

# Multi-pair Two-way Relay Networks: Interference Management Using Lattice Codes and Amplify and Compute Relaying

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**Abstract**—In this paper, we consider lattice codes for inter-pair interference cancellation in a multi-pair two-way relay network (TWRN) with amplify-and-compute relaying. We investigate a new method to separate the desired message from the interference signal by computing only two equations at the users. First, we formulate the sum rate for this lattice code based approach and find that it has better sum rate than the existing beamforming based approaches. Next, we analytically obtain the optimum power allocation coefficient for the users' and the relay's power to maximize the sum rate. We show that for larger number of user pairs, optimum power allocation allows more improvement compared to that for smaller number of user pairs.

**Index Terms**—Two-way relay network, amplify-and-compute, lattice codes, sum rate, power allocation.

## I. INTRODUCTION

Multi-pair two-way relay networks (TWRNs) allow multiple user pairs to share messages within the pairs, assisted by a single relay [1]–[7]. Multi-pair TWRNs enable the capacity and spectral efficiency benefits resulting from network coding schemes in a conventional single pair TWRN, to be harnessed in a multiple user pair scenario [8]–[11]. Potential applications of such networks could be file sharing in device to device networks where multiple devices need to communicate with their partners simultaneously.

Recently many papers have looked at different aspects of *multi-pair TWRNs*, including optimum power allocation analysis to maximize weighted sum rate [1], bit error rate (BER) analysis [2], capacity region for a deterministic channel model [4], optimum beamforming design for multiple single antenna relays [3], [5] and spectral efficiency and energy efficiency analysis [6]. In a multi-pair TWRN, each user has to cancel the interference resulting from other user pairs to extract the desired signal. Thus, *interference cancellation is a critical issue for multi-pair TWRNs*. The prior works in this respect have mainly considered beamforming approach with multiple relay nodes or multiple antennas at a single relay, increasing the signal processing complexity at the relay [3], [5], [6]. Other approaches include different spreading signatures [2], non-overlapping time slots [1] or different signal levels [4] assigned to different pairs, resulting to inefficient usage of the communication resources.

In this paper, we propose to use lattice codes in conjunction with amplify and compute relaying for interference management in multi-pair TWRNs. The key aspects of this approach are:

- 1) Lattice codes can achieve higher rates and better spectral efficiency compared to the uncoded transmissions by utilizing the property that the sum of two lattice points is another point in the same lattice [12]. While lattice codes have been successfully applied in multi-user relay networks for improving the overall throughput [13]–[16], to the best of our knowledge, they have not been considered for interference cancellation in multi-pair TWRNs. Using lattice codes, linear combinations of the desired signal and inter-pair interferences can be extracted from the received signal [15] and effectively, the interference signal can be eliminated. Thus, the users do not need separate spreading signature, or non-overlapping time slots or different signal levels to differentiate user pairs' signals, which allows efficient utilization of resources.
- 2) To enable lattice code based interference cancellation in a multi-pair TWRN, the relay can either adopt amplify and compute or compute and forward relaying protocol<sup>1</sup>. In amplify and compute relaying, the relay amplifies the received signal and the users compute the desired linear combinations of messages [14]. This is in contrast to more traditional compute and forward relaying [13], where the relay computes linear combinations of users' messages and broadcasts to users. If compute and forward protocol is adopted for a multi-pair TWRN, the relay needs to calculate separate equations for each user pair to cancel inter-pair interference. This increases the signal processing complexity at the relay and more transmissions are required from the relay to users. However, with amplify-and-compute relaying, the users in each pair computes only two equations to resolve the

<sup>1</sup>Amplify and forward (AF) and decode and forward (DF) protocols are not considered because they will need either orthogonal time slots or linear precoding at the relay for interference cancellation.

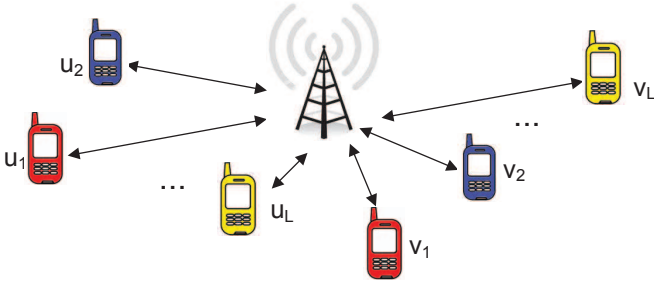


Fig. 1. System model for a multi-pair two-way relay network with  $L$  user pairs, where the users  $u$  and  $v$  in each pair exchange information via the relay. Note that the user pairs are indicated using the same colour.

received signal into desired user's message and the inter-pair interference. Thus, in this case, the relay performs only amplification, and for interference cancellation, less number of computations are required which reduces the signal processing complexity. So, we adopt amplify-and-compute relaying in this paper to investigate the lattice code based approach for interference cancellation in a multi-pair TWRN.

The main contributions of this paper are as follows:

- We investigate a new approach for interference cancellation based on lattice codes and amplify-and-compute relaying in a multi-pair TWRN that does not require multiple antennas at the relay or orthogonal time slots. We formulate the achievable sum rate for this approach, compare with that of the existing approach in [1] and find that the proposed approach outperforms the existing one.
- We analytically obtain the optimum power allocation coefficient for the users' and the relay's power that maximizes the achievable sum rate.
- We show that when large number of user pairs are involved, optimum power allocation improves the sum rate more compared to the case of small number of user pairs.

The rest of the paper is organized in the following manner. The system model of a multi-pair TWRN is presented in Section II. The proposed signal transmission protocol with amplify-and-compute relaying and lattice codes is provided in Section III. The sum rate formulation is presented in Section IV. The optimum power allocation coefficient to maximize the sum rate is derived in Section V. Section VI provides the simulation results for verification of the analytical solutions. Finally, conclusions are provided in Section VII.

## II. SYSTEM MODEL

We consider a multi-pair TWRN composed of  $L$  user pairs and a single-antenna relay, as illustrated in Fig. 1. The pre-assigned users in each pair are denoted by  $u_\ell$  and  $v_\ell$ , where  $\ell \in [1, L]$  and the relay is denoted by  $r$ . We assume that the transmissions are half duplex and the users do not have any direct link between them. The message transmission among the user pairs is completed in two phases- multiple access and broadcast phase. In the multiple access phase, all the user

pairs transmit simultaneously and the relay receives the sum of the signals. In the broadcast phase, the relay amplifies the received signal and broadcasts the messages. Each user cancels self-interference and computes two equations considering two variables- one is the desired message and the other is the sum of the inter-pair interferences. Finally, each user solves the resulting system of equations to extract the desired message.

Let the transmission power for the users and the relay be  $P$  and  $P_r$ , respectively. The channel between the  $u_\ell^{th}$  user and the relay is denoted by  $h_{u_\ell}$  and the channel between the  $v_\ell^{th}$  user and the relay by  $h_{v_\ell}$ . We make the following assumptions regarding the channel coefficients:

- The channels are assumed to be reciprocal Rayleigh fading channels which remain constant during one message packet transmission in a certain phase.
- The channel coefficients are zero mean complex-valued Gaussian random variables with variances given by  $\sigma_{h_{u_\ell}}^2$  and  $\sigma_{h_{v_\ell}}^2$ .
- The instantaneous channel state information (CSI) is assumed to be available to the relay and the users. This assumption, which is also adopted in [1], [2], [5], [17], allows benchmark results to be obtained.

In the next section, we consider signal transmission protocols in the multiple access and broadcast phases.

## III. SIGNAL TRANSMISSION PROTOCOL

In this section, we discuss lattice code based message transmission protocols at the users and the relay. First we first present the definitions of some primary operations on lattice codes in the following subsection. We follow the notations for lattice codes from [12], [13]. Further detail on lattice codes is available in [18]–[22].

### A. Preliminaries on Lattice Codes

Lattice codes are special types of structured codes that can achieve the capacity of Gaussian channels. An  $N$ -dimensional lattice is a discrete subgroup of the  $N$ -dimensional complex field  $\mathbb{C}^N$  under the normal vector addition and reflection operations and can be expressed as:

$$\Lambda = \{\lambda = \mathbf{G}_\Lambda \mathbf{c} : \mathbf{c} \in \mathbb{Z}^N\}, \quad (1)$$

where  $\mathbf{G}_\Lambda \in \mathbb{C}^{N \times N}$  is the generator matrix corresponding to the lattice  $\Lambda$  and  $\mathbb{Z}$  is the set of integers.

- Lattice quantizers are multi-dimensional generalization of uniform quantizers, which map a point  $x \in \mathbb{C}^N$  from the complex field to the nearest lattice point  $\lambda \in \Lambda$  in Euclidean distance [12]. That is,

$$Q_\Lambda(x) = \arg \min_{\lambda} \|x - \lambda\|^2. \quad (2)$$

- The modulo- $\Lambda$  operation is defined by  $x \bmod \Lambda = x - Q_\Lambda(x)$  [12]. This can be interpreted as the error in quantizing  $x$  to the closest point in the lattice  $\Lambda$ .
- The fundamental Voronoi region  $\mathcal{V}(\Lambda)$  denotes the set of all points in the  $N$ -dimensional complex field  $\mathbb{C}^N$ , which

are closest to the zero vector, i.e.,

$$\mathcal{V}(\Lambda) = \{x \in \mathbb{C}^N : Q_\Lambda(x) = 0\}. \quad (3)$$

- $\psi(\cdot)$  denotes the mapping of messages from a finite dimensional field to lattice points, i.e.,  $\psi(w) \in \Lambda$ , where  $w$  is a message from a finite dimensional field [13].
- The dither vectors  $d$  are generated independently from a uniform distribution over the fundamental Voronoi region  $\mathcal{V}(\Lambda)$ . Dithering is a well known randomization technique which is necessary for achieving statistical independence between the input vector and the error vector [12].

### B. Multiple Access Phase

In this phase, the  $u_\ell^{th}$  and the  $v_\ell^{th}$  users in each pair simultaneously transmit their messages  $W_{u,\ell}$  and  $W_{v,\ell}$  from a finite field using lattice codes  $X_{u,\ell} \in \Lambda$  and  $X_{v,\ell} \in \Lambda$ , where

$$X_{u,\ell} = (\psi(W_{u,\ell}) + d_{u,\ell}) \bmod \Lambda, \quad (4a)$$

$$X_{v,\ell} = (\psi(W_{v,\ell}) + d_{v,\ell}) \bmod \Lambda, \quad (4b)$$

where  $\Lambda$  denotes the lattice with dimension  $N$ ,  $\psi$  is the mapping function and  $d_{u,\ell}, d_{v,\ell}$  are random dithers, generated at the users and transmitted to other users.

The relay receives the sum of all the messages from all the user pairs, which is given by

$$r_{u,v} = \sqrt{P} \sum_{\ell=1}^L h_{u,\ell} X_{u,\ell} + \sqrt{P} \sum_{\ell=1}^L h_{v,\ell} X_{v,\ell} + n_1, \quad (5)$$

where  $n_1$  is the zero mean complex AWGN at the relay with noise variance  $\sigma_n^2$  per dimension.

The relay then amplifies the received signal with an amplification factor  $\alpha$  and broadcasts

$$Z_{u,v} = \alpha \left( \sqrt{P} \sum_{\ell=1}^L h_{u,\ell} X_{u,\ell} + \sqrt{P} \sum_{\ell=1}^L h_{v,\ell} X_{v,\ell} + n_1 \right), \quad (6)$$

where  $\alpha$  is chosen to minimize the noise variance.

### C. Broadcast Phase

The  $u_m^{th}$  user in the  $m^{th}$  ( $m \in [1, L]$ ) pair receives the signal

$$Y_{u,v} = \sqrt{P_r} h_{u_m} Z_{u,v} + n_2, \quad (7)$$

where  $n_2$  is the zero mean complex AWGN at the  $u_m^{th}$  user with noise variance  $\sigma_n^2 = \frac{N_0}{2}$  per dimension.

Then the  $u_m^{th}$  user removes the dithers  $d_{u,\ell}, d_{v,\ell}$  multiplied by  $\sqrt{PP_r}a_1$  for  $\ell = m$  (or by  $\sqrt{PP_r}b_1$  for  $\ell \neq m$ ), where  $a_1$  and  $b_1$  are complex integer coefficients. The resulting signal is

$$\begin{aligned} & \left[ Y_{u,v} - \sqrt{PP_r}a_1(d_{u,m} + d_{v,m}) - \sqrt{PP_r}b_1 \left( \sum_{\ell=1, \ell \neq m}^L d_{u,\ell} + d_{v,\ell} \right) \right] \bmod \Lambda \\ &= \left[ \sqrt{PP_r}a_1(\psi(W_{u,m}) + \psi(W_{v,m})) + \sqrt{PP_r}b_1 \sum_{\ell=1, \ell \neq m}^L (\psi(W_{u,\ell}) + \psi(W_{v,\ell})) + n' \right] \bmod \Lambda, \end{aligned} \quad (8)$$

where,

$$\begin{aligned} n' = & \sqrt{PP_r} \left( (\alpha h_{u_m}^2 - a_1) X_{u,m} + (\alpha h_{u_m} h_{v_m} - a_1) X_{v,m} \right. \\ & \left. + \left( \alpha h_{u_m} \sum_{\ell=1, \ell \neq m}^L h_{u,\ell} - b_1 \right) X_\ell + \left( \alpha h_{u_m} \sum_{\ell=1, \ell \neq m}^L h_{v,\ell} - b_1 \right) X_\ell \right) \\ & + \sqrt{P_r} \alpha h_{u_m} n_1 + n_2 \end{aligned} \quad (9)$$

and the coefficients  $a_1$  and  $b_1$  are chosen to minimize the variance of the noise  $n'$ . It has been shown in [13] that choosing  $a_1$  and  $b_1$  close to the channel coefficients minimizes the noise variance.

Then the  $u_m^{th}$  user decodes the signal in (8) using a lattice quantizer and obtains the estimate  $\hat{X}_{u,v}$  such that

$$\hat{X}_{u,v} = \left[ a_1 \sqrt{PP_r} \left( \psi(\hat{W}_{u,m}) + \psi(\hat{W}_{v,m}) \right) + b_1 \sqrt{PP_r} \sum_{\ell=1, \ell \neq m}^L \left( \psi(\hat{W}_{u,\ell}) + \psi(\hat{W}_{v,\ell}) \right) \right] \bmod \Lambda, \quad (10)$$

is true for large enough lattice dimension. Similarly, the  $u_m^{th}$  user extracts another equation  $\hat{X}'_{u,v}$  such that

$$\hat{X}'_{u,v} = \left[ a_2 \sqrt{PP_r} \left( \psi(\hat{W}_{u,m}) + \psi(\hat{W}_{v,m}) \right) + b_2 \sqrt{PP_r} \sum_{\ell=1, \ell \neq m}^L \left( \psi(\hat{W}_{u,\ell}) + \psi(\hat{W}_{v,\ell}) \right) \right] \bmod \Lambda \quad (11)$$

is true for large enough lattice dimension, where  $a_2$  and  $b_2$  are complex integer coefficients and they are chosen to minimize the noise variance at the  $u_m^{th}$  user.

### D. Signal Extraction

The  $u_m^{th}$  user subtracts its own signal  $W_{u,m}$  from  $\hat{X}_{u,v}$  and  $\hat{X}'_{u,v}$  after multiplication with  $\sqrt{PP_r}a_1$  and  $\sqrt{PP_r}a_2$ , respectively and solves the following equations for  $\psi(\hat{W}_{v,m})$ :

$$\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} \bmod \Lambda = \begin{bmatrix} \hat{X}_{u,v} - \sqrt{PP_r}a_1\psi(W_{u,m}) \\ \hat{X}'_{u,v} - \sqrt{PP_r}a_2\psi(W_{u,m}) \end{bmatrix} \quad (12)$$

where  $A = \sqrt{PP_r}\psi(\hat{W}_{v,m})$  and  $B = \sqrt{PP_r} \sum_{\ell=1, \ell \neq m}^L (\psi(\hat{W}_{u,\ell}) + \psi(\hat{W}_{v,\ell}))$  for large lattice dimension. The  $v_m^{th}$  user also extracts the  $u_m^{th}$  user's message.

## IV. SUM RATE

In this section, we investigate the sum rate of a multi-pair TWRN with amplify and compute relaying. The sum rate indicates the maximum throughput of the system. For a multi-pair TWRN, the sum rate can be defined as the sum of the achievable rates of the users in all the user pairs.

First, we obtain the expression of the signal to noise ratio (SNR) at the  $u_m^{th}$  user. From (8), the SNR at the  $u_m^{th}$  user for computing the first linear equation is given by:

$$\gamma_m = \frac{PP_r \min(|a_1|^2, |b_1|^2)}{N'}, \quad (13)$$

$$N' = P \left( |\alpha h_{u_m}^2 - a_1|^2 + |\alpha h_{u_m} h_{v_m} - a_1|^2 + |\alpha \sum_{\ell=1, \ell \neq m} h_{u_m} h_{u_\ell} - b_1|^2 + |\alpha \sum_{\ell=1, \ell \neq m} h_{u_m} h_{v_\ell} - b_1|^2 \right) + \alpha^2 |h_{u_m}|^2 N_0 + N_0. \quad (14)$$

where  $N'$  denotes the variance of the noise terms  $n'$  in (8) and is given by (14) at the top of the next page.

The optimum value of  $\alpha$  can be obtained by setting  $\frac{dN'}{d\alpha} = 0$  as in (15) at the top of the next page.

Now, substituting the optimum value of  $\alpha$  from (15) in to (13), the SNR at the  $u_m^{th}$  user can be expressed as:

$$\gamma_m = \frac{\min(|a_1|^2, |b_1|^2)}{N''}, \quad (16)$$

where  $N''$  is given in (17) at the top of the next page.

The achievable rate at the  $u_m^{th}$  user for computing the first linear equation is given by:

$$R_1 \leq \frac{1}{2} \log \left( \frac{\min(|a_1|^2, |b_1|^2)}{N''} \right), \quad (18)$$

where  $\log(\cdot)$  denotes logarithm to the base two and the density of  $n''$  can be upper-bounded by that of a Gaussian vector whose variance approaches  $N''$  [13]. Similarly, for computing the second linear equation, the achievable rate can be obtained as  $R_2 \leq \frac{1}{2} \log \left( \frac{\min(|a_2|^2, |b_2|^2)}{N''} \right)$ , where  $N''$  is modified by replacing  $a_1$  and  $b_1$  with  $a_2$  and  $b_2$ , respectively.

Thus, the achievable rate at the  $u_m^{th}$  user becomes  $R_{u_m} = \min(R_1, R_2)$ . Similarly, the achievable rate at the  $v_m^{th}$  user can be obtained. Then, adding the achievable rates at all the users and after some algebraic manipulations, we can obtain the sum rate for symmetric traffic as:

$$R_s = \frac{1}{2} \sum_{m=1}^L (R_{u_m} + R_{v_m}). \quad (19)$$

## V. POWER ALLOCATION

In this section, we obtain the power allocation coefficients to optimize the achievable sum rate of the network.

We assume that the average power per signal is  $P_T$  and each user has a message packet of length  $T$ . Thus, the total power for one round of communication for a user pair is  $2TP_T$ . Since, the energy is conserved during this process, we can write

$$2PT + P_r T = 2TP_T. \quad (20)$$

From (19), we can see that the sum rate is maximized when the achievable rate at the user, who has the minimum achievable rate in the system, is maximized. Since, all the users are allocated with the same transmission power, optimizing the minimum achievable rate ensures optimum performance for the overall network. Further, from (18), it can be identified that the achievable rate at the  $u_m^{th}$  user is maximized when SNR in (16) is maximized. Thus, the optimum power allocation problem can be posed as allocating the average power to users and the relay, such that the denominator in (16) is minimized,

that is:

$$\begin{aligned} & \min_{P, P_r} f_m \\ & \text{s.t. } 2P + P_r = 2P_T, \end{aligned} \quad (21)$$

where,  $f_m$  denotes the objective function at the  $u_m^{th}$  user, obtained from (17) and given by

$$f_m = L(|a_1|^2 + |b_1|^2) - \frac{F_1}{PP_r F_2 + P_r |h_{u_m}|^2 N_0} \quad (22)$$

where

$$F_1 = |h_{u_m}| (|h_{u_m}| + |h_{v_m}|) |a_1| + \sum_{\ell=1, \ell \neq m}^L (|h_{u_\ell}| + |h_{v_\ell}|) |b_1| \quad (23)$$

and

$$F_2 = |h_{u_m}|^2 (|h_{u_m}|^2 + |h_{v_m}|^2) + \sum_{\ell=1, \ell \neq m}^L (|h_{u_\ell}|^2 + |h_{v_\ell}|^2) \quad (24)$$

We assume that for the  $m^{th}$  user pair, a fraction  $\beta(m)$  of the total power is allocated to the  $u_m^{th}$  and the  $v_m^{th}$  users. Thus,

$$P = \beta(m) P_T, \quad (25a)$$

$$P_r = (1 - \beta(m)) 2P_T. \quad (25b)$$

Now, substituting (25) in (22), the optimization problem in (21) is solved to obtain  $\beta(m)$  as in the following theorem.

*Theorem 1:* The optimum fraction of the total power allocated to users in the  $m^{th}$  pair is given by:

$$\beta(m) = \frac{F_2 P_T - |h_{u_m}|^2 N_0}{2F_2 P_T} \quad (26)$$

*Proof:* The constraint in (22) can be written in terms of the power allocation coefficient  $\beta(m)$  as:

$$f_m = L(|a_1|^2 + |b_1|^2) - \frac{F_1}{\beta(m)(1 - \beta(m)) P_T^2 F_2 + (1 - \beta(m)) P_T |h_{u_m}|^2 N_0} \quad (27)$$

Now, setting  $\frac{df_m}{d\beta(m)} = 0$  and then performing some algebraic manipulations results into a quadratic equation in terms of  $\beta(m)$ . Solving the quadratic equation gives the optimum power allocation coefficient as in (26). ■

*Remark 1:* From (26), it can be noted that when the users have smaller channel gain (i.e.,  $F_2$  becomes small), less power is allocated to the users and more to the relay and vice versa. Also, it can be noted that for smaller SNR, more power is

$$\alpha = \frac{PP_r|h_{u_m}| \left( (|h_{u_m}| + |h_{v,m}|) |a_1| + \sum_{\ell=1, \ell \neq m}^L (|h_{u_\ell}| + |h_{v_\ell}|) b_1 \right)}{PP_r|h_{u_m}|^2 \left( |h_{u_m}|^2 + |h_{v_m}|^2 + \sum_{\ell=1, \ell \neq m}^L (|h_{u_\ell}|^2 + |h_{v_\ell}|^2) \right) + P_r|h_{u_m}|^2 N_0} \quad (15)$$

$$N'' = L (|a_1|^2 + |b_1|^2) - \frac{|h_{u_m}| \left( (|h_{u_m}| + |h_{v,m}|) |a_1| + \sum_{\ell=1, \ell \neq m}^L (|h_{u_\ell}| + |h_{v_\ell}|) |b_1| \right)}{PP_r|h_{u_m}|^2 \left( |h_{u_m}|^2 + |h_{v_m}|^2 + \sum_{\ell=1, \ell \neq m}^L (|h_{u_\ell}|^2 + |h_{v_\ell}|^2) \right) + P_r|h_{u_m}|^2 N_0}, \quad (17)$$

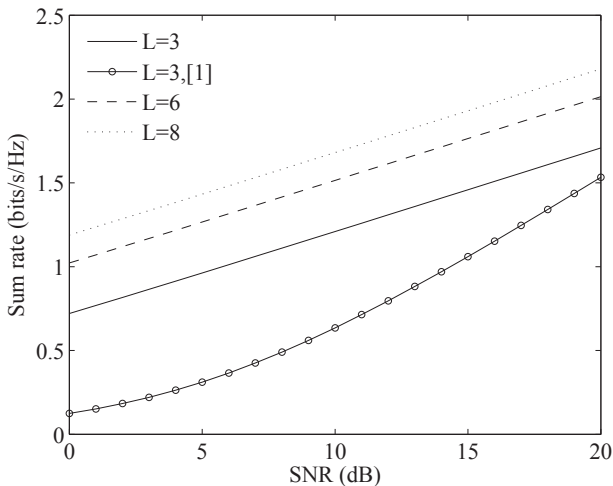


Fig. 2. Achievable sum rate in  $L = 3, 6, 8$ -pair TWRN.

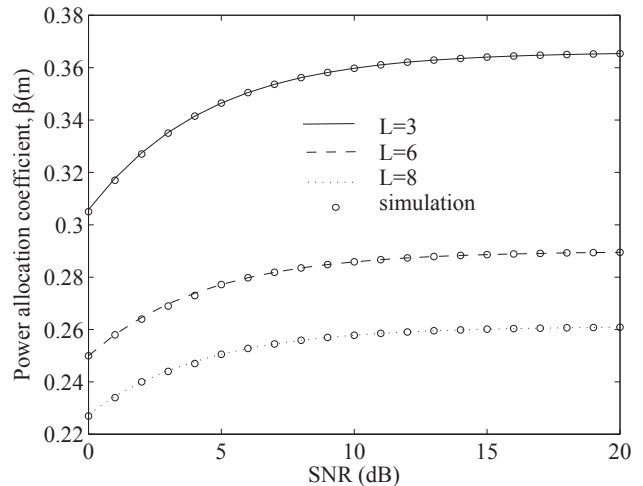


Fig. 3. Optimum power allocation coefficients for multi-pair TWRNs with  $L = 3, 6$  and  $8$  user pairs.

allocated to the relay, because the relay will need higher power to amplify the received signal.

## VI. RESULTS

In this section, we provide numerical simulation results to illustrate the sum rate and optimum power allocation results of a multi-pair TWRN with amplify-and-compute relaying. We perform Monte Carlo simulation to obtain the sum rates over different channel realizations, where each user transmits a data packet of  $T = 1000$  signals. Following [23], the average channel gain for the  $j^{\text{th}}$  users is modeled by  $\sigma_{h_{j,r}}^2 = (1/(d_j/d_0))^\nu$ , where  $d_0$  is the reference distance,  $d_j$  is the distance between the  $j^{\text{th}}$  user and the relay which is assumed to be uniformly randomly distributed between  $0$  and  $d_0$ , and  $\nu$  is the path loss exponent, which is assumed to be  $3$ . The SNR is defined as SNR per bit per user.

*Effect of different no. of user pairs:* Fig. 2 shows the achievable sum rate for multi-pair TWRNs with different number of user pairs and amplify-and-compute relaying. Here, we can see that the slope of the sum rate is unchanged with the number of user pairs. This is in line with the well known fact that the degrees of freedom (DoF) of the multi-pair TWRN with a single antenna at the relay is  $2$  [24].

We also compare the sum rate result for  $L = 3$  pair TWRNs with that provided in eq. (37) in [1] which considers

orthogonal time slots to eliminate inter-pair interference. The proposed approach based on lattice codes and amplify-and-compute relaying outperforms the approach in [1], e.g., for SNR=10 dB and  $L = 3$ , the proposed approach provides a sum rate of 1.21 bits/s/Hz, compared to only 0.64 bits/s/Hz for the scheme in [1]. The results for the scheme in [1] are not shown for  $L = 6$  and  $L = 8$  because they are almost similar to the case  $L = 3$ . This is because, [1] requires orthogonal time slots which means that with increasing number of user pairs, more time slots will be required. However, the proposed approach based on lattice codes and amplify-and-compute relaying needs only two time slots for any number of user pairs. Thus, in this approach, the sum rate increases for more number of user pairs, as expected from (19). Note that the proposed approach has higher computational complexity compared to that in [1] because of the need to solve two simultaneous equations, whereas, in [1], AF relaying is used. However, the tradeoff is smaller number of time slots leading to significant improvement in the sum rate compared to [1].

*Optimum power allocation coefficient for different no. of user pairs:* Fig. 4 shows the optimum power allocation coefficient for different number of user pairs, where the  $m^{\text{th}}$  user pair has the minimum average channel gain. Here the analytical result for the optimum power allocation coefficient  $\beta(m)$  in (26) is verified through numerical search. It can be seen that

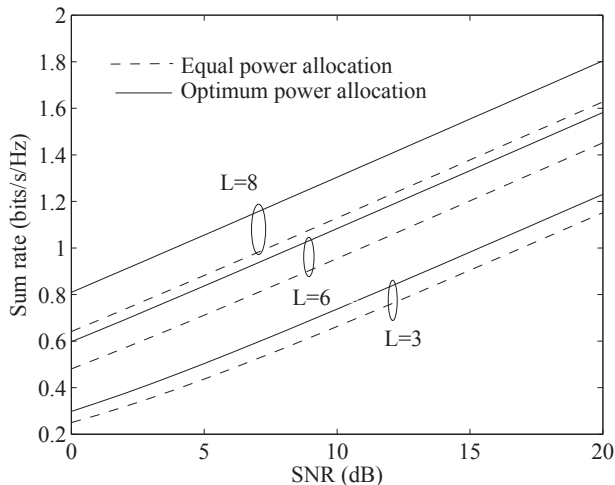


Fig. 4. Improvement in achievable sum rate from optimum power allocation for multi-pair TWRNs with  $L = 3, 6$  and  $8$  user pairs.

$\beta(m)$  increases with SNR for all the user pairs, i.e., more power is allocated to the relay at smaller SNR. This is because, when there is larger noise, the signals from the users degrade more and more power needs to be allocated to the relay for amplification. Moreover, when the number of user pairs increase,  $\beta(m)$  decreases, indicating smaller power allocated to user pairs, as expected from (26).

Effect of optimum power allocation on the sum rate: Fig. 4 shows the impact of optimum power allocation on the achievable sum rate for  $L = 3, L = 6$  and  $L = 8$ . For all the cases, optimum power allocation improves the sum rate which is expected. It can be noted that as the number of pairs increase, optimum power allocation improves the sum rate by a larger degree. That is, optimum power allocation becomes more crucial for sum rate improvement when large number of user pairs exist in the network, e.g., for SNR=10 dB and  $L = 8$ , optimum power allocation results in 18.5% improvement in the sum rate over equal power allocation. This highlights the benefit of optimum power allocation.

## VII. CONCLUSIONS

In this paper, we studied a new approach for interference cancellation in a multi-pair TWRN based on lattice codes and amplify-and-compute relaying. We formulated the achievable sum rate with this approach and showed that it performs better than the existing approach. We obtained the optimum power allocation coefficient analytically and verified using numerical search. We showed that when the users' suffer poor channel conditions, more power needs to be allocated to the relay to maintain the amplification. We observed that when more number of user pairs are involved, optimum power allocation provides greater improvement in the sum rate compared to the case when small number of user pairs are involved.

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