for the MTD and the MTD is not in channel outage.

We assume M MTDs with packet arrival in a certain frame and one MTD among them is regarded as the typical MTD. For this typical MTD, in the access barring phase, the probability that it can passes the ACB scheme is p. This corresponds to the first condition.

In the reservation phase, since each accessing MTD uniformly and randomly selects a mini-slot, the probability that one mini-slot is selected is $\frac{1}{L_C}$. Then combining with the accessing barring phase, for any MTD among the $M-1$ MTDs, we have the probability that it passes the ACB scheme and selects the same mini-slot as the typical MTD is given by $p\times\frac{1}{L_C}$. Its complementary probability, which is the probability of non-collision between this MTD and the typical MTD, is $1-\frac{p}{L_C}$. There are total number of $M-1$ MTDs and they make their own decision individually. Thus, the non-collision probability for the mini-slot selected by the typical MTD is $\left(1-\frac{p}{L_C}\right)^{M-1}$. This corresponds to the second condition.

In terms of the third condition, the probability of not being in channel outage for the typical MTD can be expressed as

$$\begin{aligned} p_{\text{suc}} &= \Pr\left(\frac{p_t g r^{-\alpha}}{\mathcal{N}} > \gamma\right) = \mathbb{E}_r\left\{\Pr\left(g > \frac{\gamma \mathcal{N} r^{\alpha}}{p_t}\right)\right\} \\ &= \int_0^R \exp\left(-\frac{\gamma\mathcal{N}r^{\alpha}}{p_t}\right)\frac{2r}{R^2}\mathrm{d}r \\ &= \frac{2}{\alpha R^2}\left(\frac{p_t}{\gamma\mathcal{N}}\right)^{\frac{2}{\alpha}}\Gamma\left[\frac{2}{\alpha},0,\frac{\gamma\mathcal{N}R^{\alpha}}{p_t}\right], \end{aligned} \tag{13}$$

where $\mathbb{E}\{\}$ is the expectation operator, $\Gamma[\cdot,\cdot,\cdot]$ is the Gamma function, and the third step comes from the fact that fading power gain g follows the exponential distribution.

From the above analysis, the conditional packet successful delivery probability, which is conditioned on the number of MTDs M, is $p\times\left(1-\frac{p}{L_C}\right)^{M-1}\times p_{\text{suc}}$. Since M is a Poisson random variable, by averaging the conditional probability over the distribution of M (i.e., (1)), we have P_{delivery} shown in (3). ■

B. Derivation of Proposition 2

Proof: The PMF of the number of served MTDs is determined by the three phases and is also related to the number of MTDs passing each phase. Let's derive the distribution of number of MTDs passing each phase.

• *The number of accessing MTDs and its PMF:* As indicated in Section II-A, in the accessing barring phase, the random number is generated independently for each MTD. Given M MTDs in a certain frame, using the Binomial theorem, the conditional PMF of the number of accessing MTDs (i.e., these MTDs pass the ACB scheme), is

$$\Pr(M_1=m_1|M)=\binom{M}{m_1}p^{m_1}(1-p)^{M-m_1}, \tag{14}$$

where M_1 is within the range of 0 and M and $\binom{\cdot}{\cdot}$ is the combinatorial operator.

• *The number of accessing MTDs without collision and its PMF:* For those accessing MTDs, their packets can be dropped in the reservation phase if their selected mini-slots are collided.

The mini-slots selection for accessing MTDs in fact belongs to a well known occupancy problem, which is about the random allocation of balls into a number of bins. Under our scenario, the balls become the accessing MTDs and the bins become the mini-slots. To determine the distribution of the number of non-collision MTDs, we leverage the results presented in [16, eqs.(3)&(4)]. By setting $r = 1$ in [16, eqs.(3)&(4)], we have the conditional distribution of M_2, given M_1 accessing MTDs, displayed as

$$\Pr(M_2 = m_2|M_1) = \frac{\binom{L_C}{m_2}\left(\prod_{k=0}^{m_2-1}(M_1-k)\right)}{L_C^{M_1}} \times G(L_C - m_2, M_1 - m_2), \tag{15}$$

where $G(U,u) = 1$ when $U = 0$ and $u \leq 0$ and

$$G(U,u) = U^u + \sum_{k=1}^{u}(-1)^k \left(\prod_{j=0}^{k-1}(u-j)(U-j)\right)\frac{(U-k)^{u-k}}{k!}. \tag{16}$$

Combining the conditional PMFs in (14) and (15), we then average the number of MTDs M, we can obtain the PMF of the number of non-collision MTDs as

$$\Pr(M_2 = m_2) = \mathbb{E}_M\left[\sum_{m_1=0}^{M}\Pr(M_2 = m_2|m_1)\Pr(M_1 = m_1|M)\right]. \tag{17}$$

• *The number of served MTDs and its PMF:* In order to become the reserved (equivalently, served) MTDs, the non-collision MTD must not be in channel outage in the reservation phase. For a non-collision MTD, the probability of not being in channel outage p_{suc} is presented in (13). Again, whether the non-collision MTD is in outage or not is an independent event. Given M_2 non-collision MTDs, using the Binomial theorem, the conditional PMF of the number of served MTDs is

$$\Pr(N = n|M_2) = \binom{M_2}{n} p_{\text{suc}}^n \left(1 - p_{\text{suc}}\right)^{M_2 - n}. \tag{18}$$

Finally, de-conditioning the above result with respect to M_2 (i.e., its PMF is shown in (17)), we obtain the PMF of the number of served MTDs as shown in (5).

■

C. Derivation of Proposition 3

Proof: Let us assume that the system contains M MTDs in a certain frame. One mini-slot is treated as a typical mini-slot among the L_C available mini-slots.

As long as either zero or one MTD chooses this typical mini-slot, this mini-slot is not collided. From the previous subsection, we find that the probability of one MTD passing the ACB scheme and selecting the typical mini-slot is $\frac{p}{L_C}$. Hence, the probability that none MTD picks the typical mini-slot is given by

$$\Pr(\text{no. select} = 0|M) = \left(1 - \frac{p}{L_C}\right)^M. \tag{19}$$

Similarly, the probability that only one MTD selects the mini-slot is

$$\Pr(\text{no. select} = 1|M) = \binom{M}{1}\frac{p}{L_C}\left(1 - \frac{p}{L_C}\right)^{M-1}. \tag{20}$$

Combining the above two probabilities, we have the conditional slot non-collision probability given by $\left(1 - \frac{p}{L_C}\right)^M + \binom{M}{1}\frac{p}{L_C}\left(1 - \frac{p}{L_C}\right)^{M-1}$. We then average this conditional probability over the number of MTDs M and have the slot non-collision probability given in (6). ■

References

[1] A. Laya, L. Alonso, and J. Alonso-Zarate, "Is the random access channel of LTE and LTE-A suitable for M2M communications? a survey of alternatives," *IEEE Commun. Surveys Tuts*, vol. 16, no. 1, pp. 4–16, Firstquarter 2014.

[2] Z. Dawy, W. Saad, A. Ghosh, J. G. Andrews, and E. Yaacoub, "Towards massive machine type cellular communications," *IEEE Wireless Commun. Mag.*, vol. 24, no. 1, pp. 120–128, Feb. 2017.

[3] J. Guo, S. Durrani, X. Zhou, and H. Yanikomeroglu, "Massive machine type communication with data aggregation and resource scheduling," 2017, to appear. [Online]. Available: http://ieeexplore.ieee.org/document/7937902/

[4] M. Hasan, E. Hossain, and D. Niyato, "Random access for machine-to-machine communication in LTE-advanced networks: issues and approaches," *IEEE Commun. Mag.*, vol. 51, no. 6, pp. 86–93, 2013.

[5] M. S. Ali, E. Hossain, and D. I. Kim, "LTE/LTE-A random access for massive machine-type communications in smart cities," *IEEE Commun. Mag.*, vol. 55, no. 1, pp. 76–83, Jan. 2017.

[6] Z. Zhao, S. S. Szyszkowicz, T. Beitelmal, and H. Yanikomeroglu, "Spatial clustering in slotted ALOHA two-hop random access for machine type communication," in *Proc. IEEE GLOBECOM*, Dec. 2016, pp. 1–6.

[7] S. Duan, V. Shah-Mansouri, Z. Wang, and V. Wong, "D-ACB: Adaptive congestion control algorithm for bursty M2M traffic in LTE networks," *IEEE Trans. Veh. Technol.*, vol. 65, no. 12, pp. 9847–9861, Dec. 2016.

[8] 3rd Generation Partnership Project, TR 37.868, "RAN improvements for machine-type communications." V11.0.0, Cedex, France, Sep. 2011.

[9] T. M. Lin, C. H. Lee, J. P. Cheng, and W. T. Chen, "PRADA: Prioritized random access with dynamic access barring for MTC in 3GPP LTE-A networks," *IEEE Trans. Veh. Technol.*, vol. 63, no. 5, pp. 2467–2472, 2014.

[10] S. Y. Lien, T. H. Liau, C. Y. Kao, and K. C. Chen, "Cooperative access class barring for machine-to-machine communications," *IEEE Trans. Wireless Commun.*, vol. 11, no. 1, pp. 27–32, Jan. 2012.

[11] O. Galinina, A. Turlikov, S. Andreev, and Y. Koucheryavy, "Stabilizing multi-channel slotted ALOHA for machine-type communications," in *Proc. IEEE ISIT*, 2013, pp. 2119–2123.

[12] M. Tavana, V. Shah-Mansouri, and V. W. S. Wong, "Congestion control for bursty M2M traffic in LTE networks," in *Proc. IEEE ICC*, 2015, pp. 5815–5820.

[13] G. Rigazzi, N. K. Pratas, P. Popovski, and R. Fantacci, "Aggregation and trunking of M2M traffic via D2D connections," in *Proc. IEEE ICC*, 2015, pp. 2973–2978.

[14] H. S. Dhillon, H. Huang, H. Viswanathan, and R. A. Valenzuela, "Fundamentals of throughput maximization with random arrivals for M2M communications," *IEEE Trans. Commun.*, vol. 62, no. 11, pp. 4094–4109, Nov. 2014.

[15] J. E. Wieselthier, A. Ephremides, and L. A. Michaels, "An exact analysis and performance evaluation of framed ALOHA with capture," *IEEE Trans. Commun.*, vol. 37, no. 2, pp. 125–137, Feb. 1989.

[16] H. Vogt, "Efficient object identification with passive RFID tags," in *Pervasive Computing*. Springer, 2002, pp. 98–113.